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The relationship between the velocity skewness and the amplitude modulation of the small scale by the large scale in turbulent boundary layers

Romain Mathis,[1,a)] Ivan Marusic,[1] Nicholas Hutchins,[1] and K. R. Sreenivasan[2]
[1]*Department of Mechanical Engineering, University of Melbourne, Victoria 3010, Australia*
[2]*Department of Physics and the Courant Institute of Mathematical Sciences, New York University, New York, New York 10012, USA*

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A defining feature of the inner-outer interactions in wall-bounded turbulent flows is the imprint of the outer large-scale motions on the inner small scale. Recently, Mathis *et al.* ["Large-scale amplitude modulation of the small-scale structures in turbulent boundary layers," J. Fluid Mech. **628**, 311 (2009)] quantified this imprint by applying the Hilbert transform to small-scale components of the fluctuating streamwise velocity, u. They found that the wall-normal profile of the amplitude modulation between the large scale and the envelope of the small scale exhibits strong resemblance to the skewness profile of u. In this study, we assess this apparent relationship and show that the Reynolds number trend in the skewness profile of u is strongly related to the amplitude modulation effect of the small scales by the large. This observation also leads to an alternative diagnostic for the amplitude modulation effect, which is one component of the skewness factor based on a scale decomposition. © *2011 American Institute of Physics*. [doi:10.1063/1.3671738]

In wall-bounded turbulent flows, the highest levels of shear and turbulence production occur close to the wall. Considerable attention has been devoted to studying the small-scale motions that populate this "inner" region. A long-standing question in the field is the nature of the interaction between this inner region and the large-scale motions in the "outer" region, with varying viewpoints on the degree of interaction.[1–5] In recent years, the discussion has gained greater clarity with the advent of high-Reynolds number facilities and improved instrumentation,[6–8] in conjunction with enhanced capabilities in numerical simulation.[9,10] Accumulating evidence supports the view that large-scale events are universally present in wall-bounded flows,[4,11–15] and that the large-scale features significantly influence the near-wall region, imposing a strong "footprint" down to the wall[12,16] (e.g., the attached-eddy hypothesis of Townsend[17]), distinctly modulating the small-scale motions in the inner region.[18] Further, it appears that the large-scale motions become increasingly energetic at higher Reynolds numbers, thus strengthening their interaction with the inner small-scale motions. A fuller discussion of these advances and other issues arising at high Reynolds numbers can be found in recent reviews.[6,19,20]

In an attempt to characterize and quantify the modulating influence, Mathis *et al.*[21] developed a procedure involving a single-point amplitude modulation coefficient, defined as the correlation coefficient between the filtered envelope of the small-scale fluctuations, $E_L\left(u_S^+\right)$, and the large-scale component, u_L^+, as

$$AM(z^+) = \frac{\overline{u_L^+ E_L(u_S^+)}}{\sqrt{\overline{u_L^{+2}}}\;\sqrt{\overline{E_L(u_S^+)^2}}}. \qquad (1)$$

Here, z is the distance normal to the wall and the subscript + indicates normalization with the friction velocity U_τ and the kinematic viscosity of the fluid, ν; for example, $u^+ = u/U_\tau$ and $z^+ = zU_\tau/\nu$. To obtain the terms in Eq. (1), the fluctuating streamwise velocity u^+ was decomposed into the large-scale u_L^+ and the small-scale $u_S^+, u^+ = u_L^+ + u_S^+$, by choosing the cut-off wavelength $\lambda_x^+ = 7000$ (where λ_x is the streamwise wavelength). The cut-off wavelength was determined by studying spectrograms,[21] and the results were insensitive to its variations within 30%.[22] The filtered envelope of the small-scale contribution was obtained via a Hilbert transformation (see Sreenivasan[23] and, for specific details, Mathis *et al.*[21]). This correlation coefficient $AM(z^+)$ is similar to the one proposed by Bandyopadhyay and Hussain,[5] who studied the interaction between small- and large-scale motions in various shear flows. It should be noted that the concept of amplitude modulation is closely related to the concept of intermittency as discussed by Sreenivasan[23] and Kholmyansky *et al.*,[24] where an enveloping function is used to modulate faster fluctuations. A typical trend of the wall-normal variation of $AM(z^+)$, along with the skewness profile $S_u(z^+)$ of the streamwise velocity component, is shown in Fig. 1 for Kármán number $Re_\tau = 2800$ (where $Re_\tau = \delta U_\tau/\nu$, with δ the boundary layer thickness). In their paper, Mathis *et al.*[21] commented that the profiles of $AM(z^+)$ and the skewness $S_u(z^+)$ resemble each other strongly but did not investigate the connection further.

Schlatter and Örlü[26] explored the resemblance between $AM(z^+)$ and $S_u(z^+)$ using experimental and synthetic signals, and urged caution on the use of the coefficient AM. Their synthetic signal was obtained by randomly shuffling amplitudes of an actual time-series signal so that the probability density function (PDF), and hence the skewness factor, was the same for both. They found that the modulation coefficient AM of

[a)]Electronic mail: rmathis@unimelb.edu.au.

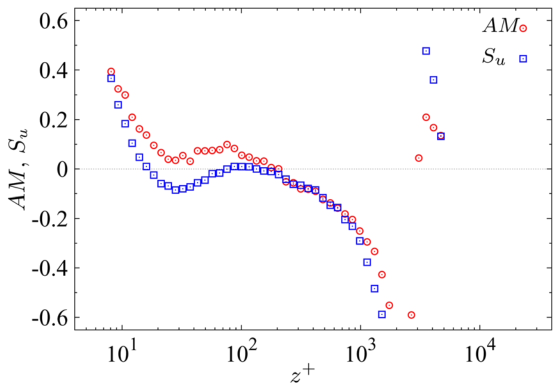

FIG. 1. (Color online) Profile of the amplitude modulation coefficient AM and skewness factor S_u; $Re_\tau = 2800$.

the synthetic signal was also close to the skewness factor S_u of the original signal. This finding led them to conclude that AM may not be an independent tool for unambiguously quantifying the effect of large-scale amplitude modulation of small scales. While their study has clearly shown that the AM coefficient and the skewness factor are related, their cautious conclusions were restricted to relatively low Reynolds numbers with limited scale-separation; more importantly, their synthetic signal is not representative of real turbulence signals as its energy is weighted towards the smaller scales as in white noise (see Mathis *et al.*, Proc. TSFP7, Ottawa, 2011). Therefore, the results from the scale-decomposition on such a signal (which is the first step in obtaining AM in Eq. (1)) are not necessarily applicable to turbulent signals, and the caution advocated by Schlatter and Örlü[26] may not apply to real signals in high-Reynolds-number turbulent flows. Bernadini and Pirozzoli[25] also considered the implications of AM and suggested instead that amplitude modulation can be more robustly captured by exploiting the covariance of u_L^+ at one wall-normal location, with $E_L(u_S^+)$ at a different wall-normal location. They again noted an increasing top-down interaction between the outer and inner regions with increasing Reynolds number.

Here, we advance our understanding of the relationship between the turbulence modulation and the skewness in wall-bounded flows, using experimental datasets obtained in the high Reynolds number boundary layer wind tunnel at the University of Melbourne. The data consist of time series of u measured across boundary layers for Kármán numbers in the range of $Re_\tau = 2800$–$19\,000$. Details of hot-wire measurements and experimental parameters are given by Hutchins *et al.*[27] and Mathis *et al.*[21]

To get an insight into the AM and S_u relationship, we consider the expansion of S_u using a scale-decomposed signal $u^+ = u_L^+ + u_S^+$, such that

$$\overline{u^{+3}} = \overline{u_L^{+3}} + 3\overline{u_L^{+2}u_S^+} + 3\overline{u_L^+u_S^{+2}} + \overline{u_S^{+3}}, \tag{2}$$

$$\text{giving}\quad S_u = \overline{\overline{u_L^{+3}}} + 3\overline{\overline{u_L^{+2}u_S^+}} + 3\overline{\overline{u_L^+u_S^{+2}}} + \overline{\overline{u_S^{+3}}}, \tag{3}$$

$$\text{with}\quad \overline{\overline{X}} = \overline{X}/\left(\overline{u^{+2}}\right)^{3/2}, \tag{4}$$

where a single overbar indicates the time average. Such a decomposition was first used by Schlatter and Örlü[26] for wall-bounded flows but was previously considered by Sreenivasan *et al.*[34] who studied the effects of large scales on the inertial range in high-Reynolds-number turbulent flows. They found that odd-order moments are affected more strongly by the large scales than even-order moments. In particular, they emphasized that a significant part of the third-order structure function comes from the straining of the small-scale kinetic energy by the large-scales. In other words, this suggests a modulation of the small-scales by the large. More recently, Schlatter and Örlü[26] also considered the two cross-terms of the scale-decomposed skewness factor to propose an alternative amplitude modulation coefficient factor (referred to as R^*), in order to demonstrate the existence of an intimate relationship between statistical asymmetry and amplitude modulation effects, as discussed above. Based on those previous results, and further discussion with Schlatter and Örlü,[35] we employ here the same scale-decomposition of the skewness factor to assess and understand the physical meaning of this apparent relationship. It is noted that the conclusions drawn from the following results would be the same if we used the third order moment $\overline{u^{+3}}$ instead of skewness S_u.

The wall-normal variations of all the terms of the scale-decomposed skewness factor are shown in Fig. 2 for $Re_\tau = 2800$. Up to approximately the middle of the log-layer, $z^+ \simeq 200$, the small-scale term $\overline{\overline{u_S^{+3}}}$ appears to account for the majority of the skewness factor, the other terms having little or negligible contributions. We do not consider the outer wake region where intermittency effects are known to affect S_u strongly.[28] It is interesting to note that, as highlighted in the inset of Fig. 2, the cross term $3\overline{\overline{u_L^+u_S^{+2}}}$ has a non-negligible contribution, whereas the two other terms, $3\overline{\overline{u_L^{+2}u_S^+}}$ and $\overline{\overline{u_L^{+3}}}$, are close to zero. It is interesting to note that, while $\overline{\overline{u_L^{+3}}}$ is nominally zero in the inner-region (see Fig. 2), the second order moment of the large-scale component, $\overline{u_L^{+2}}$, increases with increasing Re_τ.[27] Such behaviour is consistent with previous findings that elongated low-speed regions are usually flanked on either side by high-speed regions of the same magnitude.[12,29–31]

To gain a better understanding of the role of all dominant terms of the decomposed skewness factor, profiles of

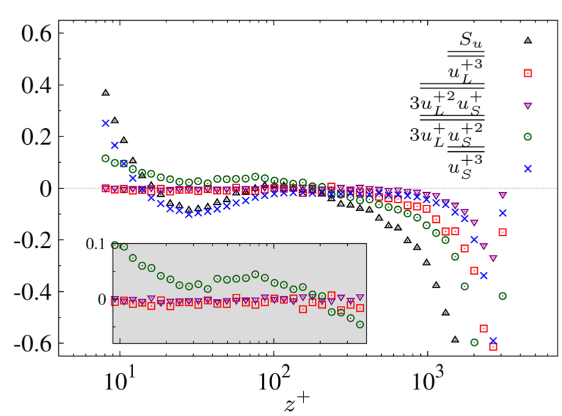

FIG. 2. (Color online) Profile of the scale-decomposed skewness factor; $Re_\tau = 2800$.

S_u, $\overline{\overline{u_S^{+3}}}$ and $\overline{\overline{3u_L^+ u_S^{+2}}}$ are plotted in Fig. 3 for several Reynolds numbers. By comparing data over a large range of Reynolds number, from laboratory facilities to atmospheric surface layers, Metzger and Klewicki[32] have previously reported an increase of S_u with Reynolds number in the near-wall region. In particular, they observed a change in the sign of the minimum occurring in the buffer region, which is consistent with the presented data shown in Fig. 3(a) (the vertical dot-dashed line marks the location of the minimum at $z^+ \simeq 30$.) The physical understanding of this change in the sign of skewness with increasing Reynolds number has been lacking and hence the observation has been somewhat controversial since the time it was first reported by Metzger and Klewicki. However, the Reynolds number trend of the dominant terms of the scale-decomposed skewness given in Figs. 3(b) and 3(c) do provide some insights. The small-scale term $\overline{\overline{u_S^{+3}}}$ appears to contribute locally to the rise of the minimum observed in the S_u profile at $z^+ \simeq 30$. In contrast, $\overline{\overline{3u_L^+ u_S^{+2}}}$ is dependent on Reynolds number across the whole boundary layer, especially in the buffer layer with values nearly four times higher for the $Re_\tau = 19\,000$ case compared with the $Re_\tau = 2800$ profile. What is also particularly notable is the high degree of resemblance of these profiles in Fig. 3(c) to the profiles of the amplitude modulation coefficient AM shown in Fig. 1. This is clearly seen in Fig. 4, where the two cases are plotted together. This correspondence suggests that the cross-term $\overline{\overline{3u_L^+ u_S^{+2}}}$ of the scale-decomposed skewness factor may be used as an alternative or complementary diagnostic tool to AM, to quantify the level of amplitude modulation of the small-scales by the large-scales. This is not a very surprising conclusion given that this term is similar to a small-scale envelope $\left(u_S^{+2}\right)$ correlated with a large-scale component $\left(u_L^+\right)$, which is intrinsically linked to the large-scale amplitude modulation effect. It should be noted that in this comparison the envelope of the small-scales, u_S^{+2}, is not filtered, and thus tracks not only the large-scale modulation but also the small-scale variations.

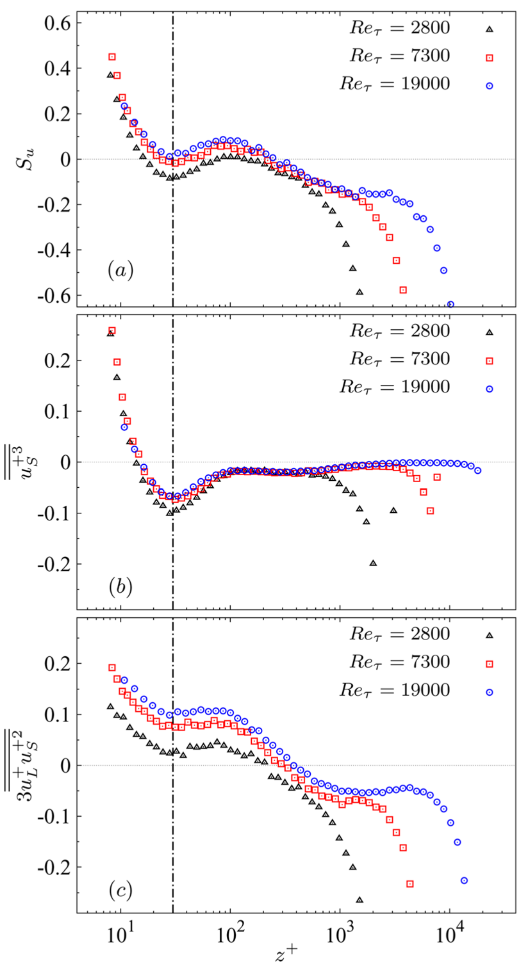

FIG. 3. (Color online) Reynolds number trends of (a) the skewness factor S_u and dominant terms of the expansion of the skewness factor, (b) $\overline{\overline{u_S^{+3}}}$, and (c) $\overline{\overline{3u_L^+ u_S^{+2}}}$. The vertical dot-dashed line marks the location of the minimum of S_u.

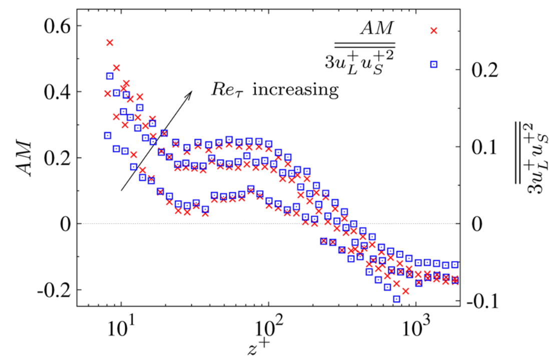

FIG. 4. (Color online) Profile of the amplitude modulation coefficient AM (the left vertical axis) and the cross term $\overline{\overline{3u_L^+ u_S^{+2}}}$ of the skewness factor expansion (the right vertical axis), for Reynolds numbers $Re_\tau = 2800$, 7300, and 19 000.

The similarity between AM and $\overline{\overline{3u_L^+ u_S^{+2}}}$ also suggests that the Reynolds number trend of the skewness factor is closely related to a rising amplitude modulation effect as Re_τ increases. Indeed, it is now known that the large-scale motions strengthen with increasing Reynolds number, as does the amplitude modulation effect[21] shown here to contribute to the rise in skewness. In fact, a reconstruction of the skewness factor without the cross-term $\overline{\overline{3u_L^+ u_S^{+2}}}$, e.g., $S_u = \overline{\overline{u_L^{+3}}} + \overline{\overline{3u_L^{+2} u_S^+}} + \overline{\overline{u_S^{+3}}}$, shows a constancy over one order of magnitude in Reynolds number, as seen in Fig. 5, as

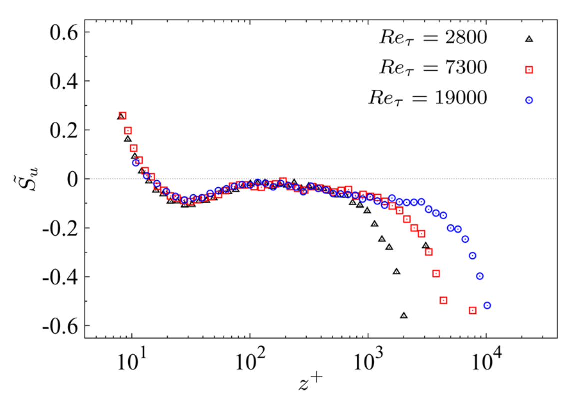

FIG. 5. (Color online) Reconstruction of the skewness factor without the cross term $\overline{\overline{3u_L^+ u_S^{+2}}}$, e.g., $\widetilde{S}_u = \overline{\overline{u_L^{+3}}} + \overline{\overline{3u_L^{+2} u_S^+}} + \overline{\overline{u_S^{+3}}}$.

opposed to the trend observed in Fig. 3(a). This feature has been observed recently by Marusic *et al.*[22,33] in their predictive model of the streamwise fluctuating velocity field of the near-wall region, which specifically takes into account the amplitude modulation effect. They noted that all odd-order moments are incorrectly predicted, particularly at high Reynolds numbers, if the amplitude modulation effect is not included.

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