Optimal Resource Allocation for Pilot Symbol Aided Multiuser Receivers in Rayleigh Faded CDMA Channels

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***Abstract*—We consider a synchronous code-division multiple-access system where each user undergoes independent frequency-flat Rayleigh fading, and where pilot symbols are periodically inserted into the data stream of each user in order to assist in the coherent demodulation of the data symbols. The motivating question for this work is: for any given set of system parameters, how often should we insert pilot symbols? Along the way to answering this question, we: 1) derive and analyze the performance of the linear minimum mean-squared-error channel estimator and 2) study the performance of a linear minimum mean-squared-error data estimator which is coupled to the channel estimator. We are able to obtain a very compact expression for the average signal-to-interference ratio in terms of the key system parameters: pilot insertion period, channel fading rate, signal-to-noise ratio, and the ratio of the number of users to the spreading gain. The average signal-to-interference ratio is numerically optimized and results are presented to illustrate the optimal rate of inserting pilot symbols for a range of system parameters.**

***Index Terms*—CDMA, channel estimation, linear receivers, multiuser detection, pilot symbols, random spreading, Rayleigh fading.**

I. Introduction

CODE-DIVISION multiple-access (CDMA) will be the dominant multiple-access technique at the air-interface of third-generation cellular networks [1]–[4]. While second-generation CDMA networks use coherent modulation only on the forward link (base to mobile), next-generation systems will employ coherent modulation on both forward and reverse links. Coherent communication on the reverse link is aided by the use of a pilot signal for each mobile; it is the allocation of resources to these pilot signals that is the subject of this paper.

Some excellent work has been carried out in this area in the case when the receiver is a single-user matched filter [5]–[7]. Third-generation standards incorporate the option of using repeated signature sequences so that more advanced multiuser receivers can be employed [8]. This paper tackles the problem of performance optimization for a pilot symbol-aided multiuser receiver in a Rayleigh faded CDMA channel. The CDMA system is assumed to be synchronous, and each user's signal undergoes independent frequency-flat Rayleigh fading as it propagates from transmitter to receiver.

Paper approved by S. L. Miller, the Editor for Spread Spectrum of the IEEE Communications Society. Manuscript received June 20, 2001; revised December 17, 2001. This paper was presented in part at the IEEE International Conference on Communications, New York, April/May 2002, and at the IEEE International Symposium on Information Theory, Lausanne, Switzerland, June/July 2002.

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Publisher Item Identifier 10.1109/TCOMM.2002.801497.

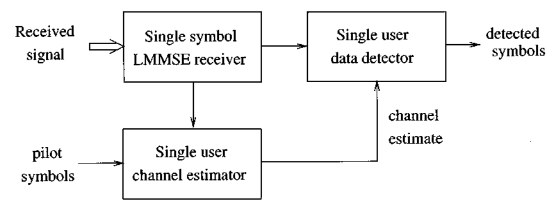

Fig. 1. Overview of receiver structure.

The rate of fading, as measured by the normalized Doppler spread, is assumed quite fast so that it is not reasonable to simply assume that the channels of all users are known perfectly. We will instead look explicitly at the channel estimation problem and assess the impact of channel estimation errors on performance.

A. Problem Statement

A receiver structure of central importance to this paper is illustrated in Fig. 1. Variations on this basic structure have been proposed by many authors developing multiuser receivers for Rayleigh fading channels [9]–[12].

To aid our discussion we briefly introduce the model for the received signal in symbol period m

$$\mathbf{r}(m) = \sqrt{P}a_1(m)b_1(m)\mathbf{s}_1 + \sum_{k=2}^{K}\sqrt{P}a_k(m)b_k(m)\mathbf{s}_k + \mathbf{w}(m) \quad (1)$$

where $a_k(m)$ is the channel of user k in symbol m, $b_k(m)$ is the data symbol for user k in symbol m, $\mathbf{w}(m)$ is a white Gaussian noise vector, and $\mathbf{s}_k$ is the signature sequence of user k, which we assume is repeated from symbol to symbol.

The received signal is first passed through a linear filter which aims to suppress interference and produce an estimate of $a_1(m)b_1(m)$. We consider a linear minimum mean-squared-error (LMMSE) receiver that requires knowledge of the signature sequences of all users, the average power of all users, and the covariance matrix of the noise. We assume that these parameters are time invariant or slowly varying, and that they can be easily estimated if not known *a priori*.

At the output of the LMMSE receiver, we are left with the task of untangling the product of the data and the channel,

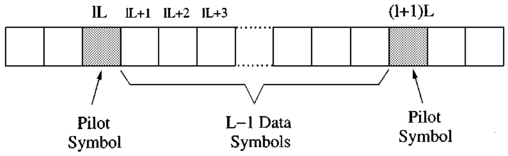

Fig. 2. Frame structure.

$a_1(m)b_1(m)$, from observations corrupted by residual interference and noise. The problem at this stage is really that of receiver design for the single-user Rayleigh fading channel, and our receiver uses well known pilot symbol-assisted techniques [13], [14]. We assume that pilot symbols (symbols known to the receiver) are periodically inserted into the stream of data symbols of user one as illustrated in Fig. 2. These pilot symbols are used in conjunction with the corresponding outputs from the LMMSE receiver to obtain estimates of the channel of user one. These estimates are then passed to a standard detector which compensates for the effect of the channel and makes decisions on the transmitted data.

Clearly, inserting pilot symbols more frequently will improve the performance of the channel estimator. However, pilot symbols take up valuable resources such as power and bandwidth. In this paper, we ask the question, how can we find the best allocation of resources to pilot symbols? To be more specific, how should we choose the pilot symbol insertion rate as a function of the key system parameters: spreading gain, number of users, signal-to-noise ratio (SNR), and the fading rate?

Before moving on, we draw the readers' attention to [15], which examines similar problems when parallel pilot channels are employed. One main difference in this work (apart from the pilot structure) is that we explicitly consider the time-variation of the channel through a specified autocorrelation function or power spectral density. This allows us to include the normalized Doppler frequency as an important system parameter.

B. Summary of Contributions

The main contributions of the paper are outlined below.

Channel Estimation: In Section III, we derive and analyze the LMMSE channel estimator for our system in the case when the estimate is based on the entire sequence of pilot symbols (and received signals at the pilot points). The resultant LMMSE estimator allows the simple implementation given in Fig. 1. We derive expressions for the mean-squared-error (MSE) of the optimal estimator for finite-size systems and also for large systems. In the latter case, the MSE is independent of the realization of the signature sequences, depending only on the number of users, the processing gain, the rate of inserting pilot symbols, and the channel parameters.

Data Detection: In Section IV, we first study the maximum-likelihood (ML) detector for a data symbol under the assumption that the detector has perfect knowledge of the channel of the user in question, but of nothing else. Under this assumption, the ML detector consists of a LMMSE front end followed by a minimum distance detector. This motivates the receiver structure of Fig. 1 when knowledge of the channel is replaced by access to the pilot symbols of the user of interest. An expression for the average signal-to-interference ratio (SIR) is derived, and by looking at a large system, we are able to get a final performance measure which is independent of the signature sequences and which depends only on the key system parameters.

Pilot Optimization: Finally, in Section V, we look at optimizing the performance (as measured by the average SIR) over the choice of pilot insertion period. Some surprising results are uncovered when we examine the behavior of the optimal pilot insertion period as the system loading is varied.

II. Rayleigh Fading CDMA Channels

The model for the received signal after down conversion and chip-matched filtering was given in (1). Referring to this model, each signature sequence is a column vector of length N (the processing gain), which is assumed known at the receiver. For performance analysis, we will assume that the entries (chips) of $\mathbf{s}_k$ are independent and identically distributed random variables with mean zero and variance $1/N$. Write $\mathbf{S} = [\mathbf{s}_1\mathbf{s}_2\ldots\mathbf{s}_K]$ for the $N \times K$ matrix of signature sequences, and $\mathbf{S}_1 = [\mathbf{s}_2\mathbf{s}_3\ldots\mathbf{s}_K]$ for the $N \times K-1$ matrix with the signature sequence of user one removed.

We further assume that

1) the channel process $\{a_k\}$ is a stationary, circularly symmetric, complex Gaussian random process with $E[a_k(m)] = 0$ and $E[a_k(m+n)a_k^*(m)] = R_a(n)$ with $R_a(0) = 1$;
2) the vector noise process $\{\mathbf{w}(m)\}$ is a stationary, circularly symmetric, complex Gaussian random process with $E[\mathbf{w}(m)] = 0$ and $E[\mathbf{w}(m+n)\mathbf{w}^*(m)] = \sigma^2\mathbf{I}\delta(n)$;
3) the data process $\{b_k\}$ is a white random process with each $b_k(m)$ selected from an M-ary phase-shift keying (PSK) alphabet with $|b_k(m)| = 1$ and $E[b_k(m)] = 0$;
4) the signature sequences, data, noise, and channel processes are independent.

Under these assumptions and conditioned on $\mathbf{S}$, the received signal is a circularly symmetric, complex random process with

$$E[\mathbf{r}(m)|\mathbf{S}] = 0$$
$$E[\mathbf{r}(m+n)\mathbf{r}^*(m)|\mathbf{S}] = \left[P\,\mathbf{S}\mathbf{S}^* + \sigma^2\mathbf{I}\right]\delta(n).$$

Observe that $a_k(m)b_k(m)$ is a circularly symmetric, complex Gaussian random variable, but that $a_k(m)b_k(m)$ and $a_k(n)b_k(n)$ are not jointly Gaussian. This means that (given $\mathbf{S}$) $\mathbf{r}(m)$ is a complex Gaussian vector, but the process $\{\mathbf{r}(m)\}$ is not a Gaussian process.

Another assumption that we have made is that the average (received) power of each user is the same (since $E[|a_k(m)|^2] = 1$ for all k). This might correspond to a situation where there is power control at the central receiver. Assume that the power control operates on a time scale which is slow compared with the Rayleigh fading, but fast compared to the changes in average power due to distance-based path loss and shadowing. Having said that, the assumption is made only for ease of exposition and all results can be readily extended to the more general situation of unequal average received powers. The term SNR is reserved for the quantity P/σ^2.

Remark 1: The model considered in this paper is one of the simplest models that includes the key ingredients of multiple-access interference and fading. The advantage of this model is that meaningful and insightful results can be obtained in a resonably straightforward manner. Importantly, the framework presented in this work could be extended to an asynchronous CDMA model and to include frequency-selective fading using techniques from [16] and [17], respectively.

Before continuing, note that all expectations in the sequel should be seen as expectations conditioned on the matrix of signature sequences. For example, $E[\cdot]$ means $E[\cdot|\mathbf{S}]$ and $E[\cdot|\cdot]$ means $E[\cdot|\cdot,\mathbf{S}]$. Throughout, we denote vectors and matrices (random and deterministic) by boldface characters and use an asterisk to denote the conjugate transpose operation.

III. Pilot Symbol-Based LMMSE Channel Estimation

We consider a structure where pilot symbols are periodically inserted into the sequence of data symbols of user one. In particular, suppose that a pilot symbol is inserted after every block of $L-1$ data symbols at locations $\ldots,-2L,-L,0,L,2L,\ldots$ as illustrated in Fig. 2.

Define the sampled and demodulated output sequence

$$\begin{aligned}\mathbf{y}(l) =& b_1^*(lL)\mathbf{r}(lL)\\ =&\sqrt{P}x(l)\mathbf{s}_1 + b_1^*(lL)\cdot\sum_{k=2}^{K}\sqrt{P}a_k(lL)b_k(lL)\mathbf{s}_k\\ &+ b_1^*(lL)\mathbf{w}(lL)\end{aligned} \quad (2)$$

where we have defined $x(l) = a_1(lL)$.

A. Estimation Based on One Pilot Symbol

To begin, we consider the simple case of linearly estimating the channel of user one, $x(l) = a_1(lL)$, based solely on $\mathbf{y}(l)$ in such a way that the MSE is minimized. To be more explicit, we are concerned with estimates of the form

$$\bar{x}(l) = \mathbf{h}\mathbf{y}(l)$$

which minimize the MSE

$$\Delta = E[(x(l)-\bar{x}(l))(x(l)-\bar{x}(l))^*]$$

where $\mathbf{h}$ is an N-dimensional row vector. Since we have already removed dependence on the pilot symbols, we can assume that $\mathbf{h}$ is time invariant without loss of generality.

Remark 2: Before proceeding, note that the channel estimates thus obtained will turn out to be a first step in obtaining estimates based on all pilot symbols. These "one-symbol" estimates should be viewed only as intermediate estimates, and not as candidates for the channel estimates to be used by the detector.

The well known LMMSE estimate of $x(l)$ is

$$\begin{aligned}\bar{x}(l) =& E\left[x(l)\mathbf{y}^*(l)\right]\left(E[\mathbf{y}(l)\mathbf{y}^*(l)]\right)^{-1}\mathbf{y}(l)\\ =&\sqrt{P}\mathbf{s}_1^*\left[P\mathbf{s}_1\mathbf{s}_1^* + P\mathbf{S}_1\mathbf{S}_1^* + \sigma^2\mathbf{I}\right]^{-1}\mathbf{y}(l).\end{aligned}$$

Making use of the Matrix Inversion Lemma, we have equivalently

$$\bar{x}(l) = \frac{\sqrt{P}}{1+\beta}\mathbf{s}_1^*\left[P\mathbf{S}_1\mathbf{S}_1^* + \sigma^2\mathbf{I}\right]^{-1}\mathbf{y}(l) \quad (3)$$

where

$$\beta = P\mathbf{s}_1^*\left[P\mathbf{S}_1\mathbf{S}_1^* + \sigma^2\mathbf{I}\right]^{-1}\mathbf{s}_1 \quad (4)$$

is the SIR for this estimate. The resulting MSE is given by

$$\begin{aligned}\Delta =& E\left[x^*(l)x(l)\right] - E\left[x(l)\mathbf{y}^*(l)\right]\\ &\left(E\left[\mathbf{y}(l)\mathbf{y}^*(l)\right]\right)^{-1}E\left[x^*(l)\mathbf{y}(l)\right]\\ =&1 - P\mathbf{s}_1^*\left[P\mathbf{S}\mathbf{S}^* + \sigma^2\mathbf{I}\right]^{-1}\mathbf{s}_1\\ =&\frac{1}{(1+\beta).}\end{aligned}$$

The performance of the channel estimator is summarized by the MSE or the SIR, which is a function of the signature sequences of all users, and the average SNR of each user (P/σ^2). If we now look at a large system and model the signature sequences as random, then the dependence on the signature sequences disappears. We have the following result [17]–[19]:

Result 1: Under our random spreading model, as $N\to\infty$ with $\alpha = K/N$ held constant, the SIR given in (4) converges (almost surely) to the nonrandom constant $\beta^\dagger$, which is the solution to

$$\beta^\dagger = \left(\frac{P}{\sigma^2}\right)\left[1+\alpha\frac{\left(\frac{P}{\sigma^2}\right)}{1+\beta^\dagger}\right]^{-1}.$$

This is a quadratic equation with desired solution

$$\begin{aligned}\beta^\dagger = \frac{1}{2}\Bigg[&(1-\alpha)\left(\frac{P}{\sigma^2}\right)-1\\ &+\left((1-\alpha)^2\left(\frac{P}{\sigma^2}\right)^2 + 2(1+\alpha)\left(\frac{P}{\sigma^2}\right)+1\right)^{1/2}\Bigg].\end{aligned} \quad (5)$$

In a system with finite spreading gain, the MSE, Δ, is a random variable because of its dependence on the signature sequences of all users. This result tells us that when N is large, almost all realizations of $\mathbf{S}$ will lead to approximately the same value of Δ. All that is important is the ratio of the number of users and the spreading gain.

The large system analysis underlying this result has been used extensively in recent years to transform somewhat cumbersome expressions (for SIR, MSE, information capacity) into very useful and compact performance measures. Examples can be found in the papers [15]–[18], [20]–[25]. The above result is all we require in order to develop large system versions of all of our performance measures.

Before proceeding, we note that $x(l)$ and $\mathbf{y}(l)$ are jointly circularly symmetric, complex Gaussian random vectors when conditioned on the pilot symbol of user one. The optimal linear estimator derived above is thus the minimum mean-squared error (MMSE) or conditional mean estimator of $x(l)$, given $\mathbf{y}(l)$, and we have the following result:

Result 2: The random variables $x(l)$ and $\bar{x}(l)$ are jointly circularly symmetric, complex Gaussian random variables with parameters $E[x(l)] = E[\bar{x}(l)] = 0$, $E[|x(l)|^2] = 1$, $E[|\bar{x}(l)|^2] = 1-\Delta$, and $E[x(l)\bar{x}^*(l)] = \Delta$.

A further consequence of this result is that the random variables $\bar{x}(l)$ and $x(l)-\bar{x}(l)$ are independent.

B. Estimation Based on All Pilot Symbols

We now wish to make use of the entire sequence of pilot symbols to obtain a channel estimate for user one at any time. We emphasize at this point that the pilot sequence is assumed to extend indefinitely in both directions (past and future). This will allow us to exploit classical linear smoothing results, and provides a bound on the performance of any linear channel estimator.

To begin, observe that we must handle the general situation of estimating the channel during a symbol that does not necessarily coincide with a pilot point. Define the shifted and sampled channel process for user one

$$x_p(l) = a_1(lL+p), \qquad 0 \le p \le L-1$$

and note that $x(l) = x_0(l)$. We wish to estimate $x_p(l)$ by linearly smoothing the (pilot) observation process $\{\mathbf{y}(l)\}$. The optimal linear smoother will not depend on l but will vary with p.

To begin, we define, for the sampled channel process, the autocorrelation function

$$\begin{aligned} R_{x_p}(n) =& E[x_p(l+n)x_p^*(l)] \\ =& E[a_1(lL+nL+p)a_1^*(lL+p)] \\ =& R_a(nL) \end{aligned}$$

and the power spectral density

$$S_{x_p}(\omega) = \sum_{n=-\infty}^{\infty} R_{x_p}(n)e^{-j\omega n} = \sum_{n=-\infty}^{\infty} R_a(nL)e^{-j\omega n}.$$

The above expressions do not depend on p, so we will write $R_{x_p}(n) = R_x(n)$ and $S_{x_p}(\omega) = S_x(\omega)$. We will also need to deal with the correlation function

$$\begin{aligned} R_{x_p x}(n) =& E[x_p(l+n)x^*(l)] \\ =& E[a_1(lL+nL+p)a_1^*(lL)] \\ =& R_a(nL+p) \end{aligned}$$

and corresponding spectral density function

$$S_{x_p x} = \sum_{n=-\infty}^{\infty} R_{x_p x}(n)e^{-j\omega n} = \sum_{n=-\infty}^{\infty} R_a(nL+p)e^{-j\omega n}.$$

Turning to the pilot observation process, we have

$$\begin{aligned} \mathbf{R}_{\mathbf{y}}(n) =& E[\mathbf{y}(l+n)\mathbf{y}^*(l)] \\ =& R_x(n)P\mathbf{s}_1\mathbf{s}_1^* + \delta(n)\left[R_a(0)P\sum_{k=2}^{K}\mathbf{s}_k\mathbf{s}_k^* + \sigma^2\mathbf{I}\right] \\ =& R_x(n)P\mathbf{s}_1\mathbf{s}_1^* + \delta(n)\left[P\mathbf{S}_1\mathbf{S}_1^* + \sigma^2\mathbf{I}\right] \\ \mathbf{R}_{x_p\mathbf{y}}(n) =& E[x_p(l+n)\mathbf{y}^*(l)] \\ =& R_{x_p x}(n)\sqrt{P}\mathbf{s}_1^*. \end{aligned} \tag{6}$$

Now observe that

$$\begin{aligned} \mathbf{S}_{\mathbf{y}}(\omega) =& \sum_{n=-\infty}^{\infty} \mathbf{R}_{\mathbf{y}}(n)e^{-j\omega n} \\ =& S_x(\omega)P\mathbf{s}_1\mathbf{s}_1^* + \left[P\mathbf{S}_1\mathbf{S}_1^* + \sigma^2\mathbf{I}\right] \end{aligned} \tag{7}$$

$$\mathbf{S}_{x_p\mathbf{y}}(\omega) = \sum_{n=-\infty}^{\infty} \mathbf{R}_{x_p\mathbf{y}}(n)e^{-j\omega n} = S_{x_p x}(\omega)\sqrt{P}\mathbf{s}_1^*. \tag{8}$$

We wish to form the estimate

$$\hat{x}_p(l) = \sum_{n=-\infty}^{\infty} \mathbf{h}_p(n)\mathbf{y}(l-n)$$

where $\mathbf{h}_p(n)$ is a row vector of length N. The optimal filter is most easily expressed in terms of its transfer function

$$\mathbf{K}_p(\omega) = \sum_{n=-\infty}^{\infty} \mathbf{h}_p(n)e^{-j\omega n}.$$

We have the following theorem which follows from *Result 3* in the Appendix and from (7) and (8).

Theorem 1: The optimal (MMSE) linear smoother is

$$\mathbf{K}_p(\omega) = K_p(\omega)\sqrt{P}\mathbf{s}_1^*\left[P\mathbf{S}_1\mathbf{S}_1^* + \sigma^2\mathbf{I}\right]^{-1}$$

where

$$K_p(\omega) = \frac{S_{x_p x}(\omega)}{1+\beta S_x(\omega)} \tag{9}$$

and β is given in (4). The corresponding MMSE is

$$\Delta_p = 1 - \frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{\beta|S_{x_p x}(\omega)|^2}{1+\beta S_x(\omega)}d\omega. \tag{10}$$

1) Structure of Optimal Linear Smoother: We can examine the structure of the optimal linear smoother using the inversion formula

$$\mathbf{h}_p(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi}\mathbf{K}_p(\omega)e^{j\omega n}d\omega$$

which tells us that

$$\mathbf{h}_p(n) = h_p(n)\sqrt{P}\mathbf{s}_1^*\left[P\mathbf{S}_1\mathbf{S}_1^* + \sigma^2\mathbf{I}\right]^{-1}$$

where

$$h_p(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi}K_p(\omega)e^{j\omega n}d\omega.$$

The optimal estimate is thus

$$\begin{aligned} \hat{x}_p(l) =& \sum_{n=-\infty}^{\infty} h_p(n)\sqrt{P}\mathbf{s}_1^*\left[P\mathbf{S}_1\mathbf{S}_1^* + \sigma^2\mathbf{I}\right]^{-1}\mathbf{y}(l-n) \\ =& \sum_{n=-\infty}^{\infty} h_p(n)\tilde{x}(n-l) \end{aligned}$$

where $\tilde{x}(n-l) = (1+\beta)\bar{x}(n-l)$ and $\bar{x}(n-l)$ is the LMMSE estimate of $x(n-l)$ based on $\mathbf{y}(n-l)$ as given in (3), and β is the one-symbol SIR of (4).

As illustrated in Fig. 3 (and also in Fig. 1), the optimal linear smoother first obtains the LMMSE estimate of the channel in each pilot symbol based only on the received vector corresponding to that symbol, and then filters the resultant scalar random process to take advantage of correlations over time. The key to the decomposition is that the independence between the data symbols of the interfering users makes the interference white over time. The only structure in the interference is within each received symbol, and a symbol-by-symbol multiuser receiver is enough to fully exploit this structure. It should be noted that the situation is very different when we assume that the pilot symbols of all users are known, or if the data symbols of the interfering users are not independent over time.

2) Large System Performance: If we consider *Theorem 1* in conjunction with *Result 1*, then we arrive at the following theorem:

Theorem 2: Under the random spreading model with $N \to \infty$ and $\alpha = K/N$ held fixed, the MSE in the channel estimate

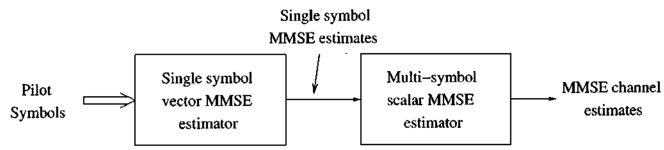

Fig. 3. Structure of optimal channel estimator.

at the output of the optimal linear smoother converges (almost surely) to the nonrandom value

$$\Delta_p^\dagger = 1 - \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{\beta^\dagger |S_{x_p x}(\omega)|^2}{1+\beta^\dagger S_x(\omega)} d\omega \tag{11}$$

where $\beta^\dagger$ is the limiting (one-symbol) SIR defined in (5).

Proof: Follows immediately from (10) and *Result 1* upon observing that Δ_p in (10) is a continuous function of β. ■

3) Spectral Density of Sampled Channel Process: The performance of the optimal linear smoother as given in (10) and (11) depends on the spectral densities, $S_x(\omega)$ and $S_{x_p x}(\omega)$. It is of interest to relate these to the power spectral density of the original channel process $\{a_1(m)\}$.

Recall that $R_x(n) = R_a(nL)$, so that the autocorrelation function of $\{x(m)\}$ results from decimating or down sampling the autocorrelation function of $\{a_1(m)\}$. The resultant power spectral density is

$$S_x(\omega) = \frac{1}{L}\sum_{m=0}^{L-1} S_a\left(\frac{\omega - m2\pi}{L}\right). \tag{12}$$

Similarly, we have

$$S_{x_p x}(\omega) = \frac{1}{L}\sum_{m=0}^{L-1} \exp\left(jp\frac{\omega - m2\pi}{L}\right) S_a\left(\frac{\omega - m2\pi}{L}\right). \tag{13}$$

These expressions can be substituted into (10) and (11) to make explicit the dependence of the MSE on the spectral density of the channel process $S_a(\omega)$, the rate of insertion of pilot symbols L, and the time shift p.

We can simplify things greatly by making some assumptions about $S_a(\omega)$ and L. In particular, suppose that

$$S_a(\omega) = 0, \qquad \omega_{ND} < |\omega| \le \pi$$

and that $L\omega_{ND} < \pi$. These conditions imply that there is no aliasing, so that

$$\begin{aligned} S_x(\omega) &= \frac{1}{L} S_a\left(\frac{\omega}{L}\right), \qquad |\omega| \le \pi \\ S_{x_p x}(\omega) &= \frac{1}{L}\exp\left(jp\frac{\omega}{L}\right) S_a\left(\frac{\omega}{L}\right), \qquad |\omega| \le \pi. \end{aligned}$$

This allows us to write

$$\begin{aligned} K_p(\omega) &= \exp\left(jp\frac{\omega}{L}\right)\frac{\frac{1}{L}S_a\left(\frac{\omega}{L}\right)}{1+\frac{\beta}{L}S_a\left(\frac{\omega}{L}\right)} \\ \Delta &= \frac{1}{2\pi}\int_{-\omega_{ND}}^{\omega_{ND}} \frac{S_a(\omega)}{1+\frac{\beta}{L}S_a(\omega)} d\omega. \end{aligned}$$

The impact of varying L on the MSE is clearly isolated in this expression. Also observe that the MSE does not depend on p, which means that the channel estimate in the middle of a frame is just as good as an estimate at a pilot symbol.

Note that the condition for no aliasing is $1/L > 2f_{ND}$, where $f_{ND} = \omega_{ND}/2\pi$. Thus, the faster the fading (the bigger f_{ND}), the more frequently we must insert pilot symbols (the smaller L).

C. Summary of Results

We now summarize the results presented in this section. To obtain the LMMSE estimate of $a_1(lL+p), 0 \le p \le L-1$, from observation of the entire sequence of pilot symbols $\{\mathbf{y}(n)\}$, we first form for all n

$$\tilde{a}_1(nL) = \sqrt{P}\mathbf{s}_1^* \left[P\mathbf{S}_1\mathbf{S}_1^* + \sigma^2\mathbf{I}\right]^{-1}\mathbf{y}(n).$$

Note that the MMSE estimate of $a_1(nL)$ from $\mathbf{y}(n)$ is $\tilde{a}_1(nL)/(1+\beta)$, where β is given in (4).

Secondly, we pass the sequence $\{\tilde{a}_1(nL)\}$ through a single-input, single-output (SISO) linear smoother to produce

$$\hat{a}_1(lL+p) = \sum_{n=-\infty}^{\infty} h_p(n)\tilde{a}_1(lL - nL).$$

The impulse response can be calculated from

$$h_p(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} K_p(\omega)e^{j\omega n} d\omega$$

where the frequency response of the smoother is

$$K_p(\omega) = \frac{S_{x_p x}(\omega)}{1+\beta S_x(\omega)}$$

and $S_x(\omega)$ and $S_{x_p x}(\omega)$ are defined in (12) and (13), respectively.

The resultant MSE is is given by (10).

In a large system with random spreading, β as given in (4) converges to $\beta^\dagger$ defined in (5), and all instances of β above can be replaced by $\beta^\dagger$. The MSE and the SISO linear smoother are thus independent of the signature sequences in a large system.

D. Special Cases and Results

In this section, we evaluate the performance of the optimal linear smoother for two channel power spectral densities.

Consider first the case when the channel is bandlimited and has a flat spectrum up to the normalized Doppler frequency ω_{ND}

$$S_a(\omega) = \begin{cases} \frac{\pi}{\omega_{ND}}, & |\omega| < \omega_{ND} \\ 0, & \omega_{ND} \ge |\omega| \le \pi. \end{cases} \tag{14}$$

Secondly, we will look at the Jakes' model where the power spectral density of the channel is given by

$$S_a(\omega) = \begin{cases} \frac{2}{\sqrt{\omega_{ND}^2 - \omega^2}}, & |\omega| < \omega_{ND} \\ 0, & \omega_{ND} \ge |\omega| \le \pi. \end{cases} \tag{15}$$

Fig. 4 shows the variation of the (normalized) MSE with frame size for two values of the time shift p. The normalized MSE is defined as $\Delta_p/R_a(0) = \Delta_p$. For these plots, the channel power spectral densities are as given in (14) and (15), with $\omega_{ND} = 0.02\pi$ and $\beta = 12$ dB. The condition for no aliasing in this case is $L < 50$.

Observe that for both channel spectra, the $p = 0$ and $p = L/2$ plots coincide up until $L = 50$. After this point, the $p = 0$ curve approaches $1/(1+\beta)$ (the MSE resulting when the estimation is based on a single pilot symbol), while the $p = L/2$ curve heads quickly toward a normalized MSE of 1 (which corresponds to a useless channel estimate).

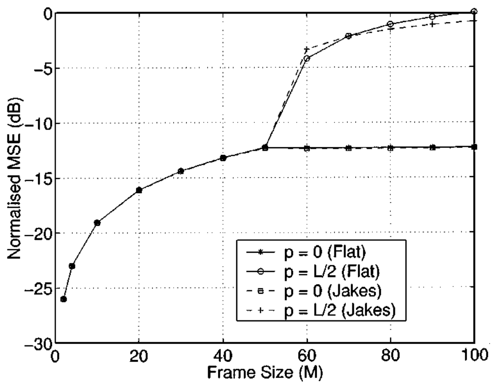

Fig. 4. Variation of (normalized) MSE $[\triangle_p/R_a(0)]$ with frame size (L) for estimation at pilot points ($p = 0$) and in the middle of a frame ($p = L/2$). Parameters are $\omega_{ND} = 0.02\pi$ and $\beta = 12$ dB.

Also observe that the performance of the channel estimator is very similar for both channel spectral densities, especially when there is no aliasing.

IV. Detection of Data Symbols

Consider our original signal model defined in (1) and repeated here for convenience

$$\mathbf{r}(m) = \sqrt{P}a_1(m)b_1(m)\mathbf{s}_1 + \sum_{k=2}^{K}\sqrt{P}a_k(m)b_k(m)\mathbf{s}_k + \mathbf{w}(m).$$

In this section, we consider the detection of $b_1(m)$, the data symbol of user one in symbol period m.

A. An Optimal Detector

Assume initially that $a_1(m)$ is known perfectly and that, as before, $a_2(m), \ldots, a_K(m)$ are independent, circularly symmetric, complex Gaussian random variables with zero mean and unit variance. With these assumptions, $\mathbf{r}(m)$ is a circularly symmetric, complex Gaussian random vector when conditioned on $b_1(m)$ and

$$E[\mathbf{r}(m)|b_1(m), a_1(m)] = \sqrt{P}a_1(m)b_1(m)\mathbf{s}_1$$
$$E[\mathbf{r}(m)\mathbf{r}^*(m)|b_1(m), a_1(m)] = P|a_1(m)|^2\mathbf{s}_1\mathbf{s}_1^* + P\mathbf{S}_1\mathbf{S}_1^* + \sigma^2\mathbf{I}.$$

This leads to the following result:

Theorem 3: The ML detector for $b_1(m)$, conditioned on $a_1(m)$, first forms the estimate

$$\hat{b}_1(m) = \sqrt{P}a_1^*(m)\mathbf{s}_1^*\left[P\mathbf{S}_1\mathbf{S}_1^* + \sigma^2\mathbf{I}\right]^{-1}\mathbf{r}(m)$$

and then chooses the data symbol from the M-ary PSK alphabet with phase closest to $\hat{b}_1(m)$.

The resultant symbol error probability is given by

$$P_M = \left(1 - \frac{1}{M}\right) - \frac{\sqrt{\beta}\sin(\frac{\pi}{M})}{\sqrt{1+\beta\sin^2(\frac{\pi}{M})}} \cdot \left[\frac{1}{2} + \frac{1}{\pi}\tan^{-1}\left(\frac{\sqrt{\beta}\cos(\frac{\pi}{M})}{\sqrt{1+\beta\sin^2(\frac{\pi}{M})}}\right)\right] \quad (16)$$

where β is the average SIR of (4).

Proof: The conditional density of $\mathbf{r}(m)$ is

$$p(\mathbf{r}(m) \mid b_1(m), a_1(m)) \propto \exp\Big[-(\mathbf{r}(m) - \sqrt{P}a_1(m)b_1(m)\mathbf{s}_1)^* \left(P|a_1(m)|^2\mathbf{s}_1\mathbf{s}_1^* + P\mathbf{S}_1\mathbf{S}_1^* + \sigma^2\mathbf{I}\right)^{-1} (\mathbf{r}(m) - \sqrt{P}a_1(m)b_1(m)\mathbf{s}_1)\Big]$$

where the constant of proportionality does not depend on $b_1(m)$. The ML detector thus maximizes the real part of

$$b_1(m)^*\left[\sqrt{P}a_1^*(m)\mathbf{s}_1^*\left(P|a_1(m)|^2\mathbf{s}_1\mathbf{s}_1^* + P\mathbf{S}_1\mathbf{S}_1^* + \sigma^2\mathbf{I}\right)^{-1}\mathbf{r}(m)\right]$$

which leads to the detector described above after application of the Matrix Inversion Lemma.

To determine the symbol error probability, we note that

$$\hat{b}_1(m) = |a_1(m)|^2\beta b_1(m) + v(m)$$

where

$$v(m) = \sqrt{P}a_1^*(m)\mathbf{s}_1^*\left[P\mathbf{S}_1\mathbf{S}_1^* + \sigma^2\mathbf{I}\right]^{-1} \cdot \left(\sum_{k=2}^{K}\sqrt{P}a_k(m)b_k(m)\mathbf{s}_k + \mathbf{w}(m)\right)$$

and β is defined in (4). Conditioned on $a_1(m)$, $v(m)$ is a circularly symmetric, complex Gaussian random variable with zero mean and variance $|a_1(m)|^2\beta$. The symbol error probability is thus exactly the symbol error rate for M-ary PSK modulation in Rayleigh fading with average SNR β (remember that $E[|a_1(m)|^2] = 1$). The required result can be found in [26]–[28]. ■

The ML detector thus first forms the LMMSE estimate of $b_1(m)$ and passes this estimate to the minimum distance detector. The key to optimality of the LMMSE receiver is that the data symbols of interfering users are absorbed by the unknown channels, and thus do not influence the likelihood function.

B. Proposed Detector

Suppose now that we drop the assumption that $a_1(m)$ is known at the receiver and instead assume that the receiver is given the entire sequence of pilot symbols and received signals at the pilot points.

Motivated by the above "optimal" detector and from practical considerations, we propose the following detector for $b_1(m)$, conditioned on the pilot information. The detector first forms

$$\hat{b}_1(m) = \sqrt{P}\hat{a}_1^*(m)\mathbf{s}_1^*\left[P\mathbf{S}_1\mathbf{S}_1^* + \sigma^2\mathbf{I}\right]^{-1}\mathbf{r}(m)$$

where $\hat{a}_1(m)$ is the LMMSE estimate of $a_1(m)$ given the pilot information. The detector then chooses the data symbol from the M-ary PSK alphabet with phase closest to $\hat{b}_1(m)$.

The pilot information is used to obtain the best possible linear estimate of $a_1(m)$ as given in Section III. Because the detector does not try to estimate the instantaneous power of the interfering users ($|a_k(m)|^2, k = 2, \ldots, K$), the front-end LMMSE receiver is time invariant. Note that both $\hat{a}_1(m)$ and $\hat{b}_1(m)$ require the same front-end multiuser receiver and the complete detector estimator can be implemented with the structure shown in Fig. 1.

C. Performance Analysis

Let us first examine the output of the front-end LMMSE receiver in Fig. 1. Denoting this output by $c_1(m)$, we have

$$\begin{aligned} c_1(m) =& \sqrt{P}\mathbf{s}_1^* \left[P\mathbf{S}_1\mathbf{S}_1^* + \sigma^2\mathbf{I}\right]^{-1}\mathbf{r}(m) \\ =& a_1(m)b_1(m)\beta + \nu(m) \end{aligned}$$

where

$$\nu(m) = \sqrt{P}\mathbf{s}_1^* \left[P\mathbf{S}_1\mathbf{S}_1^* + \sigma^2\mathbf{I}\right]^{-1} \cdot \left(\sum_{k=2}^{K} \sqrt{P} a_k(m) b_k(m)\mathbf{s}_k + \mathbf{w}(m)\right).$$

This quantity should be seen as the (scaled) LMMSE estimate of the product $a_1(m)b_1(m)$. Observe that $\nu(m)$ is a circularly symmetric, complex Gaussian random variable with $E[\nu(m)] = 0$ and $E[|\nu(m)|^2] = \beta$. This residual interference plus noise process is also white, in the sense that $E[\nu(m)\nu^*(n)] = 0$ for $m \neq n$. The finite dimensional distributions of $\{\nu(m)\}$ are not jointly Gaussian, however, and hence, the process is not Gaussian.

Suppose for the moment that the residual interference plus noise process is, in fact, Gaussian, it will then be an independent and identically distributed sequence of circularly symmetric, complex Gaussian random variables, each having mean zero and variance β. In this case, we are left with nothing other than a standard, single-user M-ary PSK system in a Rayleigh fading channel. Assuming that $m = lL + p$ with $1 < p < L$, we observe the following.

1) The LMMSE estimator of $a_1(m)$, given the pilot information $\{c_1(lL), b_1(1L) : -\infty < l < \infty\}$, is also the MMSE or conditional-mean estimator, and is precisely the estimator presented in Section III. The MSE, Δ_p, is also as given in Section III.
2) The optimal (ML) detector for $b_1(m)$ based on $c_1(m)$ and the pilot information would select the symbol from the M-ary PSK constellation with phase closest to $\hat{b}_1(m) = \hat{a}_1^*(m)c_1(m)$. This is exactly the detector proposed earlier.
3) The symbol error probability would be given by (16) with β replaced with the modified average SIR

$$\gamma = \frac{\beta(1-\Delta_p)}{1+\beta\Delta_p}. \tag{17}$$

Under the Gaussian assumption, the error probability is a monotonically decreasing function of the average SIR, γ, given in (17). Even when the residual interference plus noise process is not Gaussian, we will assume that γ is a useful performance measure. In the following section, we will look at maximizing γ over the rate of inserting pilot symbols.

We conjecture that, in a large system, the residual noise plus interference process does indeed converge to a Gaussian process. Note that [29] studies the convergence of the distribution of the residual interference at one point in time. We need to establish the convergence of all finite dimensional distributions to multivariate Gaussian distributions.

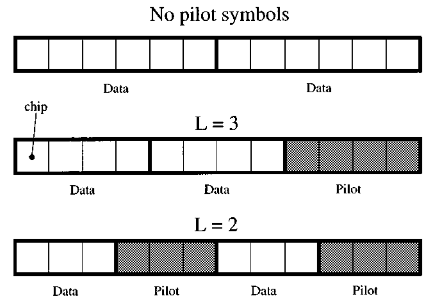

Fig. 5. Impact of frame size (L) on processing gain.

V. Pilot Symbol Optimization

We are now in a position to consider the optimization of the rate of inserting pilot symbols into the data stream of user one. We will assume that the power spectral density of the channel process $S_a(\omega)$ is bandlimited to $\omega_{ND} = 2\pi f_{ND}$, and that pilot symbols are inserted frequently enough to avoid aliasing ($2Lf_{ND} < 1$). Recall that under these conditions, the MSE in the channel estimate does not depend on the position of the data symbol in the frame.

The first question to consider is, what system parameters should be held constant as the frame size, L, is varied? It makes sense to demand that the total spread bandwidth remains fixed, so we will assume that the chip rate is constant. We would also like to fix the rate that each user transmits data symbols and the transmitted energy per data symbol.

As L is varied, the symbol period and transmitted energy per symbol will need to change (see Fig. 5). Let N_L, P_L and ω_L be, respectively, the spreading gain, the transmitted energy per symbol, and the normalized Doppler frequency with frame size L. Then $P_L = (1-1/L)P_\infty$, $N_L = (1-1/L)N_\infty$ and $\omega_L = (1-1/L)\omega_\infty$, and N_L, P_L, and ω_L all decrease with L. Note that the subscript ∞ corresponds to a reference system with no pilot symbols. We will drop the subscript for our reference system, and write $N = N_\infty$, $P = P_\infty$, and $\omega_{ND} = \omega_\infty$.

In this modeling framework, we are able to vary L while keeping the data rate of each user fixed along with the overall system bandwidth. As L decreases, however, the effective spreading gain decreases, and this lessens the ability of the receiver to mitigate interference.

Under the above assumptions, the average SIR of (17) is a function of L defined as

$$\gamma(L) = \frac{\beta_L(1-\Delta_L)}{1+\beta_L\Delta_L}$$

where

$$\Delta_L = \frac{1}{2\pi}\int_{-\omega_L}^{\omega_L} \frac{S_a(\omega)}{1+\frac{\beta_L}{L}S_a(\omega)}d\omega$$

and β_L is given in (4) with P replaced by P_L, and where the length of the signature sequences $(\mathbf{s}_k)$ is N_L.

In the large system limit, $\beta_L \to \beta_L^\dagger$ where

$$\beta_L^\dagger = \frac{1}{2}\left[(1-\alpha_L)\left(\frac{P_L}{\sigma^2}\right) - 1 + \left((1-\alpha_L)^2\left(\frac{P_L}{\sigma^2}\right)^2 + 2(1+\alpha_L)\left(\frac{P_L}{\sigma^2}\right) + 1\right)^{1/2}\right]$$

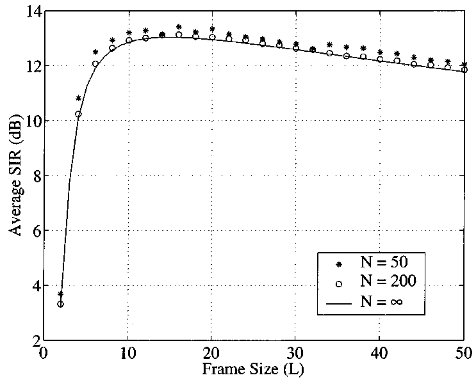

Fig. 6. Variation of average SIR (γ) with frame size (L). Parameters are $f_{ND} = 0.01, \alpha = 0.7,$ and $p/\sigma^2 = 20$ dB.

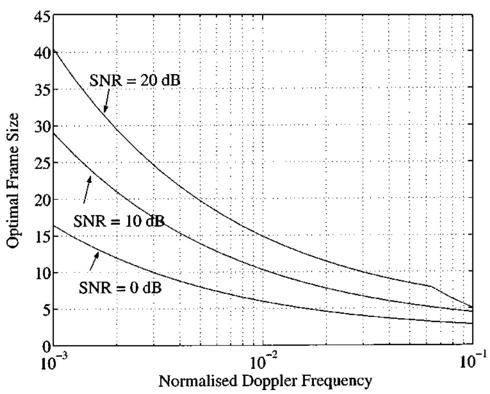

Fig. 7. Variation of optimal frame size with normalized Doppler (f_{ND}). For all plots, $\alpha = 0.7$.

and $\alpha_L = K/N_L = (1 - 1/L)^{-1}\alpha$. In a large system, we can thus express the average SIR of the proposed receiver as a function of the system loading and SNR of the reference system, the power spectral density of the channel process, and the rate of inserting pilot symbols.

To proceed further, we assume that $S_a(\omega)$ has the flat power spectral density given in (14) with ω_{ND} replaced by ω_L. In this case, the large system average SIR takes the simple form

$$\gamma^\dagger(L) = \beta_L^\dagger \left[1 + \frac{L\omega_L}{\pi}\left(1 + \frac{1}{\beta_L^\dagger}\right)\right]^{-1}.$$

Fig. 6 shows how $\gamma^\dagger(L)$ varies with L for some typical parameter values. The main feature of the curve is that performance is quite sensitive to L for values smaller than the optimum value, and relatively insensitive for larger values. Also shown in Fig. 6 are equivalent simulation points for finite size systems. These points are obtained by averaging 1000 realizations of the average SIR ($\gamma(L)$) over the random signature sequences. As the spreading gain increases, the mean value of $\gamma(L)$ is observed to approach the asymptotic value from above with close agreement even for moderate size systems ($N = 50$).

Fig. 7 illustrates the variation in the optimal value of L as a function of the speed of the fading (f_{ND}) and the SNR (P/σ^2). As one would expect, the optimal value of L decreases (more pilot symbols) as the fading rate increases. We also see that the optimal frame size increases significantly (less pilot symbols) as the SNR is increased.

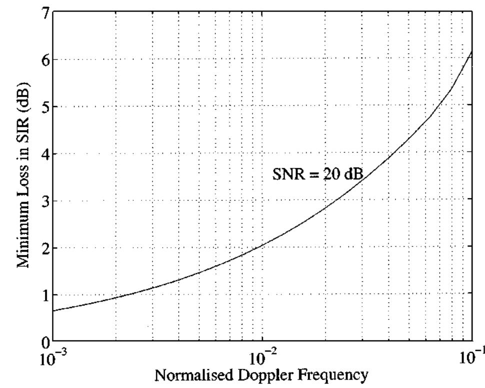

Fig. 8. Loss in average SIR from perfect knowledge case versus normalized Doppler (f_{ND}). For all plots, $\alpha = 0.7$.

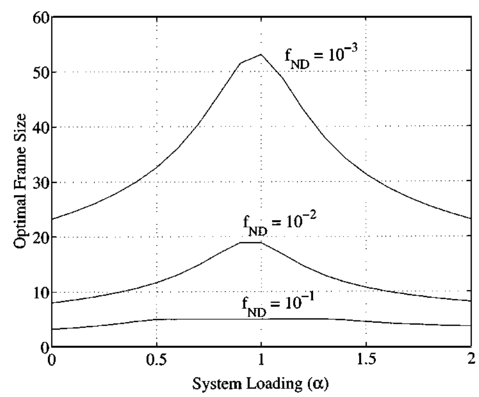

Fig. 9. Variation of optimal frame size with system loading (α). For all plots, $P/\sigma^2 = 20$ dB.

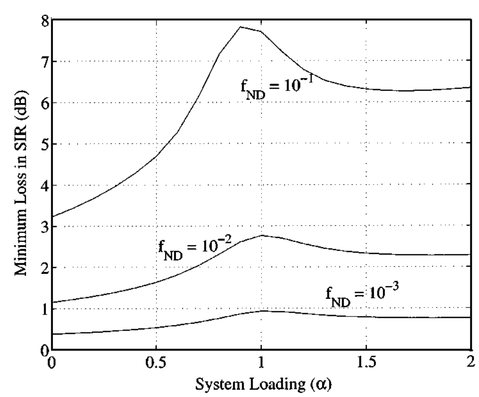

Fig. 10. Loss in average SIR from perfect knowledge case versus system loading (α). For all plots, $P/\sigma^2 = 20$ dB.

In Fig. 8 we examine the loss in average SIR for the optimized pilot scheme compared to the case when the channel of user one is known perfectly. The loss in average SIR is defined as the ratio of $\beta_\infty^\dagger$ (the average SIR with no estimation error and no pilot symbols) to $\gamma^\dagger(L)$. The loss was found to be quite insensitive to SNR, so we show the results for one value only. The performance loss is quite significant for normalized Doppler frequencies beyond 0.01.

In Figs. 9 and 10, some rather curious behavior is evident. Fig. 9 maps the variation in optimal L as a function of the system loading (α) and the normalized Doppler frequency. For all values of f_{ND}, the optimal frame size first increases and then decreases as α is increased, the turning point lying near $\alpha = 1$.

Increasing α increases the multiaccess interference, and we might guess that this would have the same effect as increasing the noise level or lowering the SNR. For $\alpha < 1$ this is clearly not the case. A larger system loading calls for a lower rate of insertion of pilot symbols. Of course, the key difference between the noise and interference is that the interference has structure, structure that our multiuser front end tries to exploit. An increase in L corresponds to an increase in effective processing gain and an increased ability to combat interference. The turning point on the plot occurs when the effective system loading $\alpha_L = 1$. It is at this value of system loading that the single-symbol SIR, β_L, is most sensitive to changes in system loading.

VI. Conclusion

We have considered a synchronous CDMA system where each user's signal propagates over a Rayleigh fading channel before reaching the receiver. In order to aid the coherent demodulation of the data symbols, prearranged pilot symbols are inserted into each user's data stream.

We assumed a receiver structure as in Fig. 1, where the front-end LMMSE receiver depends only on the signature sequences and average SNR of the users and is thus time invariant. The output of the LMMSE receiver is then treated as a single-user Rayleigh fading channel. The pilot symbols are used to obtain an estimate of the channel of the user of interest, and this estimate is passed to the data detector which makes decisions on the transmitted data symbols.

We derived an expression for the average SIR at the detector which involves the important system parameters, including the rate of inserting pilot symbols. The average SIR was then optimized over the pilot insertion rate for a range of system parameters. It was observed that more pilot symbols should be inserted as the SNR decreases, the fading rate increases, and as the system loading decreases (for loadings less than one).

Extension of this work to handle users with unequal average powers and asynchronous users is straightforward. A more challenging problem is proving the conjecture mentioned at the end of Section IV regarding the convergence of the residual noise plus interference process to a Gaussian process. We are currently tackling this problem.

Appendix
Optimal Linear Smoothing

In this appendix we give an important general result on optimal linear smoothing.

Let $\{\mathbf{x}(m)\}$ and $\{\mathbf{y}(m)\}$ be discrete time, zero mean, jointly wide-sense stationary, complex vector random processes. With $\mathbf{A}^*$ denoting the Hermitian transpose of a matrix $\mathbf{A}$, define the correlation matrices

$$\mathbf{R}_{\mathbf{y}}(n) = E[\mathbf{y}(m+n)\mathbf{y}^*(m)]$$
$$\mathbf{R}_{\mathbf{xy}}(n) = E[\mathbf{x}(m+n)\mathbf{y}^*(m)]$$

and the corresponding spectral density matrices

$$\mathbf{S}_{\mathbf{y}}(\omega) = \sum_{n=-\infty}^{\infty} \mathbf{R}_{\mathbf{y}}(n) e^{-j\omega n}$$
$$\mathbf{S}_{\mathbf{xy}}(\omega) = \sum_{n=-\infty}^{\infty} \mathbf{R}_{\mathbf{xy}}(n) e^{-j\omega n}.$$

We wish to estimate $\mathbf{x}(m)$ based on the entire observation sequence as

$$\hat{\mathbf{x}}(m) = \sum_{n=\infty}^{\infty} \mathbf{H}(n)\mathbf{y}(m-n).$$

The sequence of matrices $\ldots, \mathbf{H}(-1), \mathbf{H}(0), \mathbf{H}(1), \ldots$ is the impulse response of a multi-input, multi-output linear time-invariant (LTI) system. The frequency response of the system is

$$\mathbf{K}(\omega) = \sum_{n=\infty}^{\infty} \mathbf{H}(n) e^{-j\omega n}.$$

We wish to select the LTI filter so as to minimize the trace of the error covariance matrix

$$\Delta = E\left[(\mathbf{x}(m) - \hat{\mathbf{x}}(m))(\mathbf{x}(m) - \hat{\mathbf{x}}(m))^*\right].$$

We have the following result [30]–[32]:

A. Result 3

The LMMSE smoothing filter is the time-invariant filter

$$\mathbf{K}(\omega) = \mathbf{S}_{\mathbf{xy}}(\omega)\mathbf{S}_{\mathbf{y}}^{-1}(\omega)$$

and the corresponding error covariance matrix is

$$\Delta = \frac{1}{2\pi}\int_{-\pi}^{\pi} \left(\mathbf{S}_{\mathbf{x}}(\omega) - \mathbf{S}_{\mathbf{xy}}(\omega)\mathbf{S}_{\mathbf{y}}^{-1}(\omega)\mathbf{S}_{\mathbf{xy}}^*(\omega)\right) d\omega.$$

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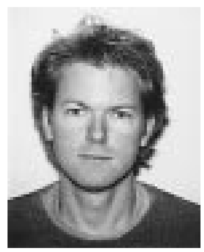

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