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An OFDM interpretation of zero padded block transmissions ☆

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Abstract

Orthogonal frequency division multiplex (OFDM) systems are particularly straightforward to understand in the frequency domain. The purpose of this paper is to draw attention to the fact that zero padded block transmissions also have a straightforward frequency domain interpretation. The advantages of this interpretation are illustrated by three additional results which further the understanding of zero padded block transmission systems; it is shown that the zero padding spreads the spectrum of the source symbols uniformly, it is explained why zero padded block transmission systems do not always outperform OFDM systems, and it is shown how pilot tones can be incorporated into zero padded block transmissions.

Keywords: Orthogonal frequency division multiplex systems; Zero padded systems; Wireless communications; Channel coding

1. Introduction

A wireless communications channel is often modelled by a finite impulse response channel whose impulse response varies with time. For high-speed transmissions, it may be assumed that n consecutive symbols can be transmitted without the impulse response changing significantly, where $n \gg 1$ (for example, $n = 64$ or higher). This paper shows that two popular and apparently distinct block based transmission systems, both of which transmit data in blocks of n symbols, are in fact more closely related than initially suspected. The usefulness of this finding is discussed at the end of this section.

The two transmission systems considered in this paper are orthogonal frequency division multiplex (OFDM) systems [[12]] and the more recently proposed zero padded systems [[10]], both of which are special cases of linearly precoded block transmission systems. These terms are defined as follows.

A *linearly precoded block transmission system* first breaks the source symbols $\{\dots, s_{-1}, s_0, s_1, \dots\}$ into blocks $\boldsymbol{s}^{(i)} = [s_{ip}, s_{ip+1}, \dots, s_{ip+p-1}]^{\mathrm{T}} \in \mathbb{C}^p$ of length p. A linear precoder matrix $P \in \mathbb{C}^{m\times p}$ is then used to encode each block prior to transmission. Assuming the system operates over a finite impulse response (FIR) channel $\boldsymbol{h} = [h_0, \dots, h_{L-1}]^{\mathrm{T}} \in \mathbb{C}^L$ of length at most L, the ith received block is

$$\boldsymbol{y}^{(i)} = \mathscr{H}^{\mathrm{IBI}} P \boldsymbol{s}^{(i-1)} + \mathscr{H}^{\mathrm{CB}} P \boldsymbol{s}^{(i)} + \boldsymbol{n}^{(i)} \in \mathbb{C}^m, \qquad (1)$$

where $\mathscr{H}^{\mathrm{CB}}$ is the $m \times m$ lower triangular Toeplitz channel matrix whose first column is $[h_0, \dots, h_{L-1}, 0, \dots, 0]^{\mathrm{T}}$, $\mathscr{H}^{\mathrm{IBI}}$ is the $m \times m$ upper triangular Toeplitz matrix with first row equal to $[0, \dots, 0, h_{L-1}, \dots, h_1]$ and $\boldsymbol{n}^{(i)} \in \mathbb{C}^m$ is additive white Gaussian noise

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(AWGN). Here, $\mathscr{H}^{\mathrm{IBI}}$ models the inter-block interference (IBI) caused by the memory of the channel while $\mathscr{H}^{\mathrm{CB}}$ models the effect of the channel on the current block.

This paper is concerned with the following special cases of (1). An *OFDM system* uses the precoder $P = CD^{\mathrm{H}}$ where D is used throughout to denote the normalized (so that $D^{\mathrm{H}}D = I$) discrete Fourier transform (DFT) matrix whose dimensions can be determined from its context and C is the cyclic prefix matrix

$$C = \begin{bmatrix} 0_{(L-1)\times(m-2L+2)} & I_{L-1} \\ \multicolumn{2}{c}{I_{m-L+1}} \end{bmatrix}, \tag{2}$$

which adds a cyclic prefix of length $L-1$. Due to the cyclic prefix, an OFDM system transmits $L-1$ extra symbols per block, that is, $m = p + L - 1$. A *channel coded OFDM system* uses a precoder of the form $P = CD^{\mathrm{H}}\tilde{P}$ for some precoder $\tilde{P} \in \mathbb{C}^{(m-L+1)\times p}$ where $m \geqslant p + L - 1$. The interpretation of $\tilde{P}$, if it has more rows than columns, is that it spreads the source symbols out over the frequency domain in each block (see Section 3). A *TZ-OFDM system* [[10]] uses $P = ZD^{\mathrm{H}}$ where Z is the zero padded matrix

$$Z = \begin{bmatrix} I_{(m-L+1)} \\ 0_{(L-1)\times(m-L+1)} \end{bmatrix}, \tag{3}$$

which adds $L-1$ trailing zeros. (As in an OFDM system, $m = p + L - 1$.) More generally, a *zero padded system* is used here to refer to any system (1) which uses a precoder P of the form $P = ZD^{\mathrm{H}}\tilde{P}$, where $\tilde{P}$ is an arbitrary precoder having equal or more rows than columns (that is, $m \geqslant p + L - 1$).

Remark. Note that in both channel coded OFDM systems and zero padded systems, $\tilde{P}$ can cancel the IDFT operation D^{H}. For example, $P = Z$ is a zero padded system since $P = ZD^{\mathrm{H}}\tilde{P}$ if $\tilde{P} = D$.

Trailing zero OFDM (TZ-OFDM) systems are so named in the literature because they replace the cyclic prefix in an OFDM system by a sequence of trailing zeros [[10]]. Ironically, this paper (see also [[2]]) shows that TZ-OFDM systems are more closely related to OFDM systems than originally suspected. Indeed, in Section 3 it is shown that the DFT of the received symbols in a zero padded system is related to the source symbols in exactly the same way as in a channel coded OFDM system. In fact, this is a consequence of the more general result, proved in Section 2, that for any zero padded system, there is an equivalent channel coded OFDM system with the same statistical performance.

These results imply that *the statistical performance of any zero padded system can be understood by investigating the statistical performance of the equivalent channel coded OFDM system*. Three examples in which this proves beneficial are given in Sections 4–6. Section 4 proves that TZ-OFDM systems spread the source symbols uniformly in the frequency domain; this is a desirable property if the channel is unknown to the transmitter. Section 5 explains intuitively why TZ-OFDM systems do not always perform better than OFDM systems. (Note though that this is no reason not to use a TZ-OFDM system; for most channels, a TZ-OFDM system performs better than an OFDM system.) Section 6 demonstrates how TZ-OFDM systems can use pilot tones to identify the channel, just as in OFDM systems. Concluding remarks are made in Section 7.

Related work: In [[8]], it was shown that it is possible to make a TZ-OFDM system resemble an OFDM system by using an appropriate reduced complexity equaliser. However, such an equaliser is statistically sub-optimal. The connection made in the present paper is stronger because it is based on statistically optimal equalisers being used for both systems. Furthermore, it is mentioned that although zero padded systems are explained in the literature as OFDM systems using a different precoder matrix [[8,11]], this does not necessarily mean zero padded systems are related to OFDM systems in any way. Indeed, without the presence of a cyclic prefix, a system cannot be called a (channel coded or linearly precoded) OFDM system.

Relevance: The design and understanding of linearly precoded systems is currently an important area of research. Prior to this work, it was not clear what the mathematical difference was of using a zero padded system instead of a cyclic prefixed system. This paper proves that the only difference is that the zero padded system inherently incorporates a precoding operation which uniformly spreads the spectrum of the source symbols. Therefore, it suffices to understand cyclic prefixed systems in order to understand zero padded systems. This is advantageous because cyclic prefixed systems have been studied extensively in the past whereas zero padded systems are relatively new.

2. Zero forcing equalisers and sufficient statistics

To understand the performance of various linearly precoded transmission systems, it is insufficient to study only the precoding operation in (1). This is because the receiver must cope with the problem of IBI (that is, the $\mathscr{H}^{\mathrm{IBI}}P\boldsymbol{s}^{(i-1)}$ term in (1)), and in particular, the best way of dealing with IBI depends on the actual precoder used. Therefore, it is necessary to study the statistical information about the source vector present in the received vector. Indeed, two systems will be defined to be equivalent if the identical statistical information is present at the receivers of the two systems.

The key result of this section is that an equivalent—but not identical—channel coded OFDM system can be associated with any zero padded system. The equivalent OFDM system has the same statistical performance as the zero padded system. The only difference is that the equivalent OFDM system introduces an extra $L-1$ zeros between each block. As clarified at the end of this section, this implies that zero padded systems can be interpreted as efficient implementations of certain channel coded OFDM systems.

The performance of a linearly precoded system (1) is studied under the following three assumptions:

A1. The receiver uses only $\boldsymbol{y}^{(i)}$ to estimate $\boldsymbol{s}^{(i)}$, and in particular, the estimate $\hat{\boldsymbol{s}}^{(i-1)}$ of the previous block $\boldsymbol{s}^{(i-1)}$ is not used to aid in the estimation of the current block $\boldsymbol{s}^{(i)}$.
A2. The receiver knows the true channel $\boldsymbol{h}$.
A3. The noise $\boldsymbol{n}^{(i)}$ has a zero mean Gaussian distribution with covariance matrix $\sigma^2 I$, where σ may or may not be known.

The first assumption holds for most block transmission systems. However, the second assumption is rarely true in practice. The justification for making the second assumption is that the effects of channel estimation errors are not important for the purposes of this paper. The third assumption is a standard one; note that the results of this paper are valid regardless of whether or not the noise power σ is known.

Before continuing further, two transmission systems being statistically equivalent is first defined rigorously. The definition is such that if two transmission systems are equivalent and a receiver structure for one of them is given, then there exists a receiver structure for the other system such that *all statistical figures of merit* (*such as bit error rate*) *are the same for both systems*.

Definition 1. (*Equivalence*). Dropping the block index in (1), let $\boldsymbol{y}$ denote the received block if $\boldsymbol{s}$ was transmitted using the precoder P. Similarly, let $\boldsymbol{y}'$ denote the received block if $\boldsymbol{s}'$ was transmitted using the precoder P'. Under assumptions A1–A3 above, the two transmission systems, P and P', are said to be equivalent if there exist sufficient statistics $\hat{\boldsymbol{s}}$ and $\hat{\boldsymbol{s}}'$ for $\boldsymbol{s}$ and $\boldsymbol{s}'$, respectively, such that $\hat{\boldsymbol{s}}$ and $\hat{\boldsymbol{s}}'$ have identical distributions.

Consider first a channel coded OFDM system given by (1) with $P=CD^{\mathrm{H}}\tilde{P}$; see Section 1 for definitions. Because the first $L-1$ symbols of $\boldsymbol{y}^{(i)}$ usually[1] depend on the previous block $\boldsymbol{s}^{(i-1)}$, in accordance with assumption A1, it is not possible for the receiver to extract any information about the source symbols from the first $L-1$ symbols of each block. This corresponds to the well-known fact that OFDM systems discard the cyclic prefix, also referred to as the guard interval, at the receiver. Thus, the receiver uses only the symbols

$$\boldsymbol{y}=\mathscr{H}CD^{\mathrm{H}}\tilde{P}\boldsymbol{s}+\boldsymbol{n} \tag{4}$$

to recover the current block $\boldsymbol{s}$, where $\mathscr{H}$ is the upper triangular $(m-L+1)\times m$ Toeplitz channel matrix with first row equal to $[h_{L-1},\ldots,h_0,0,\ldots,0]$ and $\boldsymbol{n}$ denotes AWGN. Here, (4) is obtained from (1) by omitting the first $L-1$ rows and substituting $P=CD^{\mathrm{H}}\tilde{P}$.

Given the received vector $\boldsymbol{y}$, one way of summarising all the statistical information about $\boldsymbol{s}$ in $\boldsymbol{y}$ is by using the minimum variance unbiased estimator of $\boldsymbol{s}$ given $\boldsymbol{y}$. This estimator is readily shown to be

$$\hat{\boldsymbol{s}}=(Q^{\mathrm{H}}Q)^{-1}Q^{\mathrm{H}}\boldsymbol{y},\quad Q=\mathscr{H}CD^{\mathrm{H}}\tilde{P}. \tag{5}$$

In the literature, (5) is also referred to as a zero forcing equaliser.

[1] For most precoders $\tilde{P}$ this is the case. The only exception is if some of the last $L-1$ rows of $\tilde{P}$ are zero. These exceptions are inconsequential in this paper.

Remark. Other estimators/equalisers can be used without altering the results of this paper; all that is required is that $\hat{\boldsymbol{s}}$ be a sufficient statistic for $\boldsymbol{s}$ under assumptions Al–A3, and (5) serves this purpose. (Note that $\hat{s}$ is a sufficient statistic regardless of whether or not the noise power σ is known.)

Zero padded systems are now considered. If $P = ZD^{\mathrm{H}}\tilde{P}$ (see Section 1) then the presence of the trailing zeros means that the IBI term $\mathscr{H}^{\mathrm{IBI}}P\boldsymbol{s}^{(i-1)}$ in (1) is always zero. Therefore, the receiver can use all the received symbols

$$\boldsymbol{y} = \mathscr{H}^{\mathrm{CB}}ZD^{\mathrm{H}}\tilde{P}\boldsymbol{s} + \boldsymbol{n} \tag{6}$$

to recover the current block $\boldsymbol{s}$. Here, (6) is obtained from (1) by substituting $P = ZD^{\mathrm{H}}\tilde{P}$. Analogous to (5), the minimum variance unbiased estimate (and a sufficient statistic) of $\mathbf{s}$ under assumptions Al–A3 is

$$\hat{\boldsymbol{s}} = (Q^{\mathrm{H}}Q)^{-1}Q^{\mathrm{H}}\boldsymbol{y}, \quad Q = \mathscr{H}^{\mathrm{CB}}ZD^{\mathrm{H}}\tilde{P}. \tag{7}$$

The key result of this section is that an equivalent channel coded OFDM system can be associated with any zero padded system in such a way that both systems have identical statistical performance. In order to state this result precisely, it is necessary to discriminate between variables (such as $\tilde{P}, m$ and D) pertaining to OFDM systems and to zero padded systems. Henceforth, a subscript 1 will refer to OFDM systems and a subscript 2 to zero padded systems.

Let $P_2 = ZD_2^{\mathrm{H}}\tilde{P}_2$ be an arbitrary zero padded precoder of size m_2 by p. Define an associated channel coded OFDM precoder P_1 of size $m_1 = m_2 + L - 1$ by p according to the formula

$$P_1 = CD_1^{\mathrm{H}}\tilde{P}_1, \quad \tilde{P}_1 = D_1ZD_2^{\mathrm{H}}\tilde{P}_2. \tag{8}$$

(Recall that D_1 and D_2 are both normalised DFT matrices; the subscript indicates that their sizes differ.) It is readily seen that the only difference between P_1 and P_2 is that P_1 introduces an extra $L-1$ leading zeros per block (in addition to the $L-1$ trailing zeros both precoders introduce). For example, if P_2 encodes two consecutive blocks as 1,2,3,0,4,5,6,0 then P_1 would encode the blocks as 0,1,2,3,0,0,4,5,6,0.

Although P_1 transmits more symbols per block than P_2 does, the following theorem proves that the statistical performance of the two systems is identical.

Theorem 1. *The source symbol estimates (7) of the zero padded system P_2 are identical to the source symbol estimates (5) of the associated channel coded OFDM system P_1 defined in (8). In particular, the systems P_1 and P_2 are equivalent (see Definition 1) and hence have identical statistical properties.*

Proof. The following shows that Q in (7), denoted by Q_2, is identical to Q in (5), denoted by Q_1.

$$\begin{aligned} Q_2 &= \mathscr{H}^{\mathrm{CB}}ZD_2^{\mathrm{H}}\tilde{P}_2 = \mathscr{H}CZD_2^{\mathrm{H}}\tilde{P}_2 \\ &= \mathscr{H}CD_1^{\mathrm{H}}\tilde{P}_1 = Q_1. \quad \square \end{aligned} \tag{9}$$

It is important to note that the proof of Theorem 1 shows that the received symbols $\boldsymbol{y}$ in (6) of the zero padded system P_2 are identical to the received symbols $\boldsymbol{y}$ in (4) of the associated OFDM system P_2. Even though P_1 has more rows than P_1, this is possible because the OFDM receiver (4) discards the guard interval whereas the zero padded receiver (6) does not. Going in the reverse direction, the zero padded system P_2 can be thought of as an efficient implementation of the channel coded OFDM system P_1 because they have identical statistical performance yet P_2 uses less symbols to encode each block.

The implication of Theorem 1 is that any zero padded system can be understood by studying its equivalent channel coded OFDM system. The remainder of this paper illustrates several advantages of doing so.

3. Frequency domain interpretation of zero padded systems

It is well-known that OFDM systems have a particularly simple frequency domain interpretation. This section first reviews this interpretation and then applies the results of the previous section to derive a novel frequency domain interpretation of zero padded systems.

The received symbols of a channel coded OFDM system in the frequency domain are given by taking the DFT of (4), namely:

$$D\boldsymbol{y} = (D\mathscr{H}CD^{\mathrm{H}})\tilde{P}\boldsymbol{s} + D\boldsymbol{n}. \tag{10}$$

It is a standard result that $D\mathscr{H}CD^{\mathrm{H}}$ is a diagonal matrix, a consequence of $\mathscr{H}C$ being circulant. In particular, in an OFDM system ($\tilde{P} = I$), the ith element of **s** is interpreted as being sent across the ith sub-channel [[12]]. Clearly, the effect of $\tilde{P}$ in a channel coded OFDM system is to spread the source symbols over the independent sub-channels, or in other words, $\tilde{P}$ spreads the source symbols over the frequency domain.

The traditional description of a TZ-OFDM system is that it replaces the cyclic prefix of an OFDM system by trailing zeros, and in particular, it is not apparent from this description that there is a straightforward frequency domain interpretation of TZ-OFDM systems. As is now shown though, all zero padded systems (including TZ-OFDM systems) can be understood in the frequency domain via their equivalent channel coded OFDM systems.

The received symbols of a zero padded system in the frequency domain are given by taking the DFT of (6), namely:

$$D_1\boldsymbol{y} = D_1\mathscr{H}^{\mathrm{CB}}ZD_2^{\mathrm{H}}\tilde{P}\boldsymbol{s} + D_1\boldsymbol{n}, \tag{11}$$

where D_1 and D_2 are different sized DFT matrices. In order to obtain a frequency domain interpretation of (11) it is necessary to make $\mathscr{H}^{\mathrm{CB}}$ into a diagonal matrix by multiplying it on the right and the left by appropriate matrices (cf., (10)). As in the proof of Theorem 1, $\mathscr{H}^{\mathrm{CB}}Z$ is equal to $\mathscr{H}CZ$ where $\mathscr{H}$ has the same form as the channel matrix appearing in both (4) and (10). Making this substitution in (11) shows that

$$D_1\boldsymbol{y} = D_1\mathscr{H}CZD_2^{\mathrm{H}}\tilde{P}\boldsymbol{s} + D_1\boldsymbol{n} \tag{12}$$

$$= (D_1\mathscr{H}CD_1^{\mathrm{H}})(D_1ZD_2^{\mathrm{H}}\tilde{P})\boldsymbol{s} + D_1\boldsymbol{n}. \tag{13}$$

Notice that $D_1\mathscr{H}CD_1^H$ is a diagonal matrix, and moreover, (13) is identical to (10) if $\tilde{P}$ in (10) is defined to be $D_1ZD_2^{\mathrm{H}}\tilde{P}$; compare this result with (8) and Theorem 1. That is, referring to (13), *any zero padded system* $P = ZD_2^{\mathrm{H}}\tilde{P}$ *works by first spreading the source symbols* **s** *out over the frequency domain by pre-multiplication by* $D_1ZD_2^{\mathrm{H}}\tilde{P}$ *and then sending these encoded symbols over independent sub-channels.*

4. Spectrally balanced nature of TZ-QFDM systems

As (13) shows, a zero padded system uses the matrix $D_1ZD_2^{\mathrm{H}}\tilde{P}$ to spread the source symbols over the frequency domain. This section draws attention to the fact that $D_1ZD_2^{\mathrm{H}}$ can be interpreted as spreading the symbols "uniformly", and moreover, this is a desirable property if the channel is unknown to the transmitter.

Consider the channel coded OFDM system $P = CD_1^{\mathrm{H}}\tilde{P}_1$ where $\tilde{P}_1$ spreads the spectrum of the source symbols $\boldsymbol{s}$. That is, as (10) shows, the ith element of $\tilde{P}_1\boldsymbol{s}$ is transmitted on the ith sub-channel, and in particular, the transmitted energy in each sub-channel is given by the diagonal elements of $\tilde{P}_1\boldsymbol{s}\boldsymbol{s}^{\mathrm{H}}\tilde{P}_1^{\mathrm{H}}$. If $\boldsymbol{s}$ is white noise then the expected value of the transmitted energy in each sub-channel is proportional to the diagonal elements of $\tilde{P}_1\tilde{P}_1^{\mathrm{H}}$. The off-diagonal elements of $\tilde{P}_1\tilde{P}_1^{\mathrm{H}}$ correspond to the correlation in energy between sub-channels.

For coding over unknown channels, it is known [[1,7,9]] that it is best to spread the transmitted energy evenly over all sub-channels. Since $\tilde{P}_1$ is a tall matrix in general, it is not possible to make $\tilde{P}_1\tilde{P}_1^{\mathrm{H}}$ the identity matrix. However, choosing $\tilde{P}_1\tilde{P}_1^{\mathrm{H}}$ so that its diagonal elements are equal is perhaps the next best thing. A channel coded OFDM system using such a precoder is said to be spectrally balanced.

This definition readily extends to zero padded systems by using the frequency domain interpretation (13), or equivalently, by using Theorem 1. Specifically, a zero padded system is said to be spectrally balanced if its equivalent channel coded OFDM system is spectrally balanced.

Theorem 2. *A TZ-OFDM system is spectrally balanced.*

Proof. A TZ-OFDM system is a zero padded system $P = ZD_2^{\mathrm{H}}\tilde{P}_2$ with $\tilde{P}_2 = I$. This is equivalent to the channel coded OFDM system using the precoder $CD_1^{\mathrm{H}}\tilde{P}_1$ with $\tilde{P}_1 = D_1ZD_2^{\mathrm{H}}$; see (8). It is straightforward to prove that $\tilde{P}_1\tilde{P}_1^{\mathrm{H}}$ is a circulant matrix, hence its diagonal elements are equal. □

Remark. It is clear from the proof that a "Zero Padding Only" precoder [[11]] ($P = Z$) is also spectrally balanced.

5. OFDM versus TZ-OFDM

It is proposed in [[3,4]] to measure the *intrinsic* performance of linearly precoded systems by the mean-square error $\boldsymbol{E}[\|\boldsymbol{s} - \hat{\boldsymbol{s}}\|^2]$ of the source symbol estimates if a minimum variance unbiased estimator is used; see Section 2. Although a TZ-OFDM system outperforms an OFDM system over most channels $\boldsymbol{h}$, an example of a channel over which a TZ-OFDM system performs worse than an OFDM system appears in [[5]]. This section uses the frequency domain interpretation of TZ-OFDM systems derived in Section 3 to explain this example.

Remark. Since a TZ-OFDM system has the same performance as its equivalent channel coded OFDM system, it might be conjectured that a TZ-OFDM system has a smaller MSE than an OFDM system over all channels. Theoretically then, it is interesting to know that a counter-example exists.

Consider sending $p=3$ symbols per block over the channel $\boldsymbol{h}=[1-0.6181]^{\mathrm{T}}$ of length $L=3$. In [[5]], it was proved that the mean-square error of the source symbol estimates (obtained using the optimal unbiased estimators (5) and (7)) is worse for a TZ-OFDM system than for an OFDM system. An intuitive explanation is now offered.

An OFDM system transmits the 3 symbols in each block over 3 independent sub-channels. The received symbols in the frequency domain are given by (10) with $\tilde{P}=I$, and for the particular channel $\boldsymbol{h}=[1\ \ -0.618\ \ 1]^{\mathrm{T}}$, the diagonal elements of $(D\mathscr{H}CD^{\mathrm{H}})$ are given by the 3-point DFT of $\boldsymbol{h}$, namely $\{1.4, 0.8+1.4\mathrm{j}, 0.8-1.4\mathrm{j}\}$. A TZ-OFDM system transmits the 3 source symbols in each block over 5 independent sub-channels. The received symbols in the frequency domain are given by (13) with $\tilde{P}=I$, and for the particular channel $\boldsymbol{h}=[1\ \ -0.618\ \ 1]^{\mathrm{T}}$, the diagonal elements of $(D_1\mathscr{H}CD_1^{\mathrm{H}})$ are given by the 5-point DFT of $\boldsymbol{h}$, namely $\{1.4, 0, 1.8+1.3\mathrm{j}, 1.8-1.3\mathrm{j}, 0\}$. Notice that the 2nd and 5th sub-channels attenuate the signal completely! It turns out that this attenuation is significant enough to allow the OFDM system to perform better than the TZ-OFDM system; observe that none of the 3 sub-channels in the OFDM system are significantly attenuated.

6. Pilot tones in zero padded systems

One way of allowing the receiver to estimate the channel is to transmit sinusoids at various frequencies and with known amplitudes. These sinusoids are called pilot tones. It is straightforward to generate pilot tones in an OFDM system; simply set various elements of the source symbols $\boldsymbol{s}$ to known values [[6]]. This section shows that pilot tones can also be generated in zero padded systems.

Remark. The use of pilot tones in a TZ-OFDM system has already been proposed in [[8]]. This section shows that pilot tones can be used in any zero padded system (even if an additional redundant precoder is present), and moreover, it is felt that the derivation here offers a different perspective to the one in [[8]].

Consider the zero padded system $P=ZD_2^{\mathrm{H}}\tilde{P}$. The DFT of the received block is given by (13). The aim of pilot tones is to "Probe" various elements of the diagonal matrix $D_1\mathscr{H}CD_1^{\mathrm{H}}$ in (13). To do this, it is sufficient to make various elements of $(D_1ZD_2^{\mathrm{H}}\tilde{P})\boldsymbol{s}$ known to the receiver; this is the direct analogue of pilot tones in OFDM systems.

Assume for example that it is required to send two pilot tones of unit energy, one on the 2nd sub-channel and one on the 5th sub-channel of a zero padded system. Then it suffices to restrict the source symbols s to lie in the subspace defined by the requirement that the 2nd and the 5th elements of $(D_1ZD_2^{\mathrm{H}}\tilde{P})\boldsymbol{s}$ are both unity. Note that, as in a traditional OFDM system, data can also be sent in conjunction with pilot tones because the aforementioned subspace has non-zero dimension.

7. Conclusion

OFDM systems are best understood in the frequency domain. This paper showed that zero padded systems can also be understood more easily in the frequency domain than in the time domain. Indeed, it was proved that associated with every zero padded system is a channel coded OFDM system such that both systems receive exactly the same symbols after taking into account that OFDM receivers discard the guard interval whereas zero padded systems do not. The advantages of this interpretation were demonstrated by three additional results. It was proved that TZ-OFDM systems spread the source symbols uniformly in the frequency domain, it was explained why there exist exceptional channels over which TZ-OFDM systems perform worse than OFDM systems, and it was shown how zero padded systems can transmit pilot tones.

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