On the Teletraffic Capacity of CDMA Cellular Networks

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Abstract— **The aim of this paper is to contribute to the understanding of the teletraffic behavior of code-division multiple-access (CDMA) cellular networks. In particular, we examine a technique to assess the reverse link traffic capacity and its sensitivity to various propagation and system parameters.**

We begin by discussing methods of characterizing interference from other users in the network. These methods are extremely important in the development of the traffic models of later sections. We begin with a review of several existing approaches to the problem of handling other-cell interference before presenting a novel characterization of the interference in the form of an analytic expression for the interference distribution function in the deterministic propagation environment.

We then look at extending the capacity analyses that assume a fixed and equal number of users in every cell to handle the random nature of call arrivals and departures. The simplest way to do this is by modeling each cell of the network as an independent $M/G/\infty$ queue. This allows us to replace the deterministic number of users in each cell by an independent Poisson random variable for each cell. The resulting compound Poisson sums have some very nice properties that allow us to calculate an outage probability by analyzing a single random sum. This leads to a very efficient technique for assessing the reverse link traffic capacity of CDMA cellular networks.

I. Introduction

CODE-DIVISION multiple access (CDMA) is an alternative multiple-access strategy to frequency-division and time-division multiple access. Provided the synchronization and power control problems can be overcome, CDMA is a very attractive technique for wireless communications. Its advantages over other multiple-access schemes include higher spectral reuse efficiency, greater immunity to multipath fading, gradual overload capability, simple exploitation of sectorization and voice inactivity, and more robust handoff procedures [1], [2].

As early as 1978, a CDMA system had been proposed for mobile communications [3], however, interest was limited until Qualcomm demonstrated the feasibility of implementing such a system in the late 1980's [4]–[6]. Since then, there has been an explosion in CDMA research mainly concentrating on the design and performance analysis of receivers, coding and modulation techniques and power control algorithms. Higher layer issues such as call admission control, analysis of soft handoff, and the effects of gradual overload and imperfect power control on capacity have also begun to receive attention (see [7] and [8]). Yet to be properly examined, however, is the teletraffic behavior of cellular networks employing CDMA.

The traffic modeling of orthogonally channelized reuse-based cellular systems, such as those employing frequency-division or time-division multiple access, is well developed [9]. The behavior of networks employing fixed channel assignment and dynamic channel assignment has been studied and several approaches to analyzing handoff have been put forward. Much of the success in this area results from the separation of traffic analysis from transmission issues which allows the mobile network to be treated as a conventional circuit switched or open queueing network. Unfortunately, in CDMA the separation between traffic and transmission issues is not so clear with capacity being determined by the interference caused by all transmitters in the network.

The goal of this paper is to contribute to the development of a deeper understanding of the traffic behavior of CDMA cellular networks through the determination of analytic tools for performance analysis and design of these networks. Such an understanding is vital to sensible network operation under the stochastically varying loads that characterize teletraffic.

The paper is organized as follows. In Section II, the system structure and propagation models used throughout the paper are introduced. Section III examines methods of quantifying the interference produced by mobiles in other cells of the network. The main result is an analytic expression for the distribution function of the interference from a mobile whose position is a random variable in another cell of the network.

Most of the literature on traffic modeling of CDMA cellular networks is based on modeling each cell as an independent $M/G/\infty$ queue. This literature is reviewed in Section IV before a new model based on this assumption is presented in Section V. From the network operator's point of view, the model corresponds to a system where no calls are blocked and no calls are terminated prematurely. From a mathematical point of view, the number of users in each cell becomes a Poisson random variable and the total interference can be modeled as a compound Poisson sum. Methods for approximating tail probabilities associated with these sums are discussed in Section VI. These methods lead to a very efficient technique for assessing the reverse link traffic capacity of CDMA cellular

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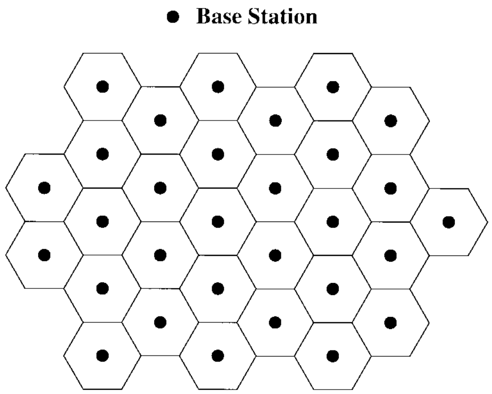

Fig. 1. Standard cellular layout.

networks and for investigating its sensitivity to propagation and system parameters. Numerical examples are presented in Section VII before the paper is concluded in Section VIII.

Before proceeding, it is important to highlight that the idea of modeling cells in a CDMA network by $M/G/\infty$ queues is not new and has indeed been proposed by several authors as discussed in Section IV. The main contribution of this paper is the concept of working with distribution functions of other-cell interference rather than just means and variances. This allows the Chernoff bound to be employed in the cellular context and provides an alternative to the Gaussian approximation. We also state and prove limit theorems for the case of a random number of mobiles in the system which demonstrate the asymptotic accuracy of the Chernoff bound and Gaussian approximation.

II. System and Propagation Issues

Throughout this work, we consider the standard uniform hexagonal layout as shown in Fig. 1 with a base station (BS) at the center of every cell. The forward and reverse links use disjoint frequency bands and can thus be analyzed independently. We only consider the reverse link as it is generally accepted to be the limiting factor in capacity calculations. In the sequel, all mention of path loss, signal to interference ratio (SIR), and capacity refers to the reverse link. We also note that we are solely concerned with direct-sequence CDMA systems.

Unless otherwise stated, a mobile connects to the BS that offers the least path loss at any given time. The chosen BS employs power control to maintain the received signal power at a constant level. We also assume the system is interference limited and that background noise is negligible. In real systems, the background noise provides the reference from which absolute signal powers are set.

Without loss of generality, we will work with normalized values of distance, power, and interference. In particular, all power and interference values are normalized to the fixed value of the target received signal power. Furthermore, all distances are normalized by the distance between closest BS's in the network of Fig. 1. Thus, the target received signal power is one (normalized) unit of power and adjacent BS's are separated by one (normalized) unit of distance.

The simplest model for the mobile radio channel is a propagation loss inversely proportional to the distance between the transmitter and the receiver raised to an exponent [10], [11]. If the transmitter and receiver are separated by d units, then the received power is given by

$$P_R(d) = P_T P_0 d^{-\gamma} \tag{1}$$

where P_T is the transmit power and P_0 and γ are independent of distance. P_0 is a function of carrier frequency, antenna heights, and antenna gains, and we assume it is constant for all paths between a mobile and a BS. γ is the path-loss exponent (PLE) which varies with antenna heights and is typically in the range two–six.

The simple model of (1) is accurate for distances from 1 to 20 km with BS antenna heights greater than 30 m and in areas with little terrain profile variation. Thus, the model is reasonable for conventional cellular systems in flat service areas but is not accurate in city microcells which employ small cells and low antennas.

Empirical results have illustrated that the deviation from (1) is normally distributed on a log–log plot [12, pp. 105–107]. The errors are due primarily to variations in terrain contour and to shadowing from buildings. Incorporation of this deviation, commonly called *lognormal shadowing*, leads to

$$P_R(d) = P_T P_0 d^{-\gamma} 10^{\zeta/10} \tag{2}$$

where P_0 and γ are as before and ζ is a zero-mean Gaussian random variable with standard deviation σ typically in the range six to twelve. $P_R(d)$ is now a random variable with lognormal density

$$f_P(z) = \frac{1}{\sigma' z\sqrt{2\pi}} e^{-(\ln z - \mu)^2/2\sigma'^2}$$

where $\mu = \ln P_T P_0 - \gamma \ln d$ and $\sigma' = \sigma \ln 10/10$.

The spatial correlation between shadowing random variables is significant over a distance of several meters [13] giving rise to a local mean over small areas. Another important propagation effect is a fast fading about this local mean. The fast fading is due to the arrival of several replicas of the signal with varying time delays and is characterized by a Rayleigh distribution for the received signal amplitude. The fading is basically independent over distances greater than half a carrier wavelength.

In this paper, we do not model multipath fading. It is generally assumed that the use of techniques such as interleaving, diversity reception and equalization, as well as the employment of a RAKE receiver, greatly mitigate fast fading. At any rate, it is reasonable to assume that the effects of the fast fading are encapsulated in the E_b/I_0 requirements of the system. This means that the propagation models used center on distance-driven path loss like (1) and the inclusion of lognormal shadowing as in (2).

III. Interference Characterization

A. Introduction and Review

In this introduction, we review several approaches to the characterization of interference in cellular CDMA networks. While simulation studies allow a great deal of complexity to be included we are solely concerned with analytical and numerical treatments of the problem.

The first paper to give an analysis of other-cell interference in spread spectrum mobile systems was [3]. Although the authors study a frequency hopped system their interference analysis in terms of SIR is applicable to direct-sequence systems. The propagation model of (1) is assumed, which coupled with perfect power control leads to a simple expression for the interference at the desired BS from a mobile station (MS) with known position in the network. The total interference from a cell other than the desired one is calculated by integrating the above interference expression mixed with a continuous and uniform user density over a circular region approximating an hexagonal cell. An analytic result is only possible for restricted values of the PLE and so the authors use numerical integration to calculate the interference levels. The overall other-cell interference results after summing the contributions from all interfering cells apart from the desired one. This paper does not deal with the randomness of the user locations and is equivalent to calculating expected values when each user is independently and uniformly distributed over the cell of concern.

In [14], a very similar analysis to the above is presented with the exception that the fixing of the PLE at four leads to analytic expressions for the interference from the circular cells. This is extended to an analytic result for the variance in [15].

An extension of [3] which includes the effects of shadowing and voice activity monitoring is found in [16]. A standard hexagonal cellular layout is assumed with the propagation model of (2) that includes lognormal shadowing taken to be independent on distinct paths. The total interference at a target BS is examined assuming that there are an equal number of users per cell (N) spread evenly and continuously over each cell. MS's are initially assumed to connect to the BS offering the least path loss. If this BS is the target then the interference is the fixed constant power specified by the power control, otherwise, the interference is a lognormal random variable with mean dependent on the position of the MS. To simplify the analysis, an MS decides between the closest BS (not including the target) and the target BS only. An expression for the interference dependent upon the MS position is then multiplied by the user density (not a probability density function) and integrated over the network to give the total other-cell interference. This total interference is a random variable due to the lognormal shadowing, and in the paper its mean and variance are calculated numerically as functions of the user density. Approximating the probability density function as a Gaussian, the other-cell interference is fully characterized.

An extension of the reverse link analysis of [16] is discussed in [17] and [18]. First, the propagation model is extended to take into consideration the dependence of the shadowing from an MS to different BS's. Second, rather than choosing between the target BS and the closest BS, an MS can connect to any of the nearest M BS's. This involves a fairly straightforward extension of the analysis in [16] although the computational complexity increases considerably to the extent that only mean values for the interference are calculated. The results show a dramatic drop in the mean other-cell interference from $M = 1$ to $M = 2$ for typical values of the shadowing variance, while the improvement is small for $M > 3$.

In [19], there is no modeling of shadowing, but more detailed and accurate versions of (1) are employed. The analysis assumes a circular target cell plus wedge-shaped adjacent cells, this geometry allowing a fairly simple investigation into the sensitivity of other-cell interference to user density profile variation. Power control errors are not examined and the results are all numerical.

In all of the above treatments, the randomness in user location within a cell has not been dealt with. Rather, some (usually uniform) continuous user density has been assumed and its product with an interference function integrated over the network. An alternative approach is to look at the interference as a function of a random position vector as in [20], where the MS location is assumed uniformly distributed over each cell. In this paper, however, only the mean and variance of the interference are required since a Gaussian approximation is used. This means the treatment is identical to [16], and it is only because of the slightly different angle taken that it is mentioned here.

Section III-B discusses a novel method of characterizing other-cell interference in CDMA cellular networks. As with the above work, Rayleigh fading is not studied and perfect power control is assumed. To begin, we work with the propagation model of (1). Given uniformly distributed users and circular approximation of the hexagonal cells, the distribution function of the interference from a MS in another cell is calculated analytically as a function of the PLE and the location of the cell. The calculation of this distribution function compares to [14] and [15], which assume a similar geometry and propagation model yet only derive an expression for the mean and variance respectively for a fixed PLE of four. The results can be extended to include lognormal shadowing similarly to [16], however, unlike it, an expression for the distribution function of the interference is constructed. This distribution function must be calculated numerically. Our numerical examples deal exclusively with the deterministic path-loss model, however, for completeness the details for the model including lognormal shadowing are given in Appendix II.

B. Deterministic Path Loss

Consider the situation shown in Fig. 2. Note that all coordinates and distances are normalized to the distance between adjacent BS's as discussed in Section II. An MS is located at (x, y) within a hexagonal cell of the standard two-dimensional (2-D) layout of Fig. 1. The MS is connected to the BS with coordinates $(0, 0)$ and causes interference to the BS at $(-a, 0)$. Based on the power control assumptions and (1), the (normalized) interference is

$$I(x,y) = \left(\frac{x^2+y^2}{(x-a)^2+y^2}\right)^{\gamma/2} \tag{3}$$

and we would like to be able to calculate the distribution function of the random variable $I(X, Y)$ given the joint distribution function $F_{X,Y}$ of the random variables X and Y.

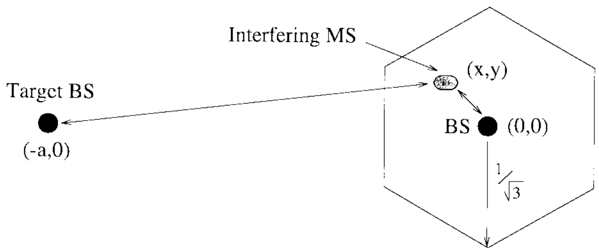

Fig. 2. Interference in 2-D network.

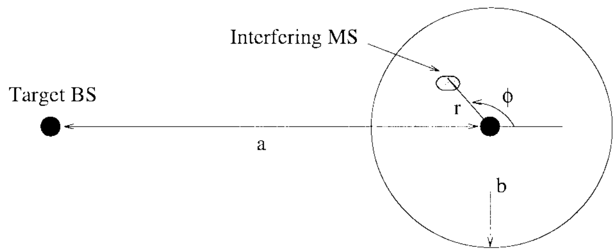

Fig. 3. Interference in 2-D network—approximation.

This is in general a complicated problem, so to simplify matters let us assume the joint density of X and Y is uniform over the hexagon. Due to the large number of possible orientations of the hexagonal cell and to the dependence of X and Y, the analysis remains exceedingly tedious. These problems can be eradicated by approximating the hexagonal cells by circles of (normalized) radius b as shown in Fig. 3. The orientation difficulty clearly vanishes and by having a and b as parameters, a great deal of flexibility results.

The derivation is carried out in the Appendix I and leads to the distribution function of the interference received at $(-a, 0)$ from an MS that has a uniformly distributed location within the circle of radius b and center the origin. The distribution function is given below

$$F_I^{a,b}(z) = \begin{cases} 0, & z < 0 \\ g_1(z), & 0 \le z < \left(\frac{a}{b}+1\right)^{-\gamma} \\ g_2(z), & \left(\frac{a}{b}+1\right)^{-\gamma} \le z < \left(\frac{a}{b}-1\right)^{-\gamma}, \quad z \neq 1 \\ g_3(z), & 1 < \left(\frac{a}{b}-1\right)^{-\gamma}, \quad z = 1 \\ 1, & \left(\frac{a}{b}-1\right)^{-\gamma} \le z \end{cases} \tag{4}$$

where

$$g_1(z) = \frac{a^2 z^{2/\gamma}}{b^2(z^{2/\gamma}-1)^2}$$

$$g_2(z) = \frac{1}{\pi}\arccos(h_1(z)) + \frac{1}{\pi}g_1(z)\Big[\pi - 2z^{2/\gamma}h_1(z)h_2(z) - \frac{a}{b}z^{2/\gamma}h_2(z) - \arccos(h_1(z)) - \arcsin(z^{1/\gamma}h_2(z))\Big]$$

$$g_3(z) = \frac{1}{\pi}\arccos\Big(-\frac{a}{2b}\Big) + \frac{a}{4\pi b^2}\sqrt{4b^2-a^2}$$

and

$$h_1(z) = \frac{-a^2 - b^2 + b^2 z^{-2/\gamma}}{2ab}$$

$$h_2(z) = \sqrt{1-h_1^2(z)}.$$

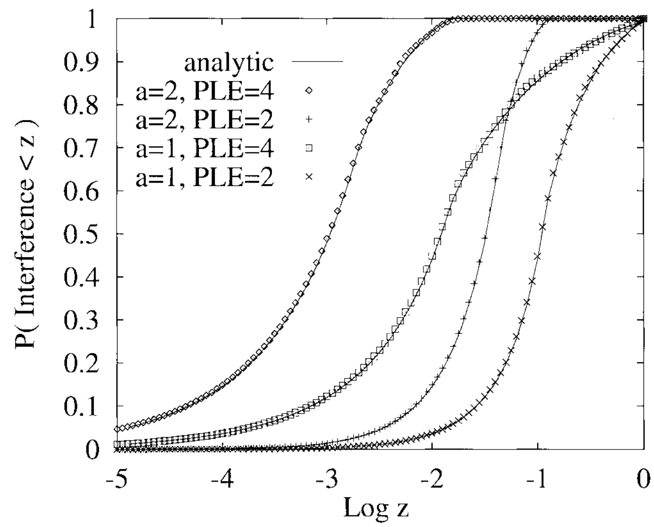

Fig. 4. Interference: approximation versus simulation.

In all future numerical calculations, b is chosen such that the areas of the hexagonal cell and approximating circle are equal which gives $b \approx 0.53$. Using this value for b, we compare the approximate analytic distribution function with that obtained via Monte Carlo simulation for the hexagonal cell in Fig. 4. The points shown on the graph are obtained from simulation while the solid lines are the corresponding distribution functions from (4). As the values of a and PLE are varied, the analytic approximation remains in good agreement with simulation.

IV. The $M/G/\infty$ Approximation

A. Introduction and Review

In this section, we present a technique that allows the teletraffic capacity of CDMA cellular networks to be estimated. The simplicity of the technique arises from modeling each cell of the network as an independent $M/G/\infty$ queue and consequently the theory of this section provides no input to the understanding of how calls should be admitted to the system. A more advanced network model which does impact on call admission control schemes is presented in [21] and [22].

We begin with a review of several papers [20], [23]–[25] that employ the $M/G/\infty$ approximation and compare them to the model presented in this paper and in [26]. Note that in these papers the generally distributed holding times are replaced by holding times with a negative exponential distribution giving $M/M/\infty$ queues. However, all their results apply in the general case since it is only the stationary distribution of the number of mobiles in the system that is used.

The first of the above papers to appear was [23]. The paper looks only at the limiting reverse link and has as its aims the development of a model that deals with variability in the number of users per cell, voice activity, and variable E_b/I_0 requirements. Concentrating on a single cell (or sector), the authors assume that no new call requests are denied and as such model the cell as an $M/M/\infty$ queue. The number of users in the cell k is thus modeled as a random variable with a Poisson distribution having mean equal to the cell offered traffic. Voice activity is included simply by assuming each mobile is gated on with probability ρ and off with probability $1-\rho$. Voice activity is thus modeled by a Bernoulli random variable ν.

Based on fitting of empirical data on the received E_b/I_0 values required to maintain frame error rates below 1%, the authors model the E_b/I_0 required by each mobile as a lognormal random variable. Since the data involves actual received values required for acceptable performance it must include the effects of imperfect power control and varying propagation conditions.

With the above aims achieved, the authors essentially define blocking to occur when the instantaneous E_b/I_0 requirements of all users cannot be met. The blocking probability is bounded for a range of cell offered traffics using a modified Chernoff bound and compared to results from a Gaussian approximation and simulation. Based on this comparison and the relative ease with which it is calculated, the Gaussian approximation is used in the extension to multiple cells.

The first assumption of the extension is that the number of users in each cell remains equal. The second assumption is that the k users in every outer cell produce a combined interference equivalent to kf users in the inner cell where f is an expected outer-cell interference fraction obtained from [17]. Accepting the assumptions the multiple cell case reduces to the single cell problem with an equivalent number of active mobiles of $(1+f)k$ where as before k is a Poisson random variable. The analysis in the cellular situation is thus a combination of a dynamic single cell capacity analysis with the static, multiple cell capacity results of [17].

In [24], a computationally intensive procedure is presented for the evaluation of the teletraffic capacity of both forward and reverse links in a CDMA cellular system. Each cell is modeled as an independent $M/M/\infty$ queue and the quality of service (QoS) criterion evaluated is the outage probability or the probability that the SIR of a link is below a certain threshold. A uniform hexagonal layout, a uniform density for the mobile location within each cell, and a propagation model including lognormal shadowing are other features of the model presented. The main disadvantage of the approach presented in this paper is the extreme computational effort required and it is debatable whether the approach is any more valuable then a straight out simulation.

In [25], a teletraffic model of the reverse link is considered. The assumptions include uniform hexagonal layout, equal traffic offered to every cell, uniform density for mobile locations within cell, two layers of interfering cells considered, deterministic propagation loss only, and perfect power control of received signal strength.

Despite the initial discussion of a model including a finite number of modems, trunk reservation for handovers, and mobility, the subsequent analysis does not allow for mobility and assumes traffic levels which reduce the new call blocking probability to a negligible figure. Thus, the system is actually modeled as a network of independent $M/M/\infty$ queues. Once more the QoS measure concerns the probability of the SIR being below a given threshold which with the assumption that each mobile is received at a fixed power level involves calculating the probability that the interference gets too large. The total interference is calculated as the sum of inner-cell interference and outer-cell interference. In line with the $M/M/\infty$ assumption, the contribution from within the desired cell is taken as a Poisson random variable with mean equal to the cell offered traffic. The outer-cell interference is approximated as a Gaussian random variable with mean and variance obtained via simulation. A numerical convolution of the Gaussian and Poisson densities then leads to the density function for the total interference. The traffic capacity corresponding to two QoS values is presented as a function of the PLE of the propagation model.

Many of the assumptions of [20] are as in [25]. Only the reverse link is considered, the QoS is based on a minimum SIR requirement, perfect power control of received signal strength is assumed, and each cell acts as an independent $M/M/\infty$ queue. The internal and external interference are again treated separately—the internal a Poisson random variable and the external a Gaussian random variable. The mean and variance of the external interference are calculated by analytical and numerical methods based on the treatment in [16], which includes lognormal shadowing in the propagation model. The blocking or outage probability is then given in the form of a convolution as in [25].

The analysis of reverse link traffic capacity for CDMA cellular networks developed in this paper and in [26] shares many of the features of the above papers. In particular, we employ the independent $M/G/\infty$ queue model for each cell, the service requirement is in terms of SIR, and each mobile is power controlled to a fixed and equal power.

In the most general development [26], arbitrary network layouts, user distributions, and traffic profiles are allowed, and lognormal shadowing is included in the propagation model. If, however, a symmetric structure is imposed, the calculation of the service measure reduces to evaluating the probability of a compound Poisson sum exceeding a certain threshold. If the propagation model does not include shadowing, an analytic expression is available for the distribution of the random summands.

The service measure is approximated using a standard Gaussian approximation and bounded with the Chernoff bound and results are presented for various propagation environments and system bandwidths.

The approach in [20] is closest in spirit to this work, but differs in several aspects. First, [20] treats internal and external interference separately thereby requiring a numerical convolution at the last step. This is avoided in our approach where there is no distinction made. Second, they give no analysis of, or justification for, the Gaussian approximation while we prove a central limit result for compound Poisson sums. Third, our analysis is strengthened with the use of the Chernoff bound and an illustration of its asymptotic accuracy. Finally, we present several results that explore how the service quality varies with the offered traffic per cell, system bandwidth, and PLE. Such results are not given in any of the above papers.

B. Model Description

In this section, we discuss the assumptions leading to, and justification for, modeling the CDMA cellular network as a collection of independent $M/G/\infty$ queues. The assumptions of the traffic model we will use are as follows.

- The call initiation processes in each cell are modeled as independent Poisson streams.
- All arriving calls are accepted into the network and remain in the network for the full call duration (no blocked or dropped calls).
- Call durations are generally distributed and independent of the arrival processes and other holding times.
- Mobility is not modeled, and, thus, the mobile is associated with the cell of its call initiation for the duration of the call.

The first and third points are standard assumptions from teletraffic engineering that have been employed for several decades to model the stochastic nature of call arrivals to telephone exchanges and their circuit holding times. The second assumption is reasonable for systems operating with CDMA since there is no theoretical hard limit on the number of quasi-orthogonal codes available to assign to users. From a more practical point of view, it is reasonable to assume that there are enough codes available so that the new call blocking probability is negligible for moderate offered traffic. The final assumption is a good approximation when the cell size is large compared to the distance a typical mobile will travel during a call.

The assumptions imply that each cell of our network behaves like an independent $M/G/\infty$ queue [27]. This is one of the most basic queueing models and has a particularly simple form for the steady-state distribution of the number of active calls. If the mean time between call arrivals is $1/\lambda$s and the mean call holding time is $1/\mu$s, then the traffic to the system is $A = \lambda/\mu$ Erlangs. Let N be the random variable representing the number of active calls in the system at steady state. Then, N has the Poisson distribution

$$P(N = n) = e^{-A}\frac{A^n}{n!}.$$

The independence of each cell in the network implies that the joint steady-state distribution for the number of active calls in each cell is simply a product of Poisson distributions.

C. Inclusion of Voice Activity Effects

Now let us suppose that once a mobile call is connected to the network the mobile user is ON with probability ψ and OFF with probability $1 - \psi$. This model results when voice activity monitoring is included and the subsequent suppression of transmission by a mobile after voice inactivity is detected. We are now interested not in the number of mobiles connected to a BS, but in the number of mobiles in a cell that are ON. Let this number be M. We have [20]

$$\begin{aligned} P(M = m) &= \sum_{n=0}^{\infty} P(M = m \mid N = n)\, P(N = n) \\ &= \sum_{n=m}^{\infty} \frac{n!}{m!(n-m)!}\psi^m (1-\psi)^{n-m} e^{-A}\frac{A^n}{n!} \\ &= e^{-\psi A}\frac{(\psi A)^m}{m!} \end{aligned}$$

which is again Poisson distributed but with reduced traffic load ψA. Thus, the gains from voice activity detection enter the formulation in a simple multiplicative manner.

Before proceeding to the next section, we make one final point. In the rest of this paper, it is assumed that the traffic offered to every cell of the network is equal. We emphasize that this equality applies to the parameters of a stochastic model and is distinctly different to assuming an equal static load in every cell. This along with the infinite, symmetrical, cellular layouts, and uniform user distributions that we have assumed allows all calculations to be performed for one cell of the network only. The extension of this work to asymmetrical layouts, offered traffic, and user distributions is straightforward from a theoretical point of view [26] and is not included here.

V. Outage Probability: Definition

In this section, we develop a simple expression for a QoS indicator which we call the outage probability. Calculation of the outage probability reduces to the evaluation of the probability that a compound Poisson random variable exceeds a given threshold. The analysis of such an expression is left to the following section.

A. Definition

The outage probability is defined as the probability that a mobile achieves an insufficient SIR. We recall that a similar performance measure is called blocking probability in [23], however, we prefer to use the term outage so as not to confuse this performance measure with that related to blocking of new call requests.

To calculate the outage probability B, we must determine the stationary probability that an arbitrary mobile anywhere in the cellular network receives a reverse link SIR that is insufficient for acceptable QoS. If certain symmetries exist, then B will be the same for mobiles at any point in the network and we may just as well consider calls that are connected to a particular BS. Associate with this target BS and its cell the index 1.

Because of the standard power control assumptions, mobiles are received at BS_1 with one unit signal power. We can thus easily translate the SIR requirement into a constraint on the total interference at BS_1. That is,

$$B = P(I > \Gamma) \tag{5}$$

where I is a random variable representing the total power received at an arbitrary BS in the network. Γ is a measure of the *capacity* of the CDMA system and is related to the system bandwidth (W Hz), the data rate (R bps), and the required bit energy to interference density ratio (E_b/I_0) by [16]

$$\Gamma = \frac{W/R}{(E_b/I_0)}.$$

B. Interference

Suppose that there are $M-1$ cells apart from cell$_1$ that generate significant interference at the target BS with labels $2,\cdots,M$. Assume that the interference random variables for mobiles in cell$_i$ are independent and identically distributed (iid) with distribution function F_i and the interference random variables from different cells are independent. Remember that for our model the possible sources of randomness in the interference include location, shadowing and voice activity. In the example of Section V-C and in the numerical results of Section VII we focus on randomness due to position only with voice activity readily incorporated as discussed in Section IV-C.

Given N_i calls in cell$_i$, the total power received at BS$_1$ is given by

$$I = \sum_{i=1}^{M}\sum_{j=1}^{N_i} I_{ij}$$

where I_{ij} is the interference from the jth mobile in cell$_i$ and the N_i are independent Poisson random variables with mean A.

It is readily shown using characteristic functions that

$$I \doteq \sum_{j=1}^{N} I_j \tag{6}$$

where N is a Poisson random variable with parameter MA and I_j is a random variable with distribution function $M^{-1}\sum_{i=1}^{M} F_i$ being a finite mixture of the original distribution functions. The symbol $\doteq$ indicates equality in distribution. In (6), the interfering cells are combined and the total traffic into the conglomeration considered.

Combining (6) and (5), we arrive at a simple expression for the outage probability in the network

$$B = P\left(\sum_{j=1}^{N} I_j > \Gamma\right). \tag{7}$$

We now present an example to illustrate and clarify the ideas of the last sections.

C. Example

Consider the standard 2-D layout of Fig. 1 with A Erlangs of traffic offered to each cell. Assume the mobile locations within each cell are iid random variables uniformly distributed over each cell and that the propagation environment is governed by (1). The interference resulting at some target BS from a mobile randomly located in a cell of the network is characterized by the approximate distribution function of (4). If only the target cell and the first two surrounding rings of cells are taken to contribute significantly to the total interference at the target BS we have $I = \sum_{j=1}^{N} I_j$, where N is a Poisson random variable with mean $19A$ and the I_j are a sequence of iid random variables with distribution function

$$F(z) = 19^{-1}\big(u(z-1) + 6F_I^{1,b}(z) + 6F_I^{\sqrt{3},b}(z) + 6F_I^{2,b}(z)\big). \tag{8}$$

In the above, $u(z)$ is the unit step function and $F_I^{a,b}(z)$ are given in (4).

To calculate the outage probability as a function of the offered traffic per cell we are faced with evaluating (7). Two methods of approximating this probability are described in the next section.

VI. Outage Probability: Approximations

In this section, we consider techniques for approximating

$$P\left(\sum_{j=1}^{N_A} X_j \geq \Gamma\right)$$

where $X_1, X_2, \cdots$ are iid random variables and N_A is a Poisson random variable with mean A that is independent of the X_j.

Let us define $S_{N_A} = \sum_{j=1}^{N_A} X_j$ and denote the mean and variance of S_{N_A} by $E[S_{N_A}] = \mu_S$ and $\mathrm{var(S_{N_A})} = \sigma_\mathrm{S}^2$, respectively. Then, if $E[X_1] = \mu_X$ and $\mathrm{var(X_1)} = \sigma_\mathrm{X}^2$

$$\mu_S = A\mu_X$$
$$\sigma_S^2 = A(\sigma_X^2 + \mu_X^2).$$

A. Normal Approximation

1) The Approximation: The normal approximation is

$$\frac{S_{N_A} - \mu_S}{\sigma_S} \doteq \mathcal{N}(0,1)$$

where $\mathcal{N}(0,1)$ is a zero-mean unit variance normal random variable. We thus have

$$P\left(\sum_{j=1}^{N_A} X_j \geq \Gamma\right) = P\big(S_{N_A} \geq \Gamma\big) \approx 0.5 - \mathrm{erf}\left(\frac{\Gamma - \mu_\mathrm{S}}{\sigma_\mathrm{S}}\right)$$

where

$$\mathrm{erf(x)} = \frac{1}{\sqrt{2\pi}}\int_0^\mathrm{x} \mathrm{e}^{-\mathrm{t}^2/2}\,\mathrm{dt}.$$

2) Asymptotic Behavior: Integral A: To examine the asymptotic behavior of this approximation as $A \to \infty$, we first assume that A takes nonnegative integral values only. Since the sum of Poisson random variables is also Poisson, we have

$$S_{N_A} = \sum_{j=1}^{N_A} X_j \doteq \sum_{j=1}^{A} Y_j$$

where $Y_1, Y_2, \cdots, Y_A$ are iid random variables and

$$Y_1 \doteq \sum_{j=1}^{N_1} X_j$$
$$E[Y_1] = \mu_Y = \mu_X$$
$$\mathrm{var(Y_1)} = \sigma_Y^2 = \sigma_X^2 + \mu_X^2.$$

The sum with a random number of summands has been transformed into a standard deterministic sum of iid random variables for which we can apply the central limit theorem (CLT) in its simplest form. The CLT [28] states that provided μ_Y is finite and σ_Y^2 is positive and finite

$$\frac{S_{N_A} - A\mu_Y}{\sigma_Y\sqrt{A}} = \frac{S_{N_A} - \mu_S}{\sigma_S} \Rightarrow \mathcal{N}(0,1) \quad \text{as } A \to \infty.$$

The symbol $\Rightarrow$ refers to convergence in distribution.

3) Asymptotic Behavior: Real A: We now consider the case when A is a positive real. As no appropriate result could be found in the literature we prove the required CLT here using characteristic functions. For convenience, we use the following notation:

$$E[X_1] = m_1$$
$$E[X_1^2] = m_2^2$$
$$\sqrt{A} = \alpha$$

so that $\mu_S = Am_1$ and $\sigma_S^2 = Am_2^2$.

Theorem 1: If m_1 is finite and m_2 is positive and finite, then

$$\frac{S_{N_A} - Am_1}{\alpha m_2} \Rightarrow \mathcal{N}(0,1) \quad \text{as } A \to \infty.$$

Proof: Let $Z_A = \frac{S_{N_A} - Am_1}{\alpha m_2}$, with

$$\begin{aligned}\phi_{Z_A}(t) &= E[e^{itZ_A}] \\ &= e^{-i\alpha m_1/m_2} e^{A(\phi_X(t/\alpha m_2)-1)}\end{aligned}$$

where $\phi_X(s) = E[e^{isX_1}]$. This follows by conditioning and use of elementary properties of characteristic functions.

Because $m_2^2 < \infty$

$$\phi_X(t/\alpha m_2) = 1 + \frac{itm_1}{\alpha m_2} - \frac{t^2}{2A} - \frac{t^2}{2Am_2^2}\epsilon_2(t/\alpha m_2)$$

where $\epsilon_2(s) \to 0$ as $s \to 0$. Thus

$$\phi_{Z_A}(t) = e^{-t^2/2} e^{\frac{t^2}{2m_2^2}\epsilon_2(t/\alpha m_2)}.$$

Fix $t \in (-\infty, +\infty)$ so that

$$\begin{aligned}\lim_{A\to\infty} \phi_{Z_A}(t) &= e^{-t^2/2} \lim_{A\to\infty} e^{\frac{t^2}{2m_2^2}\epsilon_2(t/\alpha m_2)} \\ &= e^{-t^2/2}\end{aligned}$$

by continuity of the exponential at zero.

Finally, by the Continuity Theorem for characteristic functions $Z_A \Rightarrow \mathcal{N}(0,1)$ as required. □

B. Large Deviations Bound

We now give an upper bound on the outage probability using the Chernoff bound. The asymptotic behavior of this bound is discussed in the context of elementary large deviations theory.

Consider first the case when A takes on positive integer values and the Poisson sum can be rewritten as the deterministic sum $S_{N_A} \doteq \sum_{j=1}^{A} Y_j$. The large deviation rate function is defined by

$$I(t) = \sup_\theta [\theta t - M_Y(\theta)]$$

where θ is real and

$$M_Y(\theta) = \log E[e^{\theta Y_1}] = e^{M_X(\theta)} - 1$$

is the log moment generating function (LMGF) of the Y_j which is related as shown to the LMGF of the X_j, $M_X(\theta)$.

Provided $M_Y(\theta) < \infty$ for all θ and that Y_1 is not a bounded random variable in the sense that $P(Y_1 \in (a,b)) < 1$ for all finite a and b, then from Cramer's Theorem [29]

$$\lim_{A\to\infty} \frac{1}{A} \log P\left(\frac{1}{A} S_{N_A} \geq \eta\right) = -I(\eta) \tag{9}$$

for $\eta > \mu_Y$. Moreover, for all positive integral A

$$\frac{1}{A} \log P\left(\frac{1}{A} S_{N_A} \geq \eta\right) \leq -I(\eta).$$

The above bound is commonly called the Chernoff bound and is directly applicable for any positive real value of A. To extend the limit result of (9) to the case when A is real is more involved, but is readily accomplished either by modified use of Cramer's Theorem or by direct application of the more powerful Gartner–Ellis Theorem [30], [31].

Applying the above to our problem, we have

$$\begin{aligned}\frac{1}{A} \log P(S_{N_A} \geq \Gamma) &\leq -I\left(\frac{\Gamma}{A}\right) \\ &= \inf_\theta \left[M_Y(\theta) - \theta\frac{\Gamma}{A}\right] \\ &= \inf_\theta \left[e^{M_X(\theta)} - 1 - \theta\frac{\Gamma}{A}\right]\end{aligned}$$

and from Cramer's Theorem the bound becomes tight as $A \to \infty$ with Γ/A held constant.

VII. Numerical Examples

In this section, we use our previous results to examine the traffic performance of our CDMA cellular system. After describing the network models used, we compare calculated outage probabilities by simulation with both the Chernoff bound and Gaussian approximation for some representative cases. The variation in performance with both PLE and system size (Γ) is then investigated.

A. Network Model

The network model used in calculations is as in Section V-C. In particular, the distribution function of the interferers is given by (8) with $b = 0.53$. It should be remembered that this distribution function is implicitly dependent on the PLE.

In what follows, we consider outage probabilities in the range 0.01% to 10% with the offered traffic limits altered to produce this range for each scenario considered. The two main parameters we have to vary are the PLE and Γ. The PLE used lies in the set $\{2, 3, 4, 5\}$ with four being a typical value for existing macrocellular systems. Γ takes values in $\{20\,100\,500\}$ which might correspond to systems with $E_b/I_0 = 7$ dB, $R = 10$ kbps, and $W = 1$, 5, and 25 MHz, respectively.

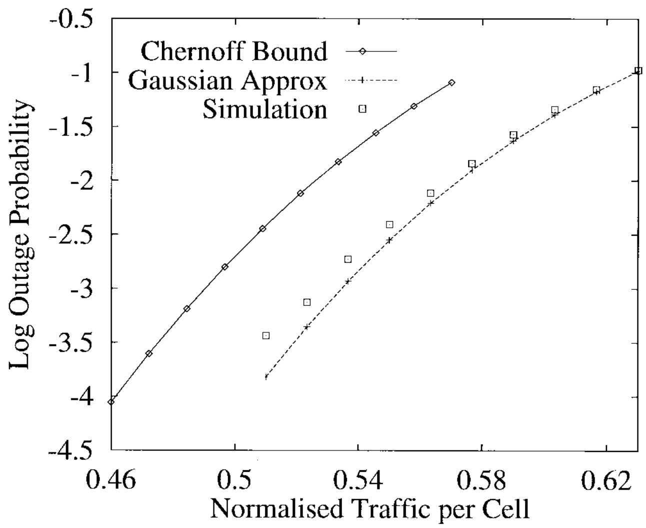

Fig. 5. Outage probability versus normalized traffic per cell ($\Gamma = 100$, $\text{PLE} = 4$).

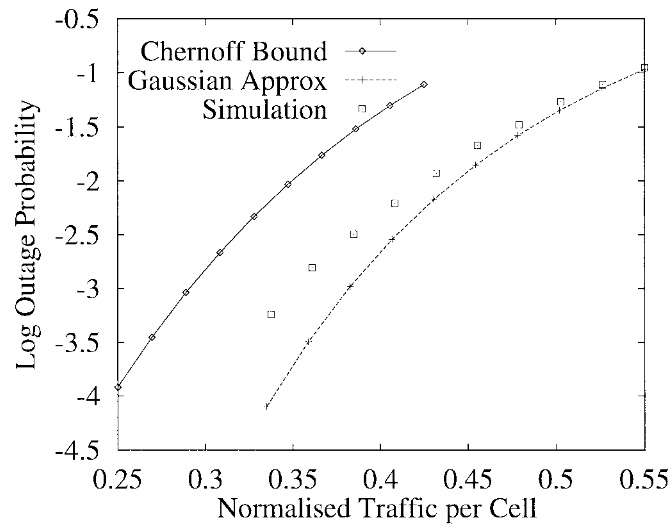

Fig. 6. Outage probability versus normalized traffic per cell ($\Gamma = 20$, $\text{PLE} = 4$).

In all plots, the ordinate represents the base 10 logarithm of outage probability while the abscissa corresponds to the offered traffic per cell divided by Γ. The traffic axis is thus normalized by the size of the system making capacity comparisons for different Γ straightforward. Simulation points are accurate to within plus or minus 20% with 95% confidence.

For a given set of system parameters and an offered traffic value, the simulation point is generated directly using a Monte Carlo technique. This involves repeatedly generating a random (Poisson) number of users for each interfering cell and a random location (uniform over each cell) for each mobile. In each trial, the total interference at the target BS is determined from which the outage condition can be checked. The simulated outage probability is then obtained by taking the ratio of the number of outage events to the total number of trials.

B. Comparison of Bound and Approximation with Simulation

In Figs. 5 and 6, we compare the Chernoff bound and Gaussian approximation to simulation for the 2-D network with $\text{PLE} = 4$, $\Gamma = 100$, and $\Gamma = 20$, respectively.

The following points are evident.

- The bound overestimates outage probability by about an order of magnitude in both cases. This translates to under estimating traffic capacity by about 10% in Fig. 5 and 15% in Fig. 6.
- The accuracy of the approximation decreases as the offered traffic, and, thus, the outage probability decreases. The effect is less severe for the larger Γ since in this case we are effectively summing a larger number of random variables and thus getting a better approximation to the tail of the sum.

The above points give some heuristic tips on when the Gaussian approximation is reasonable. Clearly, for large values of Γ and high-outage probabilities, the approximation is excellent, however, for low values of $\Gamma(<20)$ and or low-outage probabilities $(<0.1\%)$ the accuracy of the approximation may deteriorate rapidly. In the latter case, the bound is a much safer and more robust technique.

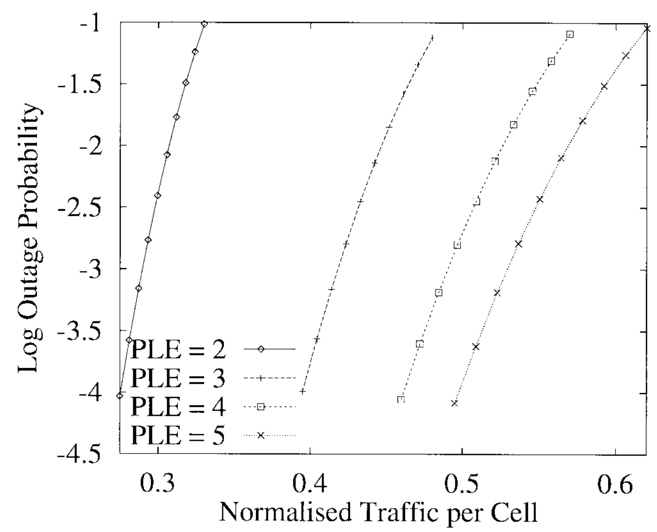

Fig. 7. Variation of outage probability with PLE ($\Gamma = 100$, Chernoff bound).

C. Variation of Bound with System Parameters

Figs. 7 and 8 show how the traffic capacity varies with PLE and Γ respectively for the 2-D network. In these plots, the Chernoff bound was used to obtain values for the outage probability. We make the following points.

- The capacity (for a fixed outage probability) is significantly reduced as the PLE decreases.
- The economy of scale for systems with large Γ results in significant increases in normalized traffic capacity. This is important in comparing narrow-band CDMA (low Γ) to wide-band CDMA (high Γ).

VIII. Conclusion

In this paper, we have presented an analysis for the reverse link traffic capacity of CDMA cellular networks.

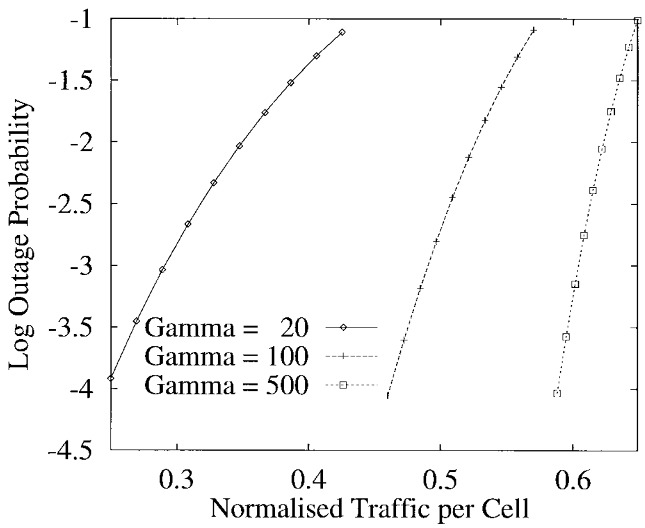

Fig. 8. Variation of outage probability with Γ (PLE $= 4$, Chernoff bound).

Initially, we provided a characterization of other-cell interference in CDMA cellular networks that was crucial to the development of the subsequent traffic analysis. The end products were expressions for the distribution functions of the interference when a mobile's location is a random variable within a cell. These expressions are analytic for the deterministic propagation environment, but involve numerical integration when shadowing is introduced.

The remaining analysis and results contributed toward the understanding the traffic capabilities of CDMA cellular networks. The key assumption was that each cell can meaningfully be modeled as an independent $M/G/\infty$ queue. After discussing the justification for and consequences of the $M/G/\infty$ model, an expression for outage probability was developed in terms of a compound Poisson random variable. Two techniques were then applied to approximate the outage probability along with corresponding asymptotic results. The numerical results gave an initial estimate of the traffic capacity of CDMA networks and demonstrated the sensitivity to propagation parameters and system processing gain.

The primary shortcoming of the preceding analysis is that it provides no information on how a network operator should control call admissions to the network so as to provide a more robust quality of service. This issue is addressed in [21] and [22].

APPENDIX I
DERIVATION OF INTERFERENCE DISTRIBUTION FUNCTION

Before proceeding, note that by symmetry it is only necessary to consider the upper semicircle in Fig. 3. Furthermore, only the ratio of a and b is relevant and in this Appendix we set $a = 1$ and $b = \beta$ without loss of generality. The final distribution function is readily transformed back in terms of a and b by setting $\beta = b/a$. It is also expedient to work in polar coordinates since the random variables R and Φ defined by $R = \sqrt{X^2 + Y^2}$ and $\Phi = \arctan(Y/X)$ are then independent. The situation is then as shown in Fig. 3. If the MS is at location (r, ϕ) in polar coordinates and is connected to the BS at the origin, the interference caused at the BS with location (a, π) is given by

$$I(r,\phi) = \left(\frac{r}{\sqrt{r^2 + a^2 + 2ar\cos\phi}}\right)^{\gamma}$$

and the problem now is to calculate the distribution function of the random variable $I(R, \Phi)$ where R and Φ are independent random variables with readily calculated distributions. The problem is formalized below.

A. Problem Formulation

Define the nonnegative, real valued function I by

$$I(r,\phi) = \left(\frac{r^2}{1 + 2r\cos\phi + r^2}\right)^{\gamma/2}, \quad \gamma > 0 \tag{10}$$

where $r \in [0, \beta]$ and $\phi \in [0, \pi]$. We will always assume that $0 < \beta < 1$.

Let R and Φ be independent random variables with marginal distribution functions

$$F_R(r) = \begin{cases} 0, & r < 0 \\ r^2/\beta^2, & 0 \le r < \beta \\ 1, & \beta \le r \end{cases} \tag{11}$$

and

$$F_\Phi(\phi) = \begin{cases} 0, & \phi < 0 \\ \phi/\pi, & 0 \le \phi < \pi \\ 1, & \pi \le \phi. \end{cases} \tag{12}$$

Aim: Find the distribution function of the random variable $I(R, \Phi)$.

Solution: Fix $\phi \in [0, \pi]$ and define $I_\phi(r) = I(r, \phi)$.

Lemma 2: $I_\phi(r)$ is strictly increasing on $(0, \beta]$ for all $\phi \in [0, \pi]$.

Proof:

$$\frac{dI_\phi(r)}{dr} = \frac{\gamma r^{\gamma-1}(1 + r\cos\phi)}{(1 + 2r\cos\phi + r^2)^{1+\gamma/2)}} > 0, \quad \text{for} \quad r \in (0, \beta].$$

□

$I_\phi(r)$ is strictly increasing from $I_\phi(0)$ to $I_\phi(\beta)$, and standard transformation techniques can thus be applied to calculate the distribution function of $I_\phi(R)$, where R is distributed as in (11). In particular, we have

$$F_{I_\phi}(z) = P(I_\phi(R) \le z) = \begin{cases} 0, & z < 0 \\ F_R\big(I_\phi^{-1}(z)\big), & 0 \le z < I_\phi(\beta) \\ 1, & I_\phi(\beta) \le z \end{cases}$$

where the inverse function $I_\phi^{-1}(z)$ is well defined on $z \in [0, I_\phi(\beta))$ because of the monotonicity of $I_\phi(r)$. It is calculated by solving

$$I_\phi(r) = \left(\frac{r^2}{1 + 2r\cos\phi + r^2}\right)^{\gamma/2} = z$$

for r taking into consideration the allowed values of the variables involved.

We are thus led to the following lemma.

Lemma 3:

$$F_{I_\phi}(z) = \begin{cases} 0, & z < 0 \\ f_1(z), & 0 \le z < I_\phi(\beta) \\ 1, & I_\phi(\beta) \le z \end{cases}$$

where

$$f_1(z) = \beta^{-2}(1-z^{2/\gamma})^{-2}(2z^{4/\gamma}\cos^2\phi + z^{2/\gamma}(1-z^{2/\gamma}) + 2z^{3/\gamma}\cos\phi(1-z^{2/\gamma}\sin^2\phi)^{1/2})$$

with the provision that when $I_\phi(\beta) > 1$, $F_{I_\phi}(1) = 1/(4\beta^2 \cos^2\phi)$.

We can consider $F_{I_\phi}(z)$ as a distribution function conditioned on the value of Φ. That is, $F_{I_\phi}(z) = F_{I|\Phi}(z \mid \phi)$. A simple unconditioning then allows us to write

$$\begin{aligned} F_I(z) &= \int_{\phi=0}^{\pi} F_{I|\Phi}(z \mid \phi) F_\Phi(d\phi) \\ &= \frac{1}{\pi}\int_{\phi=0}^{\pi} F_{I|\Phi}(z \mid \phi)\, d\phi \\ &= \begin{cases} 0, & z < 0 \\ \frac{1}{\pi}\int_0^\pi f_1(z,\phi)\, d\phi, & 0 \le z < I_0(\beta) \\ \frac{1}{\pi}\left(f_2(z) + \int_{f_2(z)}^{\pi} f_1(z,\phi)\, d\phi\right), & I_0(\beta) \le z < I_\pi(\beta) \\ 1, & I_\pi(\beta) \le z \end{cases} \end{aligned}$$

where

$$f_2(z) = \arccos\left(\frac{\beta^2(z^{-2/\gamma}-1)-1}{2\beta}\right).$$

If $I_\phi(\beta) > 1$, then

$$F_I(1) = \frac{1}{\pi}\left(f_2(1) + \int_{f_2(1)}^{\pi} 1/(4\beta^2\cos^2\phi)\, d\phi\right).$$

The integrals involve terms that can be integrated using elementary techniques. We leave the details to the interested reader. We are thus led to the following result:

Theorem 4: The distribution function of $I(R,\Phi)$, where $I(\cdot,\cdot)$ is defined in (10) and where R and Φ are independent random variables with marginal distributions given in (11) and (12), is given by

$$\begin{aligned} F_I^\beta(z) &= P(I(R,\Phi) \le z) \\ &= \begin{cases} 0, & z < 0 \\ g_1(z), & 0 \le z < I_0(\beta) \\ g_2(z), & I_0(\beta) \le z < I_\pi(\beta), \quad z \ne 1 \\ g_3(z), & 1 < I_\pi(\beta), \quad z = 1 \\ 1, & I_\pi(\beta) \le z \end{cases} \end{aligned} \tag{13}$$

where

$$\begin{aligned} g_1(z) &= \frac{z^{2/\gamma}}{\beta^2\left(z^{2/\gamma}-1\right)^2} \\ g_2(z) &= \frac{1}{\pi}\arccos(h_1(z)) \\ &\quad + \frac{1}{\pi} g_1(z)\Big[\pi - 2z^{2/\gamma} h_1(z) h_2(z) - \frac{1}{\beta} z^{2/\gamma} h_2(z) \\ &\qquad - \arccos(h_1(z)) - \arcsin(z^{1/\gamma} h_2(z))\Big] \\ g_3(z) &= \frac{1}{\pi}\arccos\left(-\frac{1}{2\beta}\right) + \frac{1}{4\pi\beta^2}\sqrt{4\beta^2-1} \end{aligned}$$

and

$$\begin{aligned} h_1(z) &= \frac{-1-\beta^2+\beta^2 z^{-2/\gamma}}{2\beta} \\ h_2(z) &= \sqrt{1-h_1^2(z)}. \end{aligned}$$

Appendix II
Inclusion of Lognormal Shadowing

In the following, we take $\mathbf{x}$ as the position vector and $\|\cdot\|$ as the Euclidean norm. Equation (3) becomes

$$I_{i,j}(\mathbf{x}) = \left(\frac{\|\mathbf{x}-\mathbf{x_i}\|}{\|\mathbf{x}-\mathbf{x_j}\|}\right)^\gamma$$

which represents the interference in the non shadowing environment to a BS at $\mathbf{x_j}$ (BS_j) from an MS at $\mathbf{x}$ connected to a BS at $\mathbf{x_i}$ (BS_i).

We are interested in extending the interference results of Section III-B and Appendix I, which were based on the propagation model of (1), to include shadowing effects as given in (2). With reference to the latter equation, we assume P_0, γ, and σ are constant over all paths and that the shadowing random variables are independent for different paths.

Initially assume that an MS at $\mathbf{x}$ connects to the closest BS, BS_i, and suppose we are interested in the subsequent interference $J_{i,j}(\mathbf{x})$, produced at a target BS, BS_j. If $\mathbf{x_i} = \mathbf{x_j}$ then the interference is clearly one unit since the MS will connect to the target BS and be power controlled to one unit signal power. If $\mathbf{x_i} \ne \mathbf{x_j}$, then

$$\begin{aligned} J_{i,j}(\mathbf{x}) &= I_{i,j}(\mathbf{x}) 10^{(\zeta_j - \zeta_i)/10} \\ &= I_{i,j}(\mathbf{x}) 10^{\zeta/10} \end{aligned} \tag{14}$$

where ζ as the difference of two independent zero-mean Gaussian random variables is a zero-mean Gaussian random variable with variance $2\sigma^2$.

Suppose, however, that rather than connecting to the closest BS, an MS is linked to the BS offering the least path loss. It is practically infeasible to allow connection to any BS in a large network and it is sensible to consider choosing between only the M closest. In any case M can be chosen so that there is an arbitrarily high probability of the *best* BS belonging to the M closest. This is the approach taken in [17] and [18] although as already mentioned, a fairly complicated analytical and numerical procedure, results only in mean values for the other cell interference. We prefer the simpler approach of [16] and [20], where the choice is made between the closest BS and the target BS. As most of the other cell interference to the target BS comes from MS's near its cell boundaries this seems a reasonable approximation.

Once more we assume that if the target BS BS_j is the closest BS, then the MS connects to it and causes one unit of interference. Failing that and with BS_i the closest BS, the interference produced by an MS at $\mathbf{x}$ is given by

$$J_{i,j}(\mathbf{x}) = \min(1, I_{i,j}(\mathbf{x}) 10^{\zeta/10}) \tag{15}$$

which is simply a truncated version of (14). One significant advantage of (15), apart from being a more accurate model of the system operation, is that the moment generating function of the interference exists only in the truncated case. The moment generating function is used for obtaining bounds on sums of interferers in later sections.

Given (15) our aim, just as in the last sections, is to allow the position vector $\mathbf{x}$ to be a random variable taking values in the cell corresponding to the BS at $\mathbf{x_i}$. Denote this random position vector $\mathbf{X}$ and assume it has distribution function $F_{\mathbf{X}}$ defined on cell_i. Our problem then is to find the distribution function of $J_{i,j}(\mathbf{X})$, which means calculating the distribution function of $I_{i,j}(\mathbf{X})10^{\zeta/10}$.

Given the position vector $\mathbf{x}$, the distribution function of the random variable $I_{i,j}(\mathbf{x})10^{\zeta/10}$, which we treat as a distribution function conditioned on $\mathbf{X}$, is

$$F(z \mid \mathbf{x}) = \frac{1}{2} + \operatorname{erf}\left(\frac{\ln \mathrm{z} - \ln \mathrm{I_{i,j}}(\mathbf{x})}{\sqrt{2}\beta\sigma}\right) \tag{16}$$

where

$$\operatorname{erf}(\mathrm{y}) = \frac{1}{\sqrt{2\pi}} \int_0^{\mathrm{y}} \mathrm{e}^{-\mathrm{t}^2/2}\,\mathrm{dt}.$$

Alternatively, we can view (16) as a distribution function conditioned on $I_{i,j}(\mathbf{X})$

$$F(z \mid I_{i,j}) = \frac{1}{2} + \operatorname{erf}\left(\frac{\ln \mathrm{z} - \ln \mathrm{I_{i,j}}}{\sqrt{2}\beta\sigma}\right). \tag{17}$$

In the above, $\beta = \ln 10/10$ and $\sqrt{2}\sigma$ is the standard deviation of ζ in (15).

Equation 16 can be unconditioned on $\mathbf{X}$ given $F_{\mathbf{X}}$

$$F(z) = \int_{\text{cell}_i} F(z \mid \mathbf{x}) F_{\mathbf{X}}(d\mathbf{x}) \tag{18}$$

while (17) can be unconditioned on $I_{i,j}(\mathbf{X})$ given its distribution function $F_{I_{i,j}}$

$$F(z) = \int_I F(z \mid I_{i,j}) F_{I_{i,j}}(dI). \tag{19}$$

In the above, $F_{I_{i,j}}$ comes from (4) (remembering that this is an approximation and is only valid when $\mathbf{X}$ is uniformly distributed) with a replaced by $\|\mathbf{x_i} - \mathbf{x_j}\|$.

The final form for the distribution function of $J_{i,j}(\mathbf{X})$ is given by

$$F_{J_{i,j}}(z) = \begin{cases} 0, & z < 0 \\ F(z), & 0 \le z < 1 \\ 1, & 1 \le z \end{cases} \tag{20}$$

and results because of the $\min$ operator in (15).

In summary, (20) gives the distribution function of the interference produced at BS_j from one MS with location in cell_i distributed as $F_{\mathbf{X}}$. It is equivalent to (3) when lognormal shadowing is modeled as above.

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Jamie S. Evans, for a photograph and biography, see this issue, p. 46.

David Everitt, for a photograph and biography, see this issue, p. 46.