Evidence of the k_1^{-1} Law in a High-Reynolds-Number Turbulent Boundary Layer

T. B. Nickels*
Department of Engineering, University of Cambridge, Cambridge CB2 1PZ, United Kingdom

I. Marusic†
Department of Aerospace Engineering and Mechanics, University of Minnesota, Minneapolis, Minnesota 55455, USA

S. Hafez‡ and M. S. Chong§
Department of Mechanical and Manufacturing Engineering, University of Melbourne, Victoria 3010, Australia
(Received 23 May 2005; published 9 August 2005)

Dimensional analysis and overlap arguments lead to a prediction of a region in the streamwise velocity spectrum of wall-bounded turbulent flows in which the dependence on the streamwise wave number, k_1, is given by k_1^{-1}. Some recent experiments have questioned the existence of this region. In this Letter, experimental spectra are presented which support the existence of the k_1^{-1} law in a high-Reynolds-number boundary layer. This Letter presents the experimental results and discusses the theoretical and experimental issues involved in examining the existence of the k_1^{-1} law and the reasons why it has proved so elusive.

DOI: 10.1103/PhysRevLett.95.074501 PACS numbers: 47.27.Ak, 47.27.Jv, 47.27.Nz

The behavior of turbulent wall-bounded flows at large-Reynolds numbers is still a matter for debate in the scientific community. One reason for this is that many experiments and direct numerical simulations are limited to relatively low Reynolds numbers. An important question is how these results can be extrapolated to higher Reynolds number flows. There are a few theoretical predictions concerning the likely characteristics of these flows at a high-Reynolds number which have been used in modeling and predicting real flows. One of these is the k_1^{-1} behavior for the power spectral density (PSD) of the streamwise fluctuating velocity component in turbulent wall-bounded flows (k_1 is the streamwise wave number). While various physical mechanisms have been associated with the k_1^{-1} law [1,2], the scaling behavior can be derived either by simple dimensional reasoning or, more rigorously, from asymptotic overlap arguments [3]. These arguments are also consistent with the attached eddy hypothesis of Townsend [4]. Essentially, the idea is that there is a range in wave number space in which the effects of both viscosity and the outer length scale (e.g., the boundary layer thickness or the pipe radius) are negligible. The essential feature of this region is that an overlap exists where both *inner scaling* and *outer scaling* are simultaneously valid. Inner scaling refers to nondimensionalization using the length scale z (the wall-normal position) and the velocity scale U_τ (the friction velocity). Outer scaling uses the length scale δ (the boundary layer thickness) and the same velocity scale U_τ. Inner scaling requires that

$$\frac{\phi_{11}(k_1 z)}{U_\tau^2} = \frac{\phi_{11}(k_1)}{zU_\tau^2} = f(k_1 z), \tag{1}$$

and outer-flow scaling requires that

$$\frac{\phi_{11}(k_1 \delta)}{U_\tau^2} = \frac{\phi_{11}(k_1)}{\delta U_\tau^2} = f(k_1 \delta), \tag{2}$$

where $\phi_{11}(k_1 z)$ is the PSD of the streamwise velocity fluctuation per unit nondimensional wave number $k_1 z$. If there is a region where these two scalings are simultaneously valid, then the streamwise spectrum (PSD) must vary as k_1^{-1}. Despite the fact that the arguments involved in deriving such a region are, at least, very plausible, the experimental evidence for the k_1^{-1} law has not always been completely convincing. This has led to questions regarding the existence of such a region. Recent analysis of data from the Princeton Superpipe Facility has suggested that no overlap region is observed and no universal k_1^{-1} region is present [5]. The authors refer to the lack of overlap of the different scalings as "incomplete similarity." Others have interpreted this lack of overlap as due to a variation of velocity scale between the largest eddies in the flow (scaling with δ) and the eddies that scale with z (the local distance from the wall) [6]. Consequently, they suggest that a logarithmic correction is needed to the k_1^{-1} law.

All of the above arguments agree that the existence of a k_1^{-1} region depends on a sufficiently large Reynolds number to ensure an overlap region. What remains unclear at this stage is what value of a Reynolds number qualifies as sufficiently large, and where in the boundary layer measurements should be made in order to discover such a region. In what follows, we present new measurements in the high-Reynolds-number boundary layer facility at the University of Melbourne both at high Reynolds numbers and close to the wall. Note that to observe this region it is not sufficient to have a high Reynolds number; it is also necessary to measure sufficiently close to the surface. The measurements show evidence of a k_1^{-1} region with complete similarity.

The measurements were conducted in the high-Reynolds-number boundary layer facility in the Walter Basset Aerodynamics Laboratory at the University of Melbourne as shown in Fig. 1. This unique facility is especially designed for the experimental study of high-Reynolds-number boundary layers with sufficient thickness to provide for good spatial resolution. The working section is the unique feature of the tunnel. It has a cross section of 2 m $\times$ 1 m (horizontal $\times$ vertical) and is 27 m in length. The long working section allows the boundary layer to grow over a long fetch and hence to produce a high-Reynolds number, thick boundary layer. The boundary layer near the end of the working section is approximately 330 mm thick, providing for good spatial resolution of the measurements. Separate experiments in another wind tunnel [7] show that the extra strain-rate effects due to finite width are negligible as long as the tunnel is wider than six boundary layer thicknesses (6δ). The free-stream turbulence intensity is nominally 0.05%. The working section is above atmospheric pressure and a zero pressure gradient is achieved by bleeding air from the tunnel ceiling through adjustable slots. The coefficient of pressure, C_p, along the working section is constant to within $\pm 1\%$.

The measurements to be presented here have been made using hot-wire probes made from platinum Wollaston wires. All measurements shown were made with wires of 2.5 μm diameter etched to a length (l) of 0.4 mm (giving a length to diameter ratio, $l/d = 160$), which corresponds with $l^+ = 17.5$ at the Reynolds number of the measurements ($\delta^+ = \delta U_\tau/\nu = 14\,380$). Significant attenuation of the streamwise stress occurs only if $l^+ > 20$–25 and only very close to the wall [8]. The wires were run at constant temperature at an overheat ratio of 1.8 using an AA-labs model AN1003 anemometer. The system was checked to ensure second order impulse response and showed a frequency response of greater than 150 kHz. The wires were calibrated statically by traversing them into the center of the tunnel in the undisturbed free stream. The signal was filtered at 35 kHz using 8th order analogue Butterworth filters, to avoid aliasing, and were sampled at 80 kHz using a 16 bit A-D converter board mounted in a personal computer. The probes could be positioned within the boundary layer with an accuracy of 3 μm. The initial position could be determined to an accuracy of ± 5 μm.

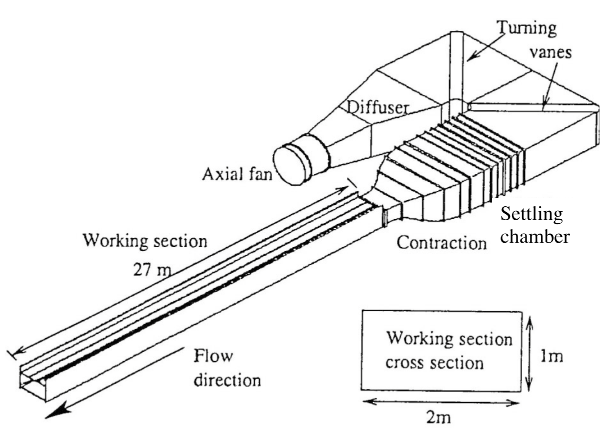

FIG. 1. Schematic of wind tunnel used in this study.

The PSD was determined by sampling bursts (or "records") of the velocity signal of 3.27 s in length at 80 kHz (262 144 points). This was determined to be sufficiently long to capture the lowest wave number motions with significant energy content. Each burst is then fast Fourier transformed online and the PSD formed from the modulus squared of the transformed signal. In order to ensure convergence of the PSD, a series of 500 bursts were taken and the PSDs were ensemble averaged to form the final PSD. Five hundred bursts corresponded with approximately 45 min total time to measure a single spectral curve (for a single wall-normal position). Extreme care was taken to ensure accurate and well-converged data for each measurement. One typical wall-normal position of interest was averaged over 5000 bursts (approximately 7.5 h) and compared with the 500 burst case to ensure that the results were equivalent in all essential details. Calibration drift was checked by recalibrating at the end of every run and proved to be negligible for all cases (the temperature of the tunnel remained constant to within $\pm 1\,^\circ$C). In order to further reduce the scatter in the data points of the original high resolution PSD file (131 072 points), the data were resampled by averaging 27 points of the original file to form 1 point in the final file (at the center of the 27 point window). This was done with nonoverlapping windows in spectral space so that each point in the file was produced from independent data. This is equivalent to the approach of using a notch filter to evaluate the PSD from an analogue velocity signal (as used before the advent of digital sampling equipment). The final file had 4800 points. It should be noted that at the very lowest wave numbers a smaller averaging window was used so as to avoid discarding data at the very low end. No other form of "smoothing" was used on the data. In order to examine this scaling it is necessary to know the mean flow parameters. These were taken from Pitot-tube measurements of the mean velocity profiles. The wall shear stress velocity, U_τ, was found using the method of Clauser, which assumes a logarithmic region in the mean velocity profile with universal constants. This technique has been shown to be accurate by comparison with direct measurements using oil-film interferometry, in the same facility, by Jones *et al.* [9], who also checked and confirmed that the mean velocity variation was logarithmic. The boundary layer thickness definition used is that of Perry *et al.* [10], i.e.,

$$\delta = \delta^* C_1/S, \tag{3}$$

where δ^* is the displacement thickness of the boundary layer, $S = U_1/U_\tau$ (U_1 is the free-stream velocity), and C_1 is a constant found from integrating the velocity-defect profile, i.e.,

$$C_1 = \int_0^\infty \frac{U_1 - \bar{U}}{U_\tau} d(z/\delta). \tag{4}$$

The streamwise wave number k_1 was found from the frequency f, using Taylor's hypothesis of frozen turbulence, i.e.,

$$k_1 = \frac{2\pi f}{U_c}, \tag{5}$$

where U_c is the convection velocity of the turbulent motions past the probe. The results shown here use the local mean velocity at the probe as the convection velocity, but the choice of external mean velocity was also examined and did not make any significant change to the conclusions presented.

The results shown were obtained at a streamwise position 22 m downstream from the exit of the contraction tripping device. The boundary layers on all four walls were tripped using grade 40 sandpaper strips located inside the contraction. All profiles were measured with a nominal free-stream velocity of 20 m/s, except for one shown in Fig. 4 (note that for runs on different days the Reynolds number per unit streamwise distance was matched by varying the tunnel velocity to compensate for viscosity variation). The Reynolds number at this station for 20 m/s is $R_\theta = 37\,540$, which corresponds with $\delta^+ = 14\,380$.

Figure 2 shows the premultiplied spectra in the vicinity of $z^+ = zU_\tau/\nu \geq 100$ scaled with inner scaling (upper plot) and outer scaling (lower plot). It is clear that there is a region where the spectra collapse with both of these scalings. This region corresponds with a plateau in the premultiplied spectra, which indicates k_1^{-1} behavior. The region is approximately 1/3 of a decade long in the profile closest to the wall. A close-up of the region is shown in Fig. 3 where every second level has been removed to avoid confusion. The behavior of the spectra in the two plots is also consistent with the expected scaling behavior, since, in the inner-scaled plot, the profiles collapse at the high end of the region and peel off at the low end as the probe moves away from the wall. This peel off is due to the outer scaling of the lower end of the region. With outer-flow scaling the spectra collapse at the low end of the region and peel off at the high end as the wall-normal distance is increased—this is the effect of changing z. The scaling behavior of the spectra and the plateau are consistent with complete similarity arguments and the existence of a k_1^{-1} region. Figure 4 shows the scaling for two tunnel velocities where U_τ^2 is different by a factor of 1.7 and the Reynolds number by 1.3 but with closely matched z/δ. Only inner scaling is shown since when the z/δ values are matched the outer-scaling plot would look the same except that the values on the wave number axis would be larger. The collapse of the two spectra shows that complete similarity still applies for the different flow conditions. This confirms that U_τ is the appropriate velocity scale since the change in U_τ does

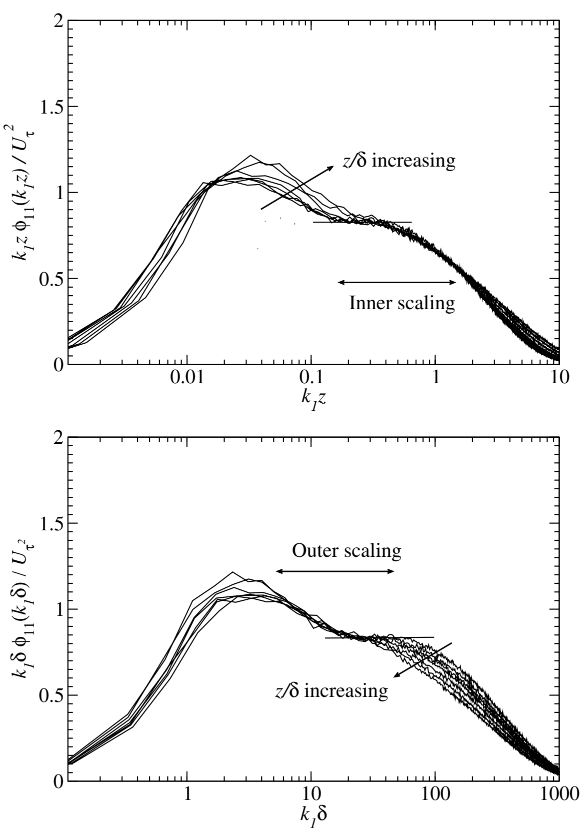

FIG. 2. Streamwise spectra in inner scaling (upper plot) and outer scaling (lower plot). Wave number calculated using U_c equal to local mean velocity at each wall-normal position. Levels shown: $z/\delta = 0.0070$, 0.0077, 0.0084, 0.0092, 0.0205, 0.0123, 0.0140. The lowest level corresponds to $z^+ = 100$.

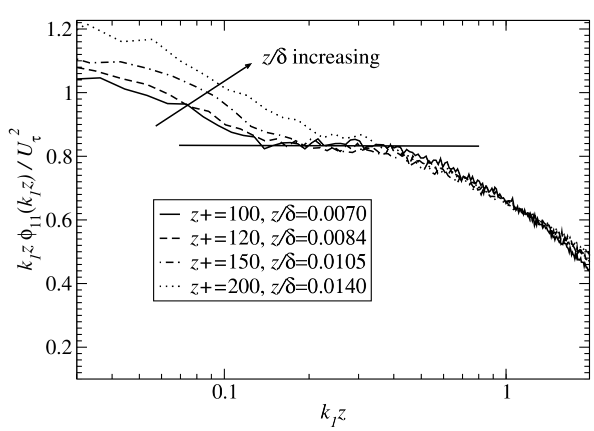

FIG. 3. Close-up of the k_1^{-1} region of spectra in premultiplied form. Wave number calculated using $U_c = \bar{U}$. Inner scaling.

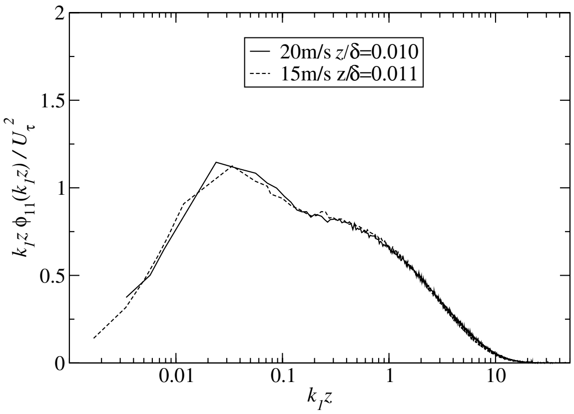

FIG. 4. Streamwise spectra in inner scaling for two tunnel velocities and virtually the same z/δ. 20 m/s: $z^+ = 150$; 15 m/s: $z^+ = 118$. Note that only inner scaling is shown. The collapse would be the same in outer scaling since the z/δ is closely matched.

not affect the collapse (the previous plots show spectra at the same station and velocity, and hence U_τ is the same for each level). It was not possible to vary the conditions much further since lower velocities have no region of similarity and higher velocities compromise spatial resolution.

The experiments shown here allow us to make some assessment of the conditions required to obtain a k_1^{-1} region in the streamwise spectrum. The outer-flow scaled plot shows that the k_1^{-1} region starts at approximately $k_1\delta = 21$ and the inner-flow scaled plot shows that the region ends at approximately $k_1 z = 0.4$. Results (not presented here) suggest that a region of complete similarity is not possible below $z^+ = 100$ due to the increased significance of viscous effects. There is still some dispute in the literature regarding the height above which viscous effects can be neglected [5,11], but in this experiment the mean flow and turbulence statistics show no significant viscous effects beyond $z^+ = 100$.

These limits allow us to make some comments on existing and future measurements. Define the highest wave number within the k_1^{-1} region as $k_{1\mathrm{up}}$ and the lowest wave number in the region as $k_{1\mathrm{low}}$. If we want to have one decade of k_1^{-1} behavior, then we need $k_{1\mathrm{up}}/k_{1\mathrm{low}} = 10$. Using the limits above, we find that this requires $z/\delta = 0.0019$. Now this value of z/δ must correspond with $z^+ \geq 100$ (to avoid viscous effects), so the Reynolds number required is $\delta^+ \geq 52\,500$ (corresponding to $R_\theta = 138\,000$). This is currently beyond the capability of the wind tunnel used in this study. It is very important to note that the length of the k_1^{-1} region depends only on z/δ with the restriction that this corresponds with a sufficiently large $z^+ (\geq 100)$.

We can also estimate the maximum z/δ beyond which no overlap region may be observed by setting $k_{1\mathrm{up}} = k_{1\mathrm{low}}$ which gives $z/\delta = 0.019$. This is likely to be the reason that no region of "complete similarity" was observed in the superpipe experiments [5] since their smallest value of z/δ was only 0.03.

Hence it may be observed that a high-Reynolds-number flow is not sufficient to observe complete similarity: it is also necessary to measure very close to the surface (and with sufficient spatial resolution). This makes the measurements difficult in small high-Reynolds-number facilities.

It should be noted that there is still controversy regarding the existence and location of the k_1^{-1} region in wall-bounded flows. Other data from different flows and facilities are still needed in order to better understand the reasons for the differences between these and previous measurements.

M. S. C. and S. H. are grateful for support from an Australian Research Council Discovery grant. I. M. would like to acknowledge support from NSF (CTS-0324898) and the Packard Foundation. T. B. N. is grateful to the EPSRC(GR/S86303/01) for support to make the measurements.

*Electronic address: tbn22@eng.cam.ac.uk
†Electronic address: marusic@aem.umn.edu
‡Electronic address: shafez@unimelb.edu.au
§Electronic address: min@unimelb.edu.au

[1] J. C. R. Hunt and P. Carlotti, Flow, Turbul. Combust. **66**, 453 (2001).
[2] A. E. Perry and M. S. Chong, J. Fluid Mech. **119**, 173 (1982).
[3] A. E. Perry, S. M. Henbest, and M. S. Chong, J. Fluid Mech. **165**, 163 (1986).
[4] A. A. Townsend, *Turbulent Shear Flow* (Cambridge University Press, Cambridge, England, 1976).
[5] J. F. Morrison, W. Jiang, B. J. McKeon, and A. J. Smits, Phys. Rev. Lett. **88**, 214501 (2002).
[6] J. C. Del Alamo, J. Jimenez, P. Zandonade, and R. D. Moser, J. Fluid Mech. **500**, 135 (2004).
[7] M. B. Jones, I. Marusic, and A. E. Perry, in *Proceedings of the 12th Australasian Fluid Mechanics Conference* (University of Sydney, Australia, 1995), pp. 473–476.
[8] P. M. Ligrani and P. Bradshaw, Exp. Fluids **5**, 6 (1987).
[9] M. B. Jones, N. Nishizawa, and M. S. Chong, in *Proceedings of the 14th Australasian Fluid Mechanics Conference* (University of Adelaide, Australia, 2001), pp. 211–214.
[10] A. E. Perry, I. Marusic, and M. B. Jones, J. Fluid Mech. **461**, 61 (2002).
[11] J. M. Österlund, A. V. Johansson, H. M. Nagib, and M. H. Hites, Phys. Fluids **12**, 1 (2000).