Optimal Power Allocation and User Loading for Multiuser MISO Channels with Regularized Channel Inversion

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***Abstract*—We consider a multiuser system where a single transmitter equipped with multiple antennas (the base station) communicates with multiple users each with a single antenna. Regularized channel inversion is employed as the precoding strategy at the base station. Within this scenario we are interested in the problems of power allocation and user admission control so as to maximize the system throughput, i.e., which users should we communicate with and what power should we use for each of the admitted users so as to get the highest sum rate. This is in general a very difficult problem but we do two things to allow some progress to be made. Firstly we consider the large system regime where the number of antennas at the base station is large along with the number of users. Secondly we cluster the downlink path gains of users into a finite number of groups. By doing this we are able to show that the optimal power allocation under an average transmit power constraint follows the well-known water filling scheme. We also investigate the user admission problem which reduces in the large system regime to optimization of the user loading in the system.**

***Index Terms*—Multiuser precoding, regularized channel inversion, power allocation, large system analysis.**

I. Introduction

MULTI-Input Multi-Output (MIMO) technologies are currently being adopted in many wireless communication standards such as the fourth generation (4G) cellular networks. In multiuser MIMO downlink transmissions or broadcast channels (MIMO-BC), the capacity region was characterized in [2] and is achieved by employing Dirty Paper Coding (DPC) at the transmitter. However, implementing this technique in practice is computationally expensive [3], [4]. Multiuser beamforming techniques such Zero Forcing (ZF) and Regularized Channel Inversion (RCI) are sub-optimal in term of the sum-rate but offer a lower complexity in the implementation. ZF can asymptotically achieve a sum rate that is close to that of DPC by appropriate power allocation and user scheduling [5].

In Multi-Input Single-Output (MISO) broadcast channels, finding the optimal power allocation policy maximizing the sum rate for ZF is a convex optimization problem and its solution follows the water-filling (WF) scheme, see e.g., [5]. In contrast, the optimal power allocation for the RCI precoder is a non-convex optimization problem with many local optima [6]–[8], even in the case of all users having the same path gain. In [6], [7], the authors investigated the sum rate maximization of MIMO broadcast channels with RCI under a total power constraint. They showed that the problem is a global difference of convex functions (d.c.) optimization problem and proposed the local gradient method to solve the problem. Their numerical results suggest that employing an RCI precoder with power allocation gives a better sum rate compared to the ZF. Reference [8] extends the previous works, but in the MISO broadcast channels setting, by putting additional quality of service (QoS) constraints where each user's data rate should be above a specified minimum rate. The authors re-cast the optimization problem as a series of geometric programming (GP) problems, called iterative GP (IGP).

As already mentioned, besides power allocation, selecting the users for transmission can improve the system performance. It has been shown in [5] that a combination of water-filling based power allocation and a user selection scheme, called semi-orthogonal user selection (SUS), in MISO BC systems with ZF precoder can approach the sum rate obtained by employing DPC when the number of users is large. A similar conclusion is also presented in [9], [10] but by using greedy search algorithms for the user selection. The performance analysis of that algorithm for the case of finite (at most two) scheduled users was carried out in [11]. The authors in [12] also proposed a greedy user selection for the RCI precoder: their algorithm is based on the closed form approximation of the expected sum rate. In [13], Dai et al. studied MISO BC systems with ZF precoder under a finite-rate or quantized feedback. The proposed power allocation scheme is binary or on/off. They showed that the feedback rate and the received SNR affect the optimal number of active ('on') users. Moreover, their scheme can be applied in heterogeneous environments where the users may have different path gains. A similar problem is also considered in [14], [15] but with different settings. Besides considering the finite-rate feedback, the authors take into account the feedback delay by using

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a Gauss-Markov model; they also assume a homogeneous environment and equal power allocation across the users. A sum rate approximation expression as a function of the number of users is derived. As a result, the number of users can be adjusted adaptively based on the feedback delay and channel quantization error (or feedback rate). This strategy is similar to the multi-mode transmission scheme considered in this paper.

In this paper, we will be considering power and user loading (also called group loading) allocations, in addition to regularization parameter optimization, in a MISO BC with heterogeneous users. As mentioned earlier, solving the optimal power allocation problem alone is a challenging task [6]–[8]. Adding the user loading allocation and the regularization parameter of the precoder into the optimization problem increases the complexity of the task. To tackle the problem, we therefore apply two simplifying strategies. Firstly, we consider the large system regime where the number of users, K, and the number of transmit antennas of the transmitter or base station (BS), N, tend to infinity with a fixed ratio. In this large system, we show how many of the related problems simplify and key insights can be obtained. Secondly, we divide all users into a finite number of groups or clusters, where all users in each group have approximately the same distance from the BS and therefore share the same distance dependent path gain. A similar grouping approach is also employed in [16].

As a result of applying the first strategy, we are able to show that in the large system regime, each user's SINR tends to a deterministic quantity, called the limiting SINR: limiting SINRs depend only on users' allocated power and path gain and not on the realization of the fast fading coefficients. Following that, under the second modeling strategy, we consider the joint optimization of the users powers and the regularization parameter. For a fixed regularization parameter, we show that the optimal power allocation problem maximizing the limiting sum rate per antenna[1] under an average power constraint is *convex* and the *optimal power allocation follows the familiar water-filling strategy* [5], [17]. By substituting back the power allocation scheme to the sum rate per antenna expression, we can derive the optimal regularization parameter. Even though it does not yield a closed form expression, this substitution leads to a one dimensional optimization problem which can be solved by standard line search algorithms.

It should be noted that the water-filling scheme may allocate zero power to some of the groups. Consequently, one may ask whether it is better to include the channel states of those groups in the precoder or not. This leads to the second part of the paper where we consider a multi-mode transmission scheme (see also [14], [15]). In this scheme, for a given total number of groups (L) and group loading of each group, we determine the optimal number of groups for the transmission and also which groups the BS should communicate with. We arrange or sort the groups based on their path gains in a descending order. We investigate two cases. In the first case, for each group, the BS can only decide between transmitting to all the users in the group or to none of them. We consider a uniform group loading over the groups. In the mode m transmission where the BS only communicates to m groups (out of L), it is optimal for the BS to transmit to the first $m \leq L$ groups. The optimal mode can then be determined by comparing the maximum sum rate per antenna of each mode. In the second case, the BS is allowed to communicate with any subset of the users in a group. We provide a necessary condition for the optimal group-loading allocation for each group. Assuming that $M \leq L$ groups are allocated positive power, the group loadings of the first $M-1$ groups should be set at their maximum value and the group loading for the M-th group can be in between zero and its maximum value. We also propose an algorithm to solve this optimization problem. Considering the group loading allocation, the algorithm offers a lower complexity in comparison to brute force search methods. In both cases, the optimal power allocation strategy and regularization parameter are also considered.

We should also note that employing precoding and power allocation in the downlink require channel state information (CSI) at the BS. The analysis in this paper assumes perfect CSI at the BS even though this is hard to obtain in practical scenarios. The impact of imperfect CSI at the BS with RCI precoder has been investigated in some work, e.g., [18], [19] and [20]. The optimization problems considered in this paper together with imperfect CSI can be a subject for future investigation.

A work closely related to ours is [19] that considers the large system analysis of MISO broadcast channels with RCI (and ZF) under a general channel model and various transmission scenarios. However, our work differs from [19] in several respects. We group together users that have approximately the same distance from the BS while the results in [19] assume all users are either identical or different (represented by the same or different correlation matrices). Moreover, the formulation of the precoding (RCI) matrix considered in [19] and in our paper is not the same. We consider a *joint* optimization of the power allocation, regularization parameter and user/group loading to maximize the sum-rate per antenna whereas in [19], the optimization of those parameters is considered separately. Furthermore, the optimal power allocation and the optimal cell-loading (assuming ZF precoder at the BS) in [19] only hold for the homogeneous[2] setting, i.e. the one-cluster setup in our model.

The rest of the paper is structured as follows. Section II presents the channel model and the SINR expressions for both finite-size and large system regimes. Section III introduces the grouping model and presents the joint optimization of the power allocation and the regularization parameter. In Section IV, multi-mode transmission is introduced and a joint optimization of the power, regularization parameter and group loading is tackled. Section V concludes the paper and some of the technical proofs are placed in the Appendix.

Throughout the paper, the following notations are used. $\mathbb{E}[\cdot]$ denotes the statistical expectation and $\xrightarrow{a.s}$ refers to almost sure convergence. $\frac{\partial f}{\partial x}$ denotes the partial derivative of f with respect to (w.r.t.) x and $\frac{\partial f(x^\star)}{\partial x}$ represents $\frac{\partial f(x)}{\partial x}$ at $x = x^\star$. The circularly symmetric complex Gaussian vector with mean $\boldsymbol{\mu}$

[1]For the rest of the paper, we omit the word 'limiting' in the limiting sum rate per antenna.

[2]Even though for the power allocation problem, each user has different channel estimation accuracy.

and covariance matrix $\mathbf{\Sigma}$ is denoted by $\mathcal{CN}(\boldsymbol{\mu}, \mathbf{\Sigma})$. $|a|$ denotes the magnitude of the complex variable a. $\|\cdot\|$ represents the Euclidean norm. $\succeq$ represents element-wise inequality for the vectors. $\text{Tr}(\cdot)$ denotes the trace of a matrix. $\mathbf{I}_N$ and $\mathbf{0}_N$ denote an $N \times N$ identity matrix and a $1 \times N$ zero entries vector, respectively. $(\cdot)^T$ and $(\cdot)^H$ refer to the transpose and Hermitian transpose, respectively. LHS and RHS refer to the left-hand and right-hand side of an equation, respectively.

II. System Model

A. Finite-size system model

We consider a MISO broadcast channel with an RCI precoder at the transmitter end. The base station has N antennas and serves K users each equipped with a single antenna. The received signal for user k is given by

$$y_k = a_k \mathbf{h}_k \mathbf{x} + w_k \tag{1}$$

where a_k^2 and $\mathbf{h}_k \in \mathbb{C}^{1\times K}$ are the slowly-varying distance-dependent path gain[3] and the fast-fading channel vector between the BS and user k, respectively. It is assumed that the entries of the row vector $\mathbf{h}_k$ are i.i.d. and $\mathbf{h}_k \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_N)$[4].

The transmitted data vector, $\mathbf{x} \in \mathbb{C}^N$, satisfies a power constraint $\mathbb{E}[\|\mathbf{x}\|_2^2] = P_d$ and it can be written as $\mathbf{x} = \mathbf{P}\mathbf{s}$, where $\mathbf{P}$ and $\mathbf{s}$ are the (linear) precoding matrix and the data symbol vector, respectively. We model the latter as $\mathbf{s} = \mathbf{\Lambda}^{1/2}\bar{\mathbf{s}}$, where $\bar{\mathbf{s}}$ is the normalized (power) data symbol vector, i.e., $\mathbb{E}[\bar{\mathbf{s}}\bar{\mathbf{s}}^H] = \mathbf{I}_K$ and $\mathbf{\Lambda} = \text{diag}(p_1, p_2, \cdots, p_K)$ where p_k denotes the power allocated to user k. Let $\mathbf{H} = [\mathbf{h}_1\ \mathbf{h}_2\ \cdots\ \mathbf{h}_K]^T$ be the channel matrix. The RCI precoder matrix, $\mathbf{P}$, takes the form $\mathbf{P} = c\left(\mathbf{H}^H\mathbf{H} + \alpha\mathbf{I}_N\right)^{-1}\mathbf{H}^H$, where α is the regularization parameter that controls the amount of interference introduced to the users and c is the normalizing constant chosen to meet the transmit power constraint $\mathbb{E}[\|\mathbf{x}\|_2^2] = P_d$, that is,

$$c^2 = \frac{P_d}{\text{Tr}\left(\mathbf{\Lambda}\mathbf{H}(\mathbf{H}^H\mathbf{H} + \alpha\mathbf{I}_N)^{-2}\mathbf{H}^H\right)}. \tag{2}$$

The receiver noise of user k, denoted by w_k, has distribution $\mathcal{CN}(0, \sigma^2)$ and is assumed to be independent of the noise of other receivers.

Based on the description above, we can rewrite (1) as follows

$$\begin{aligned} y_k &= c a_k \sqrt{p_k}\mathbf{h}_k(\mathbf{H}^H\mathbf{H} + \alpha\mathbf{I}_N)^{-1}\mathbf{h}_k^H \bar{s}_k \\ &\quad + \sum_{j\neq k}^{K} c a_k \sqrt{p_j}\mathbf{h}_k(\mathbf{H}^H\mathbf{H} + \alpha\mathbf{I}_N)^{-1}\mathbf{h}_j^H \bar{s}_j + w_k \end{aligned}$$

and the SINR attained by user k can be expressed as

$$\text{SINR}_k = \frac{c^2 a_k^2 p_k |\mathbf{h}_k(\mathbf{H}^H\mathbf{H} + \alpha\mathbf{I}_N)^{-1}\mathbf{h}_k^H|^2}{\sum_{j\neq k}^{K} c^2 a_k^2 p_j |\mathbf{h}_k(\mathbf{H}^H\mathbf{H} + \alpha\mathbf{I}_N)^{-1}\mathbf{h}_j^H|^2 + \sigma^2}. \tag{3}$$

[3] Note that a_k^2 captures the effect of geometric attenuation, which is distance based, between the BS and user k. In some papers, e.g., [13], [16], it is called the *path-loss coefficient*. We should also note that the actual channel vector for user k is represented by $a_k\mathbf{h}_k$.

[4] Even though here, we assume a specific distribution for $\mathbf{h}$, the large system analysis holds for any distribution of $\mathbf{h}_k$ if the entries of $\frac{1}{\sqrt{N}}\mathbf{h}_k$ are i.i.d. with zero mean, variance $\frac{1}{N}$ and have finite eighth moment (see e.g. [21]).

It is clear that the SINR_k is a random quantity since it depends on the propagation channels that fluctuate randomly. In the large system limit, as we will see in the next section, this randomness disappears.

B. Large-system regime SINR

The following theorem provides the convergence of the SINR_k (3) when the system dimensions, that is, K and N, grow large with their ratio fixed.

Theorem 1. *Let $\rho = \frac{\alpha}{N}$ be the normalized regularization parameter and $g(\beta,\rho)$ be the solution of $g(\beta,\rho) = \left(\rho + \frac{\beta}{1+g(\beta,\rho)}\right)^{-1}$. Let $\mathcal{P} = \lim_{K\to\infty}\frac{1}{K}\sum_{k=1}^{K} p_k$. Suppose that the limit $\mathcal{P}$ exists and is bounded. Then, as $K, N \to \infty$ with $\frac{K}{N} \to \beta$, $SINR_k$ (3) converges almost surely to a deterministic quantity, $SINR_k^\infty$, given by*

$$SINR_k^\infty = \bar{p}_k g(\beta,\rho)\frac{\gamma_k + \frac{\gamma_k\rho}{\beta}(1+g(\beta,\rho))^2}{\gamma_k + (1+g(\beta,\rho))^2}, \tag{4}$$

where $\gamma_k = \frac{P_d a_k^2}{\sigma^2}$ is defined as the effective SNR and $\bar{p}_k = \frac{p_k}{\mathcal{P}}$ is the normalized power w.r.t. $\mathcal{P}$.

Proof: Refer to Appendix A. ∎

We call SINR_k^∞ the limiting SINR of user k. Note that it is different for each user and depends on a_k and p_k. Let $f_k(\beta,\rho)$ be the RHS of (4) excluding $\bar{p}_k$. Then, we can write (4) as

$$\text{SINR}_k^\infty = \bar{p}_k f_k(\beta,\rho). \tag{5}$$

Note that f_k also depends on a_k and P_d/σ^2 via γ_k but these are assumed to be fixed throughout this paper. It is also obvious that f_k is independent of $\bar{p}_k$. This property will ease the analysis in finding the power allocation that maximizes the sum rate per antenna in the next section.

III. Optimal Power Allocation and Regularization Parameter

Let us consider the following scenario. We divide all K users into L groups, where L is finite. All users in each group are assumed to have the same path gain. For the rest of the paper, we assume that $a_1 \geq a_2 \geq \cdots \geq a_L$. The number of users in group j is denoted by K_j, with $\sum_{j=1}^{L} K_j = K$. We also assume that K_j and N tend to be large with a fixed ratio $\beta_j = \frac{K_j}{N}$. It represents the user or group loading of group j. Since the path gain and other parameters β, ρ as well as SNR are the same for all users in a group, then based on (4), we can assume that the power allocated to each user in that group is also the same. This assumption holds for the rest of the paper.

Based on the above scenario, we can define the achievable sum rate per antenna as follows

$$R_{\text{sum}}^\infty = \sum_{j=1}^{L} \beta_j \log\left(1 + \text{SINR}_j^\infty\right). \tag{6}$$

Our goal in this section is to find the optimal power allocation that maximizes R_{sum}^∞. Moreover, it is also interesting to explore how the regularization parameter of the RCI precoder adapts to different path gains and also user powers. A joint optimization problem can be formulated as follows,

$$\mathbf{P1}: \quad \max_{\bar{\mathbf{p}}\succeq \mathbf{0},\rho\geq 0} \quad R_{\text{sum}}^{\infty}$$
$$\text{s.t.} \quad \sum_{j=1}^{L}\frac{\beta_j}{\beta}\bar{p}_j = \frac{1}{\beta}\boldsymbol{\beta}^T\bar{\mathbf{p}} \leq 1. \tag{7}$$

In the above, we use lowercase bold letters to denote column vectors with size L, e.g., $\bar{\mathbf{p}} = [\bar{p}_1, \bar{p}_2, \ldots, \bar{p}_L]^T$. This notation will be used for the rest of this paper, unless otherwise stated. Note that the constraint (7) can be considered as the large system average power constraint. $\mathbf{P1}$ also requires $\bar{\mathbf{p}}$ and ρ to be non-negative.

Before addressing the solution of $\mathbf{P1}$, we characterize the objective function as a function of $\bar{p}_j$. Let $R_{\text{sum},j}^{\infty} = \beta_j \log\left(1+\text{SINR}_j^{\infty}\right)$ denote the sum rate per antenna for group j. It can be checked that it is an increasing function in p_j. Moreover, we can show that the following lemma holds.

Lemma 1. *The sum rate per antenna R_{sum}^{∞} is concave in $\bar{\mathbf{p}}$.*

Proof: The second derivative of the limiting SINR w.r.t. $\bar{p}_j$ is

$$\frac{\partial^2\text{SINR}_j^{\infty}}{\partial \bar{p}_j^2} = -\frac{f_j^2(\beta,\rho)}{(1+\bar{p}_j f_j(\beta,\rho))^2} < 0.$$

This implies that SINR_j^{∞} is concave in $\bar{p}_j$. Since the $\log$ operation does not change the concavity, therefore $R_{\text{sum},j}^{\infty}$ is also concave in $\bar{p}_j$. Moreover, R_{sum}^{∞} is a linear combination of $R_{\text{sum},j}^{\infty}$ and this operation preserves the concavity. ∎

From the lemma above, we can see that for a fixed ρ, $\mathbf{P1}$ is a convex program because $-R_{\text{sum}}^{\infty}$ is convex in $\bar{\mathbf{p}}$ and the constraints are linear. For a fixed $\bar{\mathbf{p}}$, SINR_k^{∞} is not concave in ρ but quasi-concave [22]. Since $\log$ is a non-decreasing function then $R_{\text{sum},j}^{\infty}$ is also quasi-concave (not concave) in ρ. Since a linear combination operation does not necessarily preserve the quasi-concavity, the sum rate per antenna need not be quasi-concave.

Now, let us consider the Lagrangian for $\mathbf{P1}$, as stated below[5]

$$\mathcal{L} = \sum_{j=1}^{L}\beta_j\log(1+\bar{p}_j f_j(\beta,\rho)) - \lambda\left(\frac{1}{\beta}\boldsymbol{\beta}^T\bar{\mathbf{p}}-1\right) + \boldsymbol{\xi}^T\bar{\mathbf{p}} + \kappa\rho,$$

where λ and $\boldsymbol{\xi}$ are the Lagrange multipliers for the average power and non-negative power constraints respectively, and κ is the Lagrange multiplier for the constraint $\rho \geq 0$. Let $\bar{\mathbf{p}}^{\star}, \rho^{\star}$ be the solutions for $\mathbf{P1}$. At these points, the associated Karush-Kuhn-Tucker (KKT) optimality conditions are

$$\frac{\partial\mathcal{L}}{\partial\bar{p}_j} = \beta_j\left(\frac{f_j(\beta,\rho^{\star})}{1+\bar{p}_j^{\star}f_j(\beta,\rho^{\star})} - \lambda\right) + \xi_j = 0 \tag{8}$$

$$\frac{\partial\mathcal{L}}{\partial\rho} = \sum_{j=1}^{L}\frac{\beta_j\bar{p}_j^{\star}}{1+\bar{p}_j^{\star}f_j(\beta,\rho)}\frac{\partial f_j(\beta,\rho^{\star})}{\partial\rho} + \kappa = 0, \tag{9}$$

and

$$\frac{1}{\beta}\boldsymbol{\beta}^T\bar{\mathbf{p}}^{\star} \leq 1, \lambda\left(\frac{1}{\beta}\boldsymbol{\beta}^T\bar{\mathbf{p}}^{\star}-1\right) = 0, \lambda \geq 0, \tag{10}$$

$$\bar{\mathbf{p}}^{\star} \succeq 0, \xi_j\bar{p}_j^{\star} = 0, \xi_j \geq 0, \ j = 1,\ldots,L, \tag{11}$$

$$\rho^{\star} \geq 0, \kappa\rho^{\star} = 0, \kappa \geq 0. \tag{12}$$

Recall that for a given ρ, $\mathbf{P1}$ reduces to a convex program. In this case, it is easy to show that the KKT conditions (8), (10) and (11) lead to the optimal power allocation strategy maximizing the sum rate per antenna, as presented in the following theorem.

Theorem 2. *For a fixed ρ, the optimal power allocation for the optimization problem $\mathbf{P1}$ follows the water-filling (WF) scheme and is given by*

$$\bar{p}_j^{\star} = \left[\frac{1}{\lambda} - \frac{1}{f_j(\beta,\rho)}\right]_+ \tag{13}$$

where $[x]_+ = \max(0,x)$. The constant (Lagrange multiplier) λ is the solution of

$$\sum_{j=1}^{L}\beta_j\bar{p}_j^{\star} = \beta,$$

for which the average power constraint is satisfied with equality.

In the WF scheme above, $1/\lambda$ can be perceived as the water level. It determines how power is poured to each user and is based on the value of $f_j(\beta,\rho)$. Recall that the limiting SINR is given by $\bar{p}_j^{\star}f_j(\beta,\rho)$. It can be checked that $f_j(\beta,\rho)$ is increasing in a_j. Thus, more power will be allocated for the users with better channels which can be represented by the path gains $\{a_j\}$. Note that in this case, fairness amongst users could be an issue since some users might have zero rate.

Remark 1. To find λ we can follow the following steps (see also [23]). Since we assume $a_1 \geq a_2 \geq \cdots \geq a_L$, then $\bar{p}_1^{\star} \geq \bar{p}_2^{\star} \geq \cdots \geq \bar{p}_L^{\star}$. Now let us assume that the first m groups have non-zero power. To determine λ, we just need to solve $\sum_{j=1}^{m}\beta_j\bar{p}_j^{\star} = \beta$. Using $\bar{p}_j^{\star}$ in (13), it is easy to show that

$$\lambda = \frac{\sum_{j=1}^{m}\beta_j}{\beta + \sum_{j=1}^{m}\frac{\beta_j}{f_j(\beta,\rho)}}.$$

The power allocated to group j is then given by

$$\bar{p}_j^{\star} = \frac{\beta + \sum_{j=1}^{m}\frac{\beta_j}{f_j(\beta,\rho)}}{\sum_{j=1}^{m}\beta_j} - \frac{1}{f_j(\beta,\rho)}$$

To determine m, we just need to find m such that $\bar{p}_m^{\star} > 0$ and $\bar{p}_{m+1}^{\star} \leq 0$. ∎

By using the KKT optimality conditions above, the optimal $\rho^{\star}$ can be found as stated in the following theorem.

Theorem 3. *Let $\bar{\mathbf{p}}^{\star}$ be as in* (13)*. The maximum sum rate per antenna, R_{sum}^{∞}, is obtained by choosing $\rho^{\star}$ that satisfies*

$$\sum_{j=1}^{L}\frac{\beta_j\bar{p}_j^{\star}f_j^2(\beta,\rho^{\star})}{1+\bar{p}_j^{\star}f_j(\beta,\rho^{\star})}\left(\frac{\rho^{\star}}{\beta} - \frac{1}{\gamma_j}\right) = 0. \tag{14}$$

[5]For notational simplicity, we use $\mathcal{L}$ to denote the Lagrangian $\mathcal{L}(\mathbf{x},\boldsymbol{\lambda})$, where $\mathbf{x}$ and $\boldsymbol{\lambda}$ are the optimizing variables and the Lagrange multipliers, respectively.

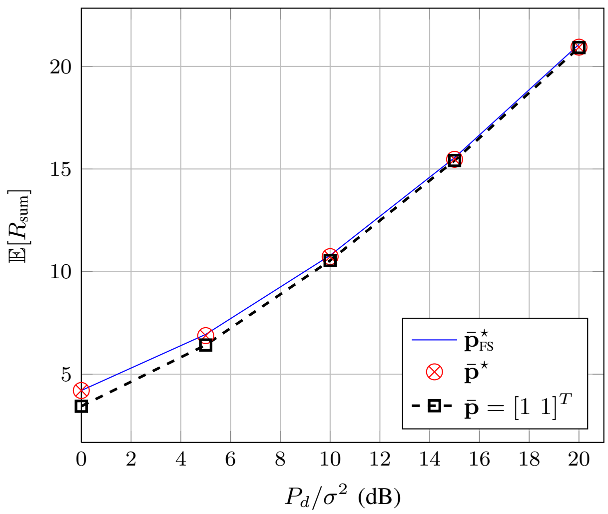

Fig. 1. Comparison of the average sum rate between using $\bar{\mathbf{p}} = \bar{\mathbf{p}}^\star_{\text{FS}}$, $\bar{\mathbf{p}} = \bar{\mathbf{p}}^\star$ and $\bar{\mathbf{p}} = [1\ 1]^T$ for $L = 2$, $\beta = 1$, $N = 8$, $\beta_j = 1/2$, $\rho = \rho^\star$ and $a_j^2 = 1/j^2$.

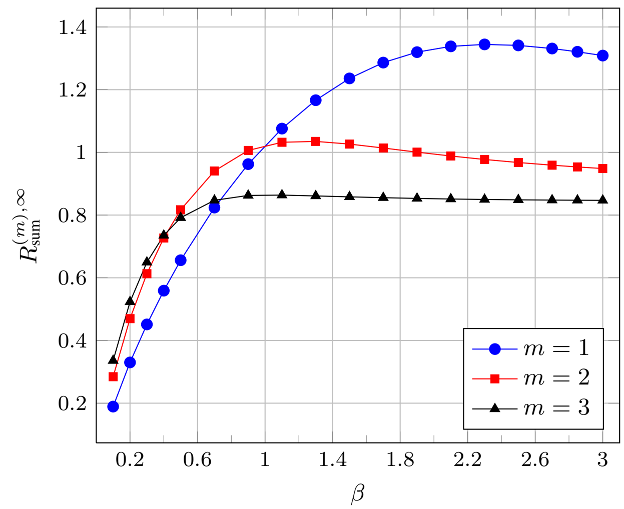

Fig. 2. Multimode Transmission for $L = 3$, $\beta_j = \beta/L$ and $a_j^2 = 1/j^2$.

and it is bounded by

$$\frac{\beta}{\gamma_1} \leq \rho^\star \leq \frac{\beta}{\gamma_L}. \tag{15}$$

Proof: See Appendix B. ■

Note that by using (13) in (14), it is straightforward to see that (14) becomes a one-dimensional zero/root-finding problem. Thus, the optimal ρ can be found by using existing line search algorithms for the interval given in (15).

Figure 1 illustrates the validity of using the large system results for the finite size system. We generate 500 channel realizations and for each realization we compute the optimal power allocation, denoted by $\bar{\mathbf{p}}^*_{\text{FS}}$, by a grid search. In the plot, we compare the average sum rate, denoted by $\mathbb{E}[R_{\text{sum}}]$, between using the power allocation $\bar{\mathbf{p}}$ in (13) and $\bar{\mathbf{p}}^*_{\text{FS}}$. The gap between the curves in the figure is very small and can be said negligible. As a comparison, we also plot the average sum-rate obtained by using an equal power allocation, that is, $\bar{\mathbf{p}} = [1\ 1]^T$ and the corresponding optimal regularization parameter. The figure shows that the optimal power allocation ($\bar{\mathbf{p}}^\star$) gives higher average sum-rates for the SNR values between 0-10 dB. As the SNR increases, we observe for the current setup that $\bar{\mathbf{p}}^\star \approx [1\ 1]^T$. Hence, for SNR > 10 dB, the optimal and equal power allocation strategies result in almost the same average sum rates.

IV. Multimode Broadcast Channels

In the previous section, we assumed that the base station communicates simultaneously with all users in all groups or clusters. This meant that the channel vector for every user was present in the precoding matrix $\mathbf{P}$. In this section we drop that assumption and seek to optimize over the number of users (or groups) to which the base station communicates, along with their powers. As an illustration, let us consider the case of $L = 3$. We set the group loading for each group to be uniform i.e., $\beta_j = \beta/L$. Figure 2 shows the sum rate per antenna obtained when the BS communicates to only the first $m \leq L$ groups, denoted by $R_{\text{sum}}^{(m),\infty}$. This means that we only include the channel of the users from these m groups in the system model and in designing the precoder. We call this scheme mode-m transmission. The figure demonstrates that for some values of cell-loading β, the maximum sum rate per antenna is achieved when $m < L$. The simulation also shows that the optimal m changes with β: we call this scheme *multi-mode transmission*.

In multimode transmission, it is clear that there are $\binom{L}{m}$ combinations of the groups that can be chosen by the base station to communicate with. The question is: which mode and group combination will give the highest sum rate per antenna? Intuition would suggest that if the base station is communicating with m groups then we would choose the m groups with the strongest channel gains in order to maximize the sum rate per antenna. We would then need to test at most L mode and group combinations. Below we show that this intuition is indeed correct for different assumptions on β_j although the proof is non-trivial.

In what follows, β_j refers to the loading of group j users that actually get served, whereas $\beta_{j,\max}$ is the total loading of group j users, i.e., the former is a quantity to be determined by the transmitter, whereas the latter is determined by the geometry and user distribution of the system. Obviously, $0 \leq \beta_j \leq \beta_{j,\max}$.

A. Binary Group Loading

Before considering the more general setup, we investigate the following optimization problem:

$$\begin{aligned} \mathbf{P2}: \quad & \underset{\bar{\mathbf{p}}\succeq\mathbf{0},\boldsymbol{\beta},\rho,\beta\geq 0}{\text{max.}} && R_{\text{sum}}^{\infty} \\ & \text{s.t.} && \frac{1}{\beta}\boldsymbol{\beta}^T\bar{\mathbf{p}} \leq 1 \\ &&& \frac{1}{\beta}\boldsymbol{\beta}^T\mathbf{1} = 1 \\ &&& \beta_j \in \{0, \beta_{j,\max}\}, \end{aligned}$$

where $\mathbf{1}$ is a column vector with all 1 entries. It can be seen that $\mathbf{P2}$ is similar to $\mathbf{P1}$, but with additional design variables: $\boldsymbol{\beta}, \beta$ and additional constraints related to them. In $\mathbf{P2}$, β_j is only allowed to have value either 0 or $\beta_{j,\max}$ and we call this scheme *binary user loading allocation*. Therefore, β_j will determine whether the BS transmits to users in group j or not. The latter occurs when $\beta_j = 0$. In that case, the channel gain matrix of the users in group j is not included in the precoder design.

First, let us investigate the optimal strategies for $\mathbf{P2}$ when $\beta_{j,\max}$ is the same for all groups, i.e., $\beta_{j,\max} = \breve{\beta}$. Let us consider the mode-m transmission. In that case, we have m groups with $\beta_j = \breve{\beta}$ and the remaining groups have $\beta_j = 0$. Let $\mathcal{G} \subset \{1, 2, \ldots, L\}$, $|\mathcal{G}| = m$ be the set of the group indexes that the BS communicates to ($\beta_j > 0, j \in \mathcal{G}$). Then, the maximum sum rate per antenna achieved for a given $\mathcal{G}$ can be obtained by solving

$$\begin{aligned} \max_{\bar{\mathbf{p}} \succeq \mathbf{0}, \rho}. \quad & R_{\text{sum}}^{(m),\infty}(\mathcal{G}) = \sum_{j \in \mathcal{G}} \beta_j \log(1 + \bar{p}_j f_j(\beta, \rho)) \\ \text{s.t.} \quad & \frac{1}{m} \sum_{j \in \mathcal{G}} \bar{p}_j \leq 1. \end{aligned} \tag{16}$$

We should note that in the average power constraint we use the fact that the total group loading β is $\sum_{j \in \mathcal{G}} \beta_j = m\breve{\beta}$. We can also see that (16) is equivalent to $\mathbf{P1}$. Thus, its solutions can be obtained by using the same strategies as in solving $\mathbf{P1}$. The maximum sum rate per antenna for mode-m transmission can be attained by evaluating (16) for every possible choice of group combinations $\mathcal{G}$, i.e.,

$$\breve{R}_{\text{sum}}^{(m),\infty} = \max_{\mathcal{G} \subset \{1,\ldots,L\}, |\mathcal{G}| = m}. R_{\text{sum}}^{(m),\infty}(\mathcal{G}). \tag{17}$$

By using the formulation (17), we can rewrite $\mathbf{P2}$ as

$$\mathbf{P2} : \max_{m \leq L}. \breve{R}_{\text{sum}}^{(m),\infty} \tag{18}$$

As mentioned earlier, for (17) there are $\binom{L}{m}$ possible choices or candidates for the optimal $\mathcal{G}$. For the problem (18), the number of candidates becomes $\sum_{i=1}^{L} \binom{L}{i} = 2^L$. In the following lemma, we show that (17) has the intuitively obvious solution mentioned above: the BS transmits to the m groups that have the largest path gains. This will reduce the number of candidate mode/group combinations for (18) to L.

Lemma 2. *$\breve{R}_{sum}^{(m),\infty}$ is achieved by choosing $\mathcal{G} = \mathcal{G}^\star$ where $\mathcal{G}^\star = \{1, 2, \ldots, m\}$.*

Proof: Let $\mathcal{G}^\star = \{1, 2, \ldots, m\}$. Also, let $\mathcal{S} \subset \{1, \ldots, L\}$ with $|\mathcal{S}| = m$ such that $\mathcal{G}^\star \neq \mathcal{S}$. Moreover, the elements of $\mathcal{S}$ are arranged in an increasing order. Let $\mathbf{a}_{\mathcal{G}^\star}$ and $\mathbf{a}_{\mathcal{S}}$ be the path gain vector for group combinations $\mathcal{G}^\star$ and $\mathcal{S}$, respectively. It is clear that $\mathbf{a}_{\mathcal{G}^\star} \succeq \mathbf{a}_{\mathcal{S}}$. Thus, for a fixed power and regularization parameter, it follows that $R_{\text{sum}}^{(m),\infty}(\mathcal{G}^\star) \geq R_{\text{sum}}^{(m),\infty}(\mathcal{S})$. Now, suppose that $\bar{\mathbf{p}}_{\mathcal{S}}^\star$ and $\rho_{\mathcal{S}}^\star$ are the optimal power allocation and regularization parameter under $\mathcal{S}$. Let us denote the corresponding sum rate per antenna as $R_{\text{sum}}^{(m),\infty}(\mathcal{S}, \rho_{\mathcal{S}}^\star, \bar{\mathbf{p}}_{\mathcal{S}}^\star)$. Under $\mathcal{G}^\star$, let us choose $\bar{\mathbf{p}}_{\mathcal{G}^\star} = \bar{\mathbf{p}}_{\mathcal{S}}^\star$ and $\rho_{\mathcal{G}^\star} = \rho_{\mathcal{S}}^\star$ for the power allocation and ρ, respectively. Even though those choices are not optimal in maximizing $R_{\text{sum}}^{(m),\infty}(\mathcal{G}^\star)$, they satisfy the constraint in (16). Since both $\mathcal{G}^\star$ and $\mathcal{S}$ have the same allocations for power and ρ, then it follows that $R_{\text{sum}}^{(m),\infty}(\mathcal{G}^\star, \rho_{\mathcal{S}}^\star, \bar{\mathbf{p}}_{\mathcal{S}}^\star) \geq R_{\text{sum}}^{(m),\infty}(\mathcal{S}, \rho_{\mathcal{S}}^\star, \bar{\mathbf{p}}_{\mathcal{S}}^\star)$. This concludes the proof. ■

It is clear from the lemma above that we greatly reduce the complexity of $\mathbf{P2}$. Now, we only need to compare L sum rates per antenna, $\breve{R}_{\text{sum}}^{(m),\infty}$. It is also easy to see that Lemma 2 also holds when $\beta_{1,\max} \geq \beta_{2,\max} \geq \cdots \geq \beta_{L,max}$. For a more general setup, we can relax the last constraint of $\mathbf{P2}$ so that $0 \leq \beta_j \leq \beta_{j,\max}$. This will be addressed in the following section.

B. Fractional Group Loading

In this section, we consider a fractional group loading scheme where β_j can take values in $[0, \beta_{j,\max}]$. This allows the BS to transmit not to all the users in the groups but some of them. In this case, $\mathbf{P2}$ becomes

$$\begin{aligned} \mathbf{P3}: \quad \max_{\bar{\mathbf{p}} \succeq \mathbf{0}, \boldsymbol{\beta}, \rho, \beta \geq 0}. \quad & R_{\text{sum}}^{\infty} \\ \text{s.t.} \quad & \frac{1}{\beta} \boldsymbol{\beta}^T \bar{\mathbf{p}} \leq 1 \\ & \frac{1}{\beta} \boldsymbol{\beta}^T \mathbf{1} = 1 \\ & \mathbf{0} \preceq \boldsymbol{\beta} \preceq \boldsymbol{\beta}_{\max}. \end{aligned}$$

To find the solution for $\mathbf{P3}$, we start by writing the Lagrangian of $\mathbf{P3}$ as follows:

$$\begin{aligned} \mathcal{L} = & \sum_{j=1}^{L} \beta_j \log(1 + \bar{p}_j f_j(\beta, \rho)) - \lambda \left(\frac{1}{\beta} \boldsymbol{\beta}^T \bar{\mathbf{p}} - 1 \right) + \boldsymbol{\xi}^T \bar{\mathbf{p}} \\ & + \mu \left(\frac{1}{\beta} \boldsymbol{\beta}^T \mathbf{1} - 1 \right) + \boldsymbol{\nu}^T \boldsymbol{\beta} - \boldsymbol{\eta}^T (\boldsymbol{\beta} - \boldsymbol{\beta}_{\max}) + \kappa\rho + \eta\beta, \end{aligned}$$

where $\lambda, \kappa, \mu, \eta, \boldsymbol{\xi}, \boldsymbol{\nu}, \boldsymbol{\eta}$ are the Lagrange multipliers for the constraints of $\mathbf{P3}$. Let $\bar{\mathbf{p}}^\star, \boldsymbol{\beta}^\star, \rho^\star, \beta^\star$ be the (candidate) solutions for $\mathbf{P3}$. The KKT necessary optimality conditions are

$$\frac{\partial \mathcal{L}}{\partial \rho} = \sum_{j=1}^{L} \frac{\beta_j^\star \bar{p}_j^\star}{1 + \bar{p}_j^\star f_j(\beta^\star, \rho^\star)} \frac{\partial f_j(\beta^\star, \rho^\star)}{\partial \rho} + \kappa = 0 \tag{19}$$

$$\frac{\partial \mathcal{L}}{\partial \bar{p}_j} = \beta_j^\star \left(\frac{f_j(\beta^\star, \rho^\star)}{1 + \bar{p}_j^\star f_j(\beta^\star, \rho^\star)} - \lambda \right) + \xi_j = 0 \tag{20}$$

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \beta_j} = & \log(1 + \bar{p}_j^\star f_j(\beta^\star, \rho^\star)) - \lambda(\bar{p}_j^\star - 1) \\ & + \nu_j - \eta_j + \mu = 0 \end{aligned} \tag{21}$$

$$\frac{\partial \mathcal{L}}{\partial \beta} = \sum_{j=1}^{L} \frac{\beta_j^\star \bar{p}_j^\star}{1 + \bar{p}_j^\star f_j(\beta^\star, \rho^\star)} \frac{\partial f_j(\beta^\star, \rho^\star)}{\partial \beta} - \mu + \eta = 0 \tag{22}$$

with primary constraints:

$$\begin{aligned} & \frac{1}{\beta^\star} \boldsymbol{\beta}^{\star T} \bar{\mathbf{p}}^\star - 1 \leq 0, \frac{1}{\beta^\star} \boldsymbol{\beta}^{\star T} \mathbf{1} - 1 = 0, \\ & \mathbf{0} \preceq \boldsymbol{\beta}^\star \preceq \boldsymbol{\beta}_{\max}, \bar{\mathbf{p}}^\star \succeq \mathbf{0}, \beta^\star \geq 0, \rho^\star \geq 0, \end{aligned}$$

dual constraints:

$$[\lambda \ \kappa \ \eta]^T \succeq 0, \boldsymbol{\xi} \succeq 0, \boldsymbol{\nu} \succeq 0, \boldsymbol{\eta} \succeq 0,$$

and slackness:

$$\begin{aligned} & \lambda \left(\frac{1}{\beta^\star} \boldsymbol{\beta}^{\star T} \bar{\mathbf{p}}^\star - 1 \right) = 0, \eta_j(\beta_j^\star - \beta_{j,\max}) = 0, \\ & \xi_j \bar{p}_j^\star = 0, \nu_j \beta_j^\star = 0, \eta\beta^\star = 0, \kappa\rho^\star = 0, \end{aligned}$$

for all $j = 1, \ldots, L$.

Let us consider the stationarity condition (19). In solving $\mathbf{P1}$, we have shown that $f_j(\beta, \rho)$ is increasing in ρ up to

$\rho = \beta/\gamma_j$ and then decreasing. Thus, the optimal ρ can not be zero ($\kappa = 0$) and at the optimum,

$$\sum_{j=1}^{L} \frac{\beta_j^\star \bar{p}_j^\star}{1+\bar{p}_j^\star f_j(\beta^\star,\rho^\star)} \frac{\partial f_j(\beta^\star,\rho^\star)}{\partial \rho} = 0. \tag{23}$$

Looking at (20), one can see that when $\bar{p}_j > 0$ ($\xi_j = 0$), it satisfies

$$\bar{p}_j^\star = \left[\frac{1}{\lambda} - \frac{1}{f_j(\beta^\star,\rho^\star)}\right]_+$$

which has a similar form to the solution for $\mathbf{P1}$. Since $a_1 \geq \ldots \geq a_L$, then $\bar{p}_1^\star \geq \cdots \geq \bar{p}_L^\star$. At the optimum, the following holds

$$\sum_{j=1}^{L} \beta_j^\star \left(\left[\frac{1}{\lambda} - \frac{1}{f_j(\beta^\star,\rho^\star)}\right]_+ - 1\right) = 0$$

and it can be used to determine λ.

Exploring the stationary condition (21) will lead us to the following result.

Lemma 3. *The optimal $\{\beta_j\}$ allocation is such that*

(i) *the first M groups, for some $M \leq L$, will be allocated non-zero power,*
(ii) *$\beta_1^\star, \beta_2^\star, \ldots, \beta_{M-1}^\star$ are all at the maximum possible values,*
(iii) $0 \leq \beta_M^\star \leq \beta_{M,\max}$,
(iv) *the remaining groups are allocated zero power.*

Proof: See Appendix C. ■

We should note that in the lemma above, we do not know the optimal value of M maximizing the sum rate per antenna since there are several values of M that satisfy the lemma. Let $R_{\text{sum}}^{(i),\infty}$ be the achieved sum rate per antenna with $M = i$. Let $\mathcal{M} = \{1, 2, \ldots, L\}$ be the set of possible values for M. Then, the optimal M is given by

$$M^\star = \arg\max_{i \in \mathcal{M}} R_{\text{sum}}^{(i),\infty}. \tag{24}$$

We should note that in evaluating $R_{\text{sum}}^{(i),\infty}$, we use $\{\beta_j^\star\}$ allocation scheme in Lemma 3, $\beta^\star = \sum_{j=1}^{i} \beta_j^\star$ and also the stationary conditions in (19) and (20) to determine $\rho^\star$ and $\bar{\mathbf{p}}^\star$ respectively. The value for $\beta_M^\star$ must satisfy (21) with $\nu_M = 0$ and $\eta_M = 0$, i.e.,

$$\log(1 + \bar{p}_M^\star f_M(\beta^\star,\rho^\star)) - \lambda(\bar{p}_M^\star - 1) + \mu = 0, \tag{25}$$

where μ is given by (40). Thus, solving (24) correspondingly solves $\mathbf{P3}$. The steps in solving it are presented in Algorithm 1.

We have L iterations where in a particular iteration, say iteration j, the first j groups are considered. Assuming those groups each have their group loading at its maximum value, the corresponding optimal power allocation (i.e., solving $\mathbf{P1}$) is computed. Then, the value of $M \leq j$ for that iteration can be determined by using the fact that $\bar{p}_{M+1}^\star = 0$. We should note that different js may give the same M and hence, we need only to consider one of them. After obtaining M, we can set $\beta_{M+1}^\star = \cdots = \beta_j^\star = 0$. To determine the optimal value for $\beta_M^\star$, we need to compute η_M. If $\eta_M > 0$, $\beta_M^\star = \beta_{M,\max}$ (we already set this in the first step). Otherwise, $0 \leq \beta_M^\star \leq \beta_{M,\max}$. In the latter case, we need to solve $\mathbf{P1}$ and (25) simultaneously. Then, we can update the value for $\{\beta_j^\star\}_{j=1}^M$ and $\beta^\star$, and also compute the corresponding sum rate per antenna. In the final steps, we compare the sum rates (per antenna) for different M and the maximum is the solution of $\mathbf{P3}$.

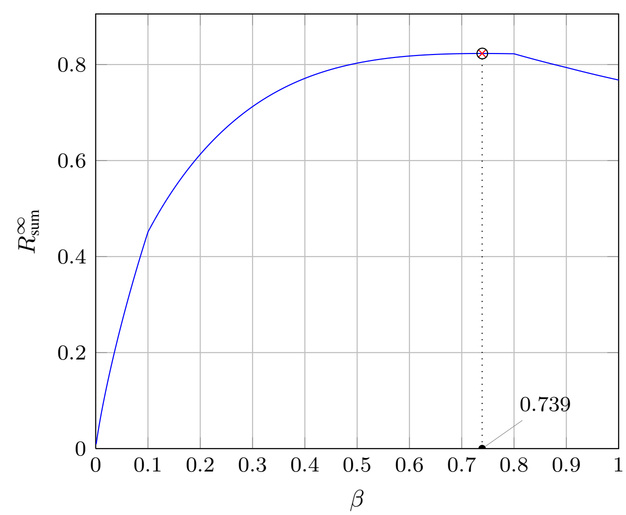

Fig. 4. The maximum sum rate per antenna obtained from the grid search (×) and Algorithm 1 (○).

Figure 3 illustrates the implementation results of algorithm 1 for the case: $L = 5$, $a_j^2 = 1/j^2$, $j = 1, \ldots, L$, $\boldsymbol{\beta}_{\max} = [0.1\ 0.7\ 0.1\ 0.05\ 0.05]^T$ where the j-th element corresponds to $\beta_{j,\max}$ and $P_d/\sigma^2 = 10$ dB. From the (upper-left) plot, we can see that we only have three possible values for M, i.e., $\mathcal{M} = \{1, 2, 3\}$. For $M = 1$, we have a positive η_M while for $M = 2$ and $M = 3$, η_M is negative. We should note that for $M = 3$, η_2 is slightly above zero (0.0028). Executing step 14 in Algorithm 1 yields[6] $\beta_2^\star \approx 0.64$ and $\beta_3^\star = 0$ for $M = 2$ and $M = 3$, respectively. Even though $M = 2$ and $M = 3$ have the same two groups with positive group loading, they have different total group loadings, i.e., 0.74 and 0.8, respectively and consequently different sum rates (per antenna). The last plot in the bottom-right shows that the maximum sum rate per antenna is achieve when $M = 2$. To validate the result from Algorithm 1, we perform a grid search where β takes values between 0 and 1 with 0.001 increment. For each value of β, the corresponding sum rate per antenna is computed. The results are plotted in Figure 4. The plot shows that the maximum sum rates (per antenna) and the optimal β obtained from the grid search and Algorithm 1 are identical. This confirms our theoretical analysis and the proposed algorithm. We should note that even though the line around the optimal β looks flat, closer inspection of the numerical values of the sum rates per antenna in that region reveals that the sum rate per antenna is actually increasing until reaching the optimal β and then decreasing.

We can also observe from the results of Algorithm 1 in Figure 3 that we can stop the iterations once an iteration for which $\eta_M < 0$ (and $\beta_M^\star \in [0, \beta_{M,\max}]$) is reached. Intuitively, if this occurs, only part of the last group receives non-zero

[6] In a real system we would then use an integer either side of $\beta_j^\star N$ to determine how many users should be active.

Algorithm 1 Algorithm for Solving **P3**

1: $\mathcal{M} = \{\}$ ▷ Contain possible values for M
2: **for** $j = 1$ **to** L **do**
3: $\quad \beta_i^\star = \beta_{i,\max}, \forall\ i = 1, \ldots, j$ ▷ Assume that $\beta_j^\star = \beta_{j,\max}$
4: $\quad \lambda, \rho^\star, [\bar{p}_1^\star\ \ldots \bar{p}_j^\star]^T \leftarrow$ Solving **P1** with $\beta^\star = \sum_i^j \beta_i^\star$
5: $\quad$ Determine M s.t. $\bar{p}_M^\star > 0$ and $\bar{p}_{M+1}^\star = \cdots = \bar{p}_j^\star = 0$ ▷ $M \geq 1$
6: $\quad$ **if** $M \in \mathcal{M}$ **then**
7: $\qquad$ **continue** ▷ Skip the remaining steps and go to the next iteration (Step 2)
8: $\quad$ **end if**
9: $\quad \mathcal{M} \leftarrow M$
10: $\quad \beta_{M+1}^\star = \cdots = \beta_j^\star = 0$
11: $\quad$ Compute μ according to (40)
12: $\quad \eta_M = \log(1 + \bar{p}_M^\star f_M(\beta^\star, \rho^\star)) - \lambda(\bar{p}_M^\star - 1) + \mu$
13: $\quad$ **if** $\eta_M < 0$ **then**
14: $\qquad \beta_M^\star \in [0, \beta_{M,\max}] \leftarrow$ Solving **P1** and (25) with $\beta^\star = \sum_i^{M-1} \beta_{i,\max} + \beta_M^\star$
15: $\quad$ **end if**
16: $\quad$ Compute $R_{\text{sum}}^{(M),\infty}$ with the updated β and $\{\beta_j\}$
17: **end for**
18: $M^\star \leftarrow$ Solving (24)

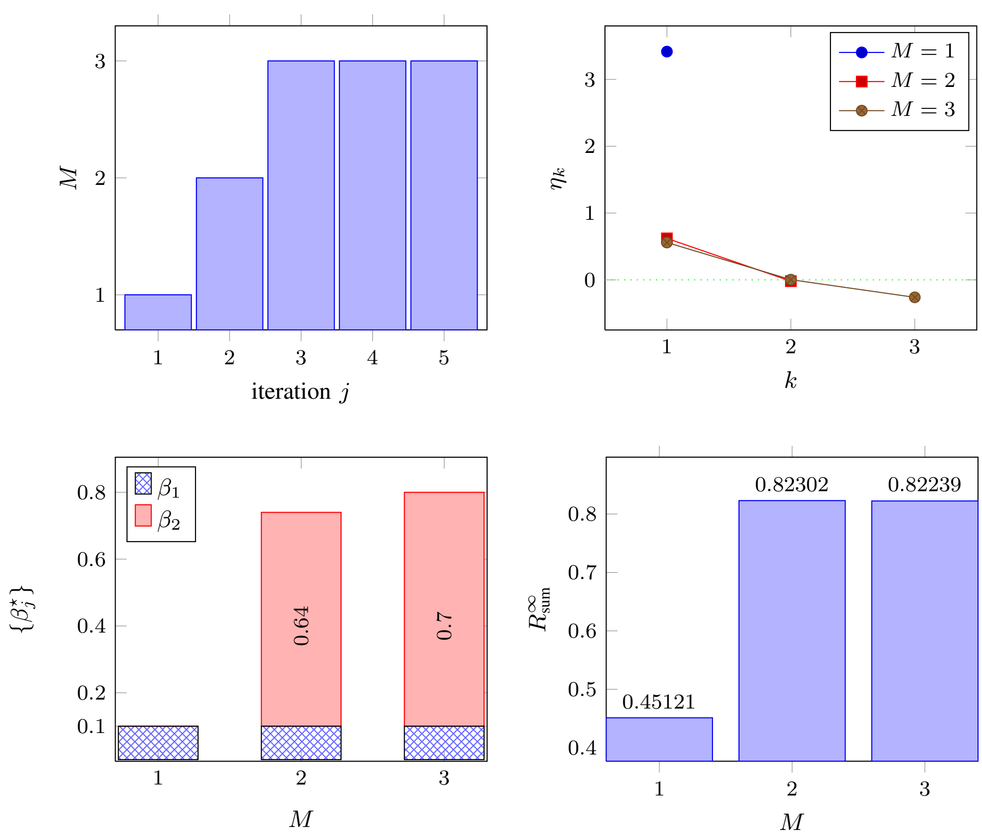

Fig. 3. Algorithm 1 implementation for $L = 5$, $\boldsymbol{\beta}_{\max} = [0.1\ 0.7\ 0.1\ 0.05\ 0.05]^T$, $a_j^2 = 1/j^2$ and $P_d/\sigma^2 = 10$ dB.

power, i.e. is served and it seems unlikely that adding groups with even weaker channels will change that and lead to an increase in the sum rate. This can be justified by realizing that the sum rate per antenna obtained by increasing $\beta_M^\star$ by, say β_δ, will be greater or equal to that obtained by adding one more group with group loading β_δ. Moreover, increasing $\beta_M^\star$ still gives a negative η_M which does not satisfy the KKT necessary condition ($\eta_M \geq 0$). Thus, we can modify Algorithm 1 by adding a '**break**' instruction after line 14. That will stop the iteration and jump directly to line 18. This will reduce the number of iterations and computations.

The applicability of the optimal user loading obtained from Algorithm 1 to a finite-size system is described in Figure 5. The simulation setup considers $N = 10$ and $K = 8$ users which are divided into two groups with $a_1^2 = 1, a_2^2 = 0.24$, $\beta_{1,\max} = 0.1, \beta_{2,\max} = 0.7$. The simulation is run by generat-

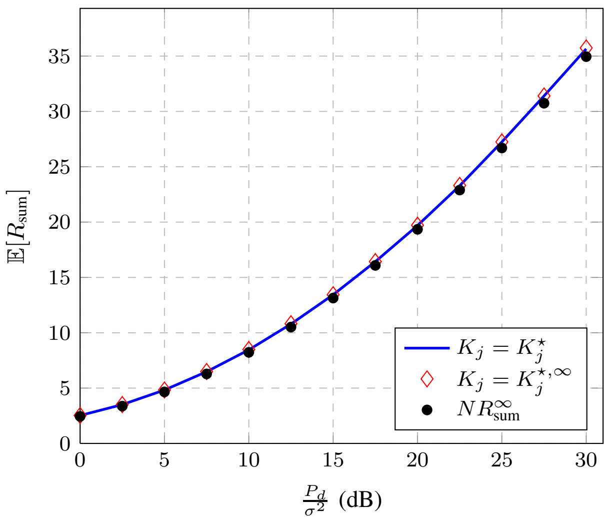

Fig. 5. The comparison of the average sum rates obtained by using the optimal number of users from Algorithm 1 and the exhaustive search.

ing 10^4 channel realizations and uses $\rho = \rho^\star, \bar{\mathbf{p}} = \bar{\mathbf{p}}^\star$ obtained from **P1**. From Algorithm 1, we obtain $\beta_j^\star, j = 1, 2$ and the corresponding optimal number of users is $K_j^{\star,\infty} = [N\beta_j^\star]$, where $[x]$ rounds x to the nearest integer. The average sum rate by employing $K_j^{\star,\infty}$ is depicted by the solid line (–). As a comparison, we also plot the average sum rates (◇) by using the optimal number of users obtained from exhaustive search (denoted by $K_j^\star$). As a reference, we also add the equivalent average sum-rate (•) by using R_{sum}^∞ from Algorithm 1. From Figure 5, we can see that those plots almost coincide and this indicates the applicability of Algorithm 1 to finite-size system designs.

V. Conclusion

In this paper, we have investigated problems related to determining the optimal power allocation, regularization parameter and group loadings of a finite number of groups of users so as to maximize the sum rate (per antenna) of MISO broadcast channels with RCI precoder. Even though the analysis was performed in the large system limit, our numerical simulations show its validity for finite-size system designs. Considering the power allocation problem only, we show that the optimal strategy follows the water-filling scheme. For some cases considered in this paper we show that it is optimal for the BS to communicate to some groups having best channels (highest path gains). We also provide the KKT necessary conditions and propose an algorithm for the optimal group-loading allocation when the BS is allowed to transmit to only subsets of the users in the groups.

Appendix A
Proof of Theorem 1

Here, we present the proof briefly since we repeat the same techniques as we have used in deriving the results in [1], [22] that are based on some well-known results in large random matrix theory (see e.g., [21], [24]). An alternative method can be found in [19, App. II]. Let $A_k = \frac{1}{N}\mathbf{h}_k\left(\frac{1}{N}\mathbf{H}_k^H\mathbf{H}_k + \rho\mathbf{I}_N\right)^{-1}\mathbf{h}_k^H = \frac{1}{N}\mathbf{h}_k\mathbf{M}_k\mathbf{h}_k^H$, where $\rho = \alpha/N$, $\mathbf{M}_k = \left(\frac{1}{N}\mathbf{H}_k^H\mathbf{H}_k + \rho\mathbf{I}_N\right)^{-1}$ and $\mathbf{H}_k$ is $\mathbf{H}$ with the k-th row removed. Then, by employing the matrix inversion lemma (MIL), $\mathbf{h}_k(\mathbf{H}^H\mathbf{H}+\alpha\mathbf{I}_N)^{-1}\mathbf{h}_k^H$ in the numerator of (3) can be written as $\frac{A_k}{1+A_k}$. By using the results [25, Lemma 1] or [26, Lemma 5.1], it follows that $A_k - \frac{1}{N}\text{Tr}\left(\mathbf{M}_k\right) \xrightarrow{a.s} 0$. By using the rank-1 perturbation lemma (R1PL), see e.g. [21, Theorem 3.9, Lemma 14.3], $\frac{1}{N}\text{Tr}\left(\mathbf{M}_k\right)$ converges almost surely to $\frac{1}{N}\text{Tr}\left(\mathbf{M}\right)$ with $\mathbf{M} = (\mathbf{H}^H\mathbf{H}+\alpha\mathbf{I}_N)^{-1}$. We can also show that $\frac{1}{N}\text{Tr}\left(\mathbf{M}\right) \xrightarrow{a.s} g(\beta,\rho)$ (see the results in e.g. [25, Theorem 7]) where $g(\beta,\rho)$ is the solution of $g(\beta,\rho) = \left(\rho + \frac{\beta}{1+g(\beta,\rho)}\right)^{-1}$. Thus, $A_k - g(\beta,\rho) \xrightarrow{a.s} 0$. Now considering the denominator, we can write $|\mathbf{h}_k(\mathbf{H}^H\mathbf{H}+\alpha\mathbf{I}_N)^{-1}\mathbf{h}_j^H|^2$ as $\frac{\frac{1}{N}}{(1+A_k)^2}I_j$ where $I_j = \frac{1}{N}\mathbf{h}_k\mathbf{M}_k\mathbf{h}_j^H\mathbf{h}_j\mathbf{M}_k\mathbf{h}_k^H$. By [26, Lemma 5.1], we have

$$\max_{j\leq K}\left|I_j - \frac{1}{N}\text{Tr}\left(\mathbf{M}_k\mathbf{h}_j^H\mathbf{h}_j\mathbf{M}_k\right)\right| \xrightarrow{a.s} 0.$$

The matrix inside the trace has rank one. Thus, the second term on the RHS becomes

$$\frac{1}{N}\mathbf{h}_j\mathbf{M}_k^2\mathbf{h}_j^H = \frac{1}{(1+A_{j,kj})^2}\frac{1}{N}\mathbf{h}_j\mathbf{M}_{kj}^2\mathbf{h}_j^H,$$

where the RHS is obtained by the MIL, $\mathbf{M}_{kj} = (\frac{1}{N}\mathbf{H}_{kj}^H\mathbf{H}_{kj} + \rho\mathbf{I}_N)^{-1}$, $A_{j,kj} = \frac{1}{N}\mathbf{h}_j\mathbf{M}_{kj}\mathbf{h}_j^H$, $\mathbf{H}_{kj}$ is $\mathbf{H}_k$ with row j removed. Then, it follows $\max_{j\leq K}\left|\frac{1}{N}\mathbf{h}_j\mathbf{M}_{kj}^2\mathbf{h}_j^H - \frac{1}{N}\text{Tr}\left(\mathbf{M}_{kj}^2\right)\right| \xrightarrow{a.s} 0$. We can show that the second term on the LHS is equal to $-\frac{\partial}{\partial\rho}\frac{1}{N}\text{Tr}\left(\mathbf{M}_{kj}\right)$. We also have that $\max_{j\leq K}\left|A_{j,kj} - \frac{1}{N}\text{Tr}\left(\mathbf{M}_{kj}\right)\right| \xrightarrow{a.s} 0$. By applying R1PL twice, $\frac{1}{N}\text{Tr}\left(\mathbf{M}_{kj}\right) \xrightarrow{a.s} g(\beta,\rho)$. Suppose that $\mathcal{P} = \lim_{K\to\infty}\frac{1}{K}\sum_{j=1}^K p_j$ exists and is bounded. Note that it can be interpreted as the empirical mean of the users' power or just average power. Thus, by combining the large system results, we obtain

$$\sum_{j\neq k} p_j|\mathbf{h}_k(\mathbf{H}^H\mathbf{H}+\alpha\mathbf{I}_N)^{-1}\mathbf{h}_j^H|^2 + \beta\mathcal{P}\frac{\frac{\partial g(\beta,\rho)}{\partial\rho}}{(1+g(\beta,\rho))^4} \xrightarrow{a.s} 0. \tag{26}$$

By following the same steps as in obtaining (26), we can establish that

$$c^2 + \frac{P_d(1+g(\beta,\rho))^2}{\beta\mathcal{P}\frac{\partial}{\partial\rho}g(\beta,\rho)} \xrightarrow{a.s} 0.$$

Hence, by using this last result, we can conclude that the signal and interference energy converges almost surely to $-P_d p_k a_k^2 g^2(\beta,\rho)\left(\beta\mathcal{P}\frac{\partial}{\partial\rho}g(\beta,\rho)\right)^{-1}$ and $P_d a_k^2(1+g(\beta,\rho))^{-2}$, respectively. Recalling the definitions of γ_k and $\bar{p}_k$ in the statement of Theorem 1, and using the fact that

$$\frac{\partial}{\partial\rho}g(\beta,\rho) = -\frac{g(\beta,\rho)(1+g(\beta,\rho))^2}{\beta+\rho(1+g(\beta,\rho))^2}, \tag{27}$$

(4) follows immediately. This concludes the proof.

APPENDIX B
PROOF OF THEOREM 3

Related to the KKT (stationary) conditions (19), for a given $\bar{\mathbf{p}}$, we have

$$\frac{\partial R^\infty_{\text{sum}}}{\partial \rho} = \sum_{j=1}^{L} \frac{\beta_j \bar{p}_j}{1+\bar{p}_j f_j(\beta,\rho)} \frac{\partial f_j(\beta,\rho)}{\partial \rho},$$

where

$$\begin{aligned}\frac{\partial f_j(\beta,\rho)}{\partial \rho} &= \frac{\gamma_j^2}{[\gamma_j + (1+g)^2]^2} 2g(1+g)\left(\frac{\rho}{\beta} - \frac{1}{\gamma_j}\right)\frac{\partial g}{\partial \rho} \\ &= f_j^2(\beta,\rho) \frac{2\left(\frac{1}{g}+1\right)}{[1+\frac{\rho}{\beta}(1+g)^2]^2}\left(\frac{\rho}{\beta} - \frac{1}{\gamma_j}\right)\frac{\partial g}{\partial \rho},\end{aligned}$$

and g represents $g(\beta,\rho)$. Thus,

$$\frac{\partial R^\infty_{\text{sum}}}{\partial \rho} = \frac{2\left(\frac{1}{g}+1\right)}{[1+\frac{\rho}{\beta}(1+g)^2]^2} \sum_{j=1}^{L} \frac{\beta_j \bar{p}_j f_j^2(\beta,\rho)}{1+\bar{p}_j f_j(\beta,\rho)} \left(\frac{\rho}{\beta} - \frac{1}{\gamma_j}\right)\frac{\partial g}{\partial \rho}.$$

Recall that $\frac{\partial g}{\partial \rho} < 0$ (see (27)). Let $q_j = \frac{\rho}{\beta} - \frac{1}{\gamma_j}$. It is also obvious that q_j is decreasing in j. Thus, for $q_L > 0$, $\frac{\partial R^\infty_{\text{sum}}}{\partial \rho}$ is negative. This implies that $\frac{\partial R^\infty_{\text{sum}}}{\partial \rho}$ can not be zero for $\rho > \frac{\beta}{\gamma_L}$. For $q_1 < 0$, $\frac{\partial R^\infty_{\text{sum}}}{\partial \rho}$ is positive and consequently, can not be zero for $\rho < \frac{\beta}{\gamma_1}$. Therefore, the optimal ρ must be in the interval of

$$\frac{\beta}{\gamma_1} \le \rho^\star \le \frac{\beta}{\gamma_L}.$$

as in (15). When we only have one group then ρ^* is the same as the one obtained in [22], [27]. We can also remove the boundary point $\rho = 0 < \frac{\beta}{\gamma_1}$ (related to the case $\kappa > 0$, that is, the constraint $\rho \ge 0$ is inactive) since as previously discussed, $\frac{\partial R^\infty_{\text{sum}}}{\partial \rho} > 0$ at that point. Thus, from (9) with $\kappa = 0$ or by evaluating $\frac{\partial R^\infty_{\text{sum}}}{\partial \rho} = 0$, $\rho^\star$ must satisfy (14) at $\bar{\mathbf{p}} = \bar{\mathbf{p}}^\star$.

APPENDIX C
PROOF OF LEMMA 3

In the first part, we will prove part (i) - (iii) of the lemma. We show those by considering any two groups l and j such that $l < j$, such that the current allocation has $\beta_j > 0$ and $\bar{p}_j > 0$ and proving that we can improve performance by having β_l at its maximum possible value. Let us assume an assignment $(\beta_l, \bar{p}_l)$ and $(\beta_j, \bar{p}_j)$ such that $\beta_l \le \beta_{l,\max}$ and $\beta_j \le \beta_{j,\max}$. In that case, the combined group loading is $\beta_l + \beta_j$. Now, let x_l be the new group loading allocation for group l and y_l be the corresponding assigned power. In the following we will show that the optimal x_l maximizing the sum rate of of users in group j and l is $\beta_{l,\max}$ by solving the following optimization problem

$$\begin{aligned}\max_{x_l,y_l,y_j}.\quad & x_l \log(1+y_l f_l(\beta,\rho)) \\ & \quad +(\beta_l+\beta_j-x_l)\log(1+y_j f_j(\beta,\rho)) \\ \text{s.t.}\quad & \max(0,\beta_l+\beta_j-\beta_{j,\max}) \le x_l, \\ & \min(\beta_l+\beta_j, \beta_{l,\max}) \ge x_l, \\ & y_l x_l + y_j(\beta_l+\beta_j-x_l) \le \beta_l\bar{p}_l + \beta_j\bar{p}_j, \\ & y_l \ge 0, y_j \ge 0.\end{aligned}$$

The Lagrangian is given by

$$\begin{aligned}\mathcal{L} &= x_l \log(1+y_l f_l(\beta,\rho)) + (\beta_l+\beta_j-x_l)\log(1+y_j f_j(\beta,\rho)) \\ &\quad + \mu_{x_l}(x_l - \max(0,\beta_l+\beta_j-\beta_{j,\max})) \\ &\quad + \nu_{x_l}(\min(\beta_l+\beta_j,\beta_{l,\max}) - x_l) \\ &\quad + \lambda(\beta_l\bar{p}_l + \beta_j\bar{p}_j - y_l x_l - y_j(\beta_l+\beta_j-x_l)) \\ &\quad + \mu_{y_l} y_l + \mu_{y_j} y_j,\end{aligned}$$

where $\mu_{x_l}, \nu_{x_l}, \mu_{y_l}, \mu_{y_j}, \lambda$ are the Lagrange multipliers associated to the constraints on x_l, y_l, y_j and the second constraint, respectively. The stationary conditions for the solution candidates are then given by[7]

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial x_l} &= \log(1+y_l f_l(\beta,\rho)) - \log(1+y_j f_j(\beta,\rho)) \\ &\qquad + \mu_{x_l} - \nu_{x_l} - \lambda(y_l - y_j) = 0, \qquad (28)\\ \frac{\partial \mathcal{L}}{\partial y_l} &= \frac{x_l f_l(\beta,\rho)}{1+y_l f_l(\beta,\rho)} + \mu_{y_l} - \lambda x_l = 0, \qquad (29)\\ \frac{\partial \mathcal{L}}{\partial y_j} &= (\beta_l+\beta_j-x_l)\frac{f_j(\beta,\rho)}{1+y_j f_j(\beta,\rho)} \\ &\qquad + \mu_{y_j} - \lambda(\beta_l+\beta_j-x_l) = 0. \qquad (30)\end{aligned}$$

From (29) and (30), it follows that

$$y_l = \left[\frac{1}{\lambda} - \frac{1}{f_l(\beta,\rho)}\right]_+, \qquad (31)$$

$$y_j = \left[\frac{1}{\lambda} - \frac{1}{f_j(\beta,\rho)}\right]_+. \qquad (32)$$

One can check that $y_l = 0$ will never be the optimal solution. For $y_l > 0$, two cases arise depending on whether y_j is strictly positive or not.

- Case $y_j = 0$. To satisfy the KKT conditions, the second constraint is met with equality, for $\lambda > 0$. Thus, we have $y_l = \frac{\beta_l\bar{p}_l + \beta_j\bar{p}_j}{x_l}$. From (31), we can express

  $$\frac{1}{\lambda} = \frac{\beta_l\bar{p}_l + \beta_j\bar{p}_j}{x_l} - \frac{1}{f_l(\beta,\rho)}.$$

  When $y_j = 0$, it also holds $1/\lambda - 1/f_j(\beta,\rho) \le 0$. Consequently, from the equation above, we can write

  $$\frac{\beta_l\bar{p}_l + \beta_j\bar{p}_j}{\frac{1}{f_j(\beta,\rho)} - \frac{1}{f_l(\beta,\rho)}} \le x_l\ .$$

  From (28), we can obtain

  $$\begin{aligned}&\log\left(1 + \left(\frac{\beta_l\bar{p}_l + \beta_j\bar{p}_j}{x_l}\right) f_l(\beta,\rho)\right) \\ &\qquad - \frac{1}{1+\frac{x_l}{\beta_l\bar{p}_l+\beta_j\bar{p}_j}\frac{1}{f_l(\beta,\rho)}} = \nu_{x_l} - \mu_{x_l}. \qquad (33)\end{aligned}$$

  The LHS of (33) is a function of the form $f(x) = \log(1+x) - \frac{x}{1+x}$, which can be easily shown to be strictly increasing in x. Moreover, at $x = 0$, $f(x) = 0$. So, the LHS of (33) is positive. Thus, ignoring the constraint on x_l, the objective function is strictly increasing for $\frac{\beta_l\bar{p}_l + \beta_j\bar{p}_j}{\frac{1}{f_j(\beta,\rho)} - \frac{1}{f_l(\beta,\rho)}} \le x_l$.

[7]Here, we do not use superscript $\star$ for the solution candidates

- Case $y_j > 0$. For $\gamma > 0$, the average power constraint is met with equality and we have

$$y_j = \frac{\beta_l \bar{p}_l + \beta_j \bar{p}_j - (y_l - y_j)x_l}{\beta_l + \beta_j} = \frac{\beta_l \bar{p}_l + \beta_j \bar{p}_j - \left(\frac{1}{f_j(\beta,\rho)} - \frac{1}{f_l(\beta,\rho)}\right)x_l}{\beta_l + \beta_j}.$$

Then, we can express

$$\frac{1}{\lambda} = \frac{1}{f_j(\beta,\rho)} + \frac{\beta_l \bar{p}_l + \beta_j \bar{p}_j - \left(\frac{1}{f_j(\beta,\rho)} - \frac{1}{f_l(\beta,\rho)}\right)x_l}{\beta_l + \beta_j}.$$

Since for $y_j > 0$, $\frac{1}{\lambda} > \frac{1}{f_j(\beta,\rho)}$, then we obtain

$$\frac{\beta_l \bar{p}_l + \beta_j \bar{p}_j}{\frac{1}{f_j(\beta,\rho)} - \frac{1}{f_l(\beta,\rho)}} > x_l \ . \quad (34)$$

Using the expression for $1/\lambda$, we can rewrite (28) as

$$\nu_{x_l} - \mu_{x_l} = \log\left(\frac{f_l(\beta,\rho)}{f_j(\beta,\rho)}\right) - \frac{\frac{1}{f_j(\beta,\rho)} - \frac{1}{f_l(\beta,\rho)}}{\frac{1}{f_j(\beta,\rho)} + \frac{\beta_l \bar{p}_l + \beta_j \bar{p}_j - \left(\frac{1}{f_j(\beta,\rho)} - \frac{1}{f_l(\beta,\rho)}\right)x_l}{\beta_l + \beta_j}}. \quad (35)$$

It is clear that the LHS of (35) is decreasing in x_l. Moreover, for $x_l \to \infty$, its value is $\log\left(\frac{f_l(\beta,\rho)}{f_j(\beta,\rho)}\right) > 0$. Therefore, without the constraints on x_l, the objective function is also strictly increasing in x_l when the condition (34) holds.

Combining the two cases, the optimal x_l is equal to its maximum allowable value. By using this fact repeatedly, starting from group 1, we establish (i)-(iii).

Now, it remains to show that if no power is allocated to a group, it must be that the corresponding $\beta_j = 0$ (see (iv)). Let us consider the stationary conditions for β_j and β which are given by (21) and (22), respectively. We can rewrite them as

$$\log(1 + \bar{p}_j f_j) - \lambda(\bar{p}_j - 1) + \nu_j + \mu = \eta_j \quad (36)$$

and

$$\sum_{j=1}^{L} \frac{\beta_j \bar{p}_j}{1 + \bar{p}_j f_j(\beta,\rho)} \frac{\partial f_j(\beta,\rho)}{\partial \beta} = \mu, \quad (37)$$

respectively. In obtaining (37), we use the fact that β must be positive, i.e., $\eta = 0$. The first derivative of $f_j(\beta,\rho)$ over β in (37) can be shown to take the form

$$\frac{\partial f_j(\beta,\rho)}{\partial \beta} = -\frac{f_j(\beta,\rho)}{\beta}\left[1 + \frac{g}{1 + \frac{\rho}{\beta}(1+g)^2} + \frac{2g(1+g)^2(\frac{\rho}{\beta}\gamma_j - 1)}{[\gamma_j + (1+g)^2][1 + \frac{\rho}{\beta}(1+g)^2]^2}\right], \quad (38)$$

where for brevity we denote $g = g(\beta,\rho)$. The derivative of $f_j(\beta,\rho)$ w.r.t. ρ in (19) can be written as follows

$$\frac{\partial f_j(\beta,\rho)}{\partial \rho} = -\frac{f_j(\beta,\rho)}{\beta} \frac{2g(1+g)^3(\frac{\rho}{\beta}\gamma_j - 1)}{[\gamma_j + (1+g)^2][1 + \frac{\rho}{\beta}(1+g)^2]^2}.$$

So we can rewrite (38) in terms of $\frac{\partial f_j(\beta,\rho)}{\partial \rho}$ as

$$\frac{\partial f_j(\beta,\rho)}{\partial \beta} = -\frac{f_j(\beta,\rho)}{\beta}\left[1 + \frac{g}{1 + \frac{\rho}{\beta}(1+g)^2}\right] + \frac{1}{1+g}\frac{\partial f_j(\beta,\rho)}{\partial \rho}. \quad (39)$$

Recall that $1 + p_j f_j(\beta,\rho) = f_j(\beta,\rho)/\lambda$. Substituting (39) into (37) yields

$$\mu = -\frac{\lambda}{\beta}\left[1 + \frac{g}{1 + \frac{\rho}{\beta}(1+g)^2}\right]\sum_{j=1}^{L}\beta_j \bar{p}_j + \frac{1}{1+g}\sum_{j=1}^{L}\frac{\beta_j \bar{p}_j}{1 + \bar{p}_j f_j(\beta,\rho)}\frac{\partial f_j(\beta,\rho)}{\partial \rho} \stackrel{(a)}{=} -\lambda\left[1 + \frac{g}{1 + \frac{\rho}{\beta}(1+g)^2}\right] \quad (40)$$

where in (a) we use the fact that $\sum_{j=1}^{L}\beta_j \bar{p}_j = \beta$ and the second term of the RHS is zero due to (19). Moreover, (a) gives the expression for μ at the optimal operating points. Plugging (a) into (36) with $p_j = 0$, we obtain

$$-\lambda\frac{g}{1 + \frac{\rho}{\beta}(1+g)^2} + \nu_j = \eta_j.$$

As a result, ν_j must be strictly positive. This implies that $\beta_j = 0$ and the proof is completed.

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