Reynolds Number Invariance of the Structure Inclination Angle in Wall Turbulence

Ivan Marusic*
Department of Mechanical and Manufacturing Engineering, University of Melbourne, Victoria 3010, Australia

Weston D. C. Heuer
Department of Aerospace Engineering and Mechanics, University of Minnesota, Minneapolis, Minnesota 55455, USA
(Received 31 May 2007; published 14 September 2007)

Cross correlations of the fluctuating wall-shear stress and the streamwise velocity in the logarithmic region of turbulent boundary layers are reported over 3 orders of magnitude change in Reynolds number. These results are obtained using hot-film and hot-wire anemometry in a wind tunnel facility, and sonic anemometers and a purpose-built wall-shear stress sensor in the near-neutral atmospheric surface layer on the salt flats of Utah's western desert. The direct measurement of fluctuating wall-shear stress in the atmospheric surface layer has not been available before. Structure inclination angles are inferred from the cross correlation results and are found to be invariant over the large range of Reynolds number. The findings justify the prior use of low Reynolds number experiments for obtaining structure angles for near-wall models in the large-eddy simulation of atmospheric surface layer flows.

DOI: 10.1103/PhysRevLett.99.114504 PACS numbers: 47.27.Jv, 92.60.Fm

Skin friction drag on aircraft and other vehicles and the performance of many aerodynamic and hydrodynamic components rely on the behavior of the wall-bounded region of turbulence adjacent to the surface. How this turbulent boundary layer behaves at large Reynolds numbers is still a matter for debate in the scientific community [1]. This is of considerable importance since many flows in practical applications are at high Reynolds number, while most laboratory experimental facilities are restricted to 2–3 orders of magnitude lower Reynolds number. Purpose-built facilities have been constructed to achieve high Reynolds numbers, such as the Princeton superpipe [2] or using cryogenics [3]. However, while much has been learned from these facilities, high Reynolds numbers are obtained by using fluids with low kinematic viscosity. Consequently, access to, and measurements of, the near-wall region and the lower portion of the logarithmic region are restricted because of the very small viscous length scales involved.

An alternative approach to achieving high Reynolds numbers, under incompressible conditions, is to use large length scales. Here we adopt this strategy and obtain high Reynolds numbers by conducting experiments in the lower portion of the atmospheric surface layer (ASL) at the Surface Layer Turbulence and Environmental Science Test (SLTEST) facility in western Utah. The SLTEST facility is located on extremely flat and barren salt flats that extend over 240 km north to south and 48 km east to west. Under carefully monitored neutrally buoyant conditions, the SLTEST has been successfully used by a number of investigators [4–10] to study near-wall turbulence physics, near-wall models for large-eddy simulations, and other boundary layer phenomena.

One parameter, which is of fundamental importance to turbulent boundary layer theory, is the time-varying wall-shear stress, and to date no previous measurements of this quantity have been available at high Reynolds numbers. Here we report measurements of fluctuating wall-shear stress measurements in the ASL using a purpose-built sensor for the SLTEST site. From these measurements we obtain cross correlations of streamwise fluctuating wall-shear stress τ, with u, the fluctuating component of streamwise velocity for different wall-normal positions in the logarithmic region of the turbulent boundary layer. The cross correlation is defined as

$$R_{\tau u}(\Delta t) = \frac{\langle \tau(t) u(t + \Delta t)\rangle}{\sqrt{\langle \tau^2\rangle \langle u^2\rangle}}, \qquad (1)$$

where Δt is time delay and angle brackets indicate long-time averages. This is a fundamental parameter that has been extensively documented in laboratory facilities. Brown and Thomas [11] used a hot-film wall-shear stress sensor in conjunction with a wall-normal array of four hot wires, above the shear-stress sensor, to measure fluctuating velocities. They found that the measured cross correlations supported the hypothesis that an organized structure exists in the boundary layer, and the structure is at an oblique angle to the free stream. Knowledge of this inclination angle θ has been used to develop models of the near-wall region for large-eddy simulations of turbulent boundary layers [12,13]. Here we define θ as

$$\theta = \arctan(z/\Delta x^*), \qquad (2)$$

where z is the wall-normal position (where velocity is measured) and Δx^* is the spatial delay corresponding to a peak in $R_{\tau u}$. Here we have converted from time to space using Taylor's hypothesis of frozen turbulence. That is, $\Delta_x = U\Delta t$, where the convection velocity U is taken to be the mean velocity at the corresponding wall-normal position.

We report the results from two separate experiments that span 3 orders of magnitude change in Reynolds number. The first experiment was conducted at the University of

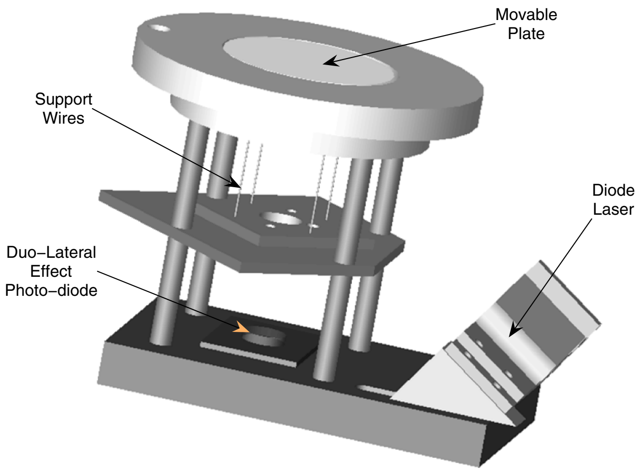

FIG. 1 (color online). Schematic of wall-shear stress sensor (taken from [15]). The sensor is encapsulated and buried, with the top surface of the sensor flush with the salt flats.

Minnesota in a boundary layer wind tunnel with a 4 m long working section, using surface hot-film and hot-wire anemometry for the τ and u measurements, respectively. Full details of the experimental procedures and apparatus are given in Refs. [13,14]. The second experiment was conducted at the SLTEST site in Utah using a wall-normal array of five sonic anemometers. The anemometers are Campbell Scientific CSAT3 and measure all three components of velocity. The wall-shear stress was simultaneously measured using a custom-made sensor, shown schematically in Fig. 1. The sensor was flush mounted into the salt flats playa and measures the two components of wall-shear stress by using a duolateral photodiode detector to monitor the deflection of a laser beam reflected from a 50 mm diameter floating element, which is suspended by four thin support wires. This enables accurate measurement of the small forces that are encountered during the ASL measurements [$O(100\ \mu$N)]. Full details of the sensor and its calibration procedure are given in Ref. [15].

The ASL measurements were conducted on May 24, 2004. Table I gives the wall-normal positions and measured mean velocities for each of the sonics. The friction velocity U_τ (velocity based on mean wall-shear stress [16]) was 0.28 m/s. The conditions were nominally neutrally buoyant with the Monin-Obukov length L equal to 129 m giving a stability parameter [16] at the top positioned sonic of $z/L = -0.02$. The top of the surface layer was estimated to be 100 m (based on previous mini-Sodar measurements)

TABLE I. Wall-normal position of sonic anemometers and measured mean streamwise velocities.

Sonic number	1	2	3	4	5
z (m)	0.24	0.50	0.91	1.65	2.93
U (m/s)	5.3	5.8	6.2	6.7	7.2

and we use this as an equivalent boundary layer thickness δ.

Figure 2 shows a sample 25 s record of simultaneously sampled fluctuating wall-shear stress from the new sensor and fluctuating velocities from the five sonic anemometers. A high degree of correlation is observed, at least, for the first wall-normal position u and τ. The resulting cross correlations, from data acquired over 2400 s and sampled at 50 Hz, are shown in Fig. 3. Also shown in the figure are the wind tunnel results for friction Reynolds number $\mathrm{Re}_\tau = 1350$. ($\mathrm{Re}_\tau = \delta U_\tau/\nu$ where ν is kinematic viscosity.) All profiles correspond to velocity measurements nominally in the logarithmic region of the boundary layer. Significantly higher levels of cross correlations are observed for the ASL but they appear to be consistent with δ scaling, that is, with the changing level of z/δ from the wind tunnel to ASL range. Tests were made of the validity of the cross correlation results due to the finite sampling time and sampling volume of the sonics. For example, the sonics measure velocities over a spatial dimension of 0.1 m and hence an effective low-pass filtering is taking place. To test for these effects we use the results from a separate experiment conducted in 2005 where u time series were measured in the ASL using hot wires with 1 mm sensing lengths, and sampled for 900 s at 10 kHz sampling rate. The hot wires were positioned at $z = 0.12$ and 1.15 m, respectively. Down sampling these data to mimic the 50 Hz response of the sonics showed negligible effect (within the accuracy of the measurements). Figure 4 shows the effect of varying sampling volume on the cross correlations between the two fluctuating velocities. Here the two u signals were filtered by performing a convolution with

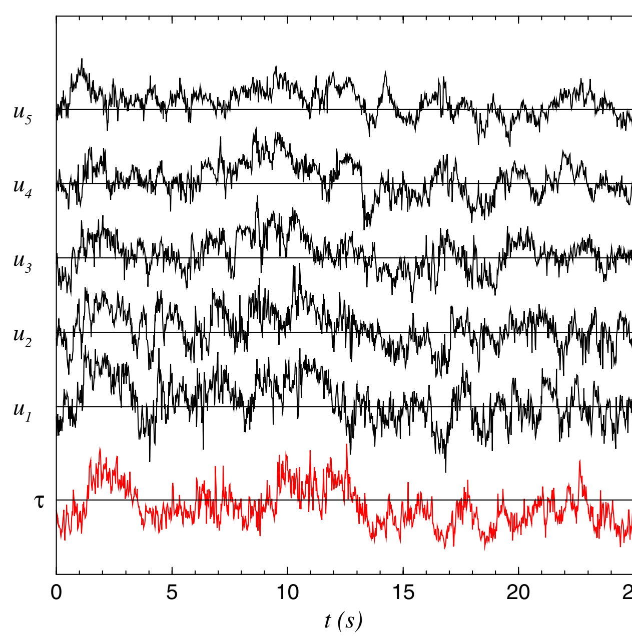

FIG. 2 (color online). Sample record of simultaneous signals from wall-shear stress sensor and sonic anemometers in the ASL experiment.

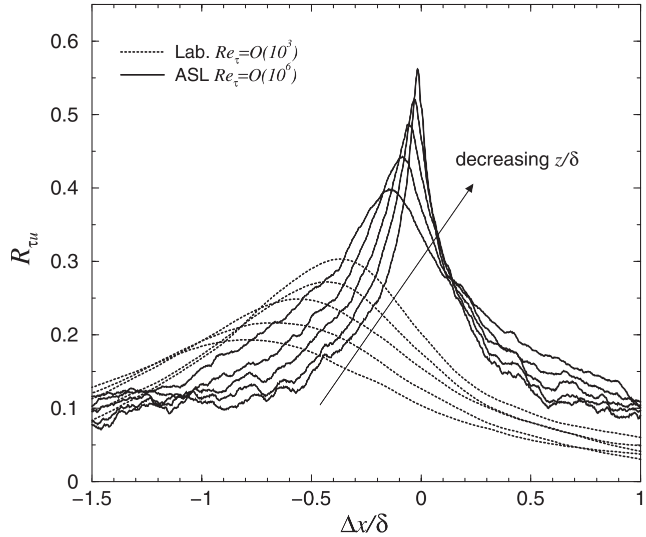

FIG. 3. Cross correlation of τ and u results. Solid lines are the ASL measurements for $z/\delta = 0.0024$, 0.005, 0.0091, 0.0165, and 0.0293. Dotted lines correspond to laboratory wind tunnel results for $z/\delta = 0.073$, 0.091, 0.115, 0.145, and 0.183.

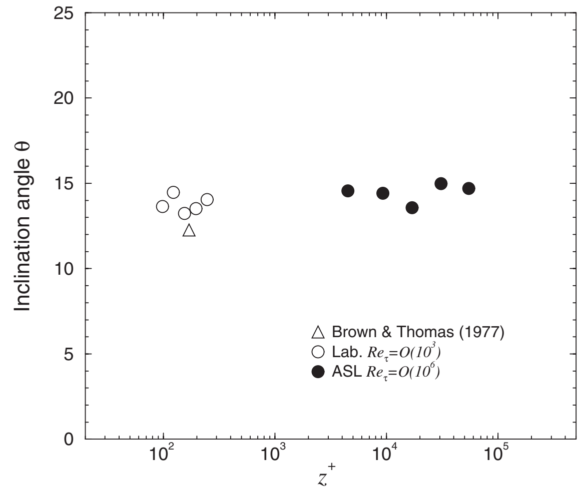

FIG. 5. Structure inclination angle results.

boxcar filters of varying lengths. The results show that only minor changes in the level of cross correlation occur with a filter length of 0.1 m, and a negligible shift in the location of the peak cross correlation level is observed. (The profiles in Fig. 4 are observed to be not as well converged compared to Fig. 3 due to the smaller sample size used. However, this does not affect the conclusions regarding the effect of spatial resolution.)

The structure inclination angle results corresponding to the $R_{\tau u}$ results are shown in Fig. 5 versus z^+ (zU_τ/ν). The wind tunnel results show no discernible trend with changing wall-normal position in the log layer with an average θ of 13.8°. This compares reasonably well with the experiment of Brown and Thomas for $\mathrm{Re}_\tau = 3400$ with $\theta = 12.3°$, while the ASL experiment at $\mathrm{Re}_\tau = O(10^6)$ has an average $\theta = 14.4°$ over the five positions. Other experiments at $\mathrm{Re}_\tau = O(10^3)$ [17] report angles of nominally 16°. Therefore, within the small scatter of the results, the experiments show strong support for invariance of the structure inclination angle over 3 orders of magnitude change in Reynolds number.

The invariance of θ is good news for workers who have relied on laboratory scale measurements to determine what inclination angle to use in wall-layer models for large-eddy simulations of atmospheric boundary layers [18,19]. Prior investigations have been done in the ASL using wall-normal arrays of sonic anemometers, but they have relied on inferred wall-shear stress and not on direct measure-

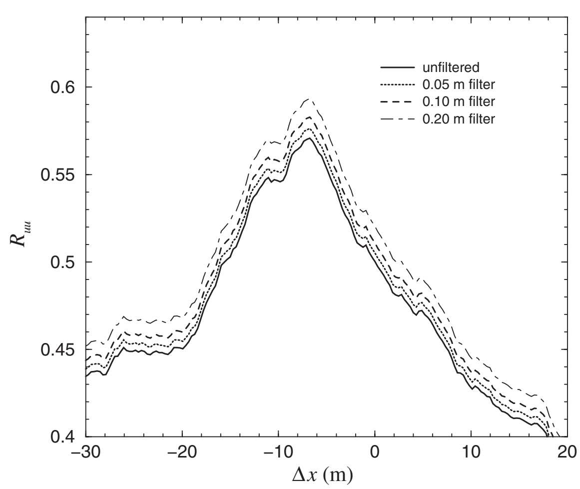

FIG. 4. Cross correlations from highly spatially resolved hot-wire measurements in the atmospheric surface layer with varying levels of prefiltering applied. Only small deviations are seen for a filter length of 0.1 m, which corresponds to the spatial resolution of the sonic anemometers.

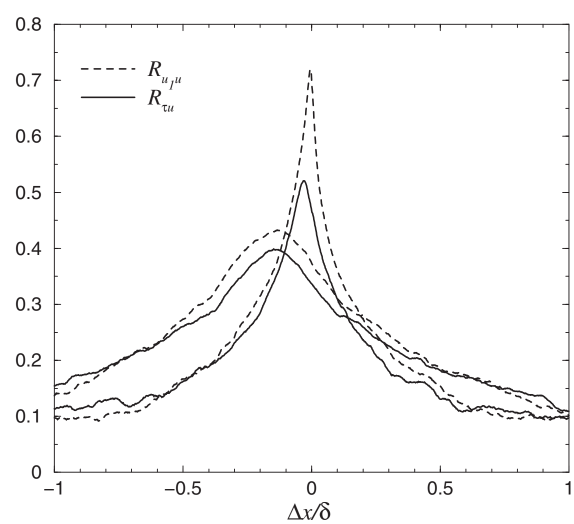

FIG. 6. Comparison of cross correlations between τ and u, and u_1 (the fluctuating velocity from lowest sonic) and u, from sonic 2 and sonic 5. The profiles with lower peak values correspond to u from sonic 5.

TABLE II. Peak values of cross correlation and inclination structure angles corresponding to profiles in Fig. 6.

	$(R_{\tau u})_{\max}$	$(R_{u_1 u})_{\max}$	θ^{a}	$\theta_{u_1 u}{}^{b}$
Sonic 2 ($z = 0.50$ m)	0.52	0.72	14.4	27.5
Sonic 5 ($z = 2.93$ m)	0.40	0.43	14.7	11.8

[a]Using $R_{\tau u}$.
[b]Using $R_{u_1 u}$.

ments. The most common approach is to use the cross correlation of the lowest u sensor to the surface with velocities at higher wall-normal positions. Using this, Carper and Porte-Agel [6] obtained a structure inclination of 16° for salt flat experiments, while Boppe *et al.* [20] obtained approximately $\theta = 15°$ in the near-surface region of the marine atmospheric surface layer. While these results agree well with the present study, some caution is required if using velocity measurements alone as there is likely to be a dependence on the actual locations of probes used. To demonstrate this, Fig. 6 shows a comparison between cross correlations $R_{\tau u}$ and that inferred using the u_1 signal for the wall shear stress ($R_{u_1 u}$). The differences between the estimates of inferred structure angle and peak level of cross correlation are given in Table II. The lowest sonic results ($z = 0.50$ m) show significant differences in the peak values and structure angles between $R_{\tau u}$ and $R_{u_1 u}$ (38% and 91%, respectively), while for the highest sonic ($z = 2.93$ m) the profiles agree somewhat better with the peak cross correlation overestimated by 7% for $R_{u_1 u}$, and an inclination angle from $R_{u_1 u}$ underestimated by 25%.

Table III shows how the inferred structure angle varies for cross correlations based on the u signals from the first four sonics and the fifth highest sonic. A significant variation is observed, with higher angles inferred when the sonics are closely spaced. This again indicates that caution is needed when inferring θ from R_{uu} results. The results in Table III are also consistent with a scenario where closed-spaced sensors likely better reflect the local inclination angle of individual structures while sonics that are far apart will average across a larger range of different scaled structures. These results are consistent with the prior studies in laboratory turbulent boundary layers [21] that indicate the average coherent structure in the logarithmic region has a low inclination angle (akin to a ramp) but to be made up of an organized array, or packet, of individual vortex structures (or attached eddies) [22–24].

TABLE III. Peak cross correlations and inclination structure angles for u signals from highest sonic (5) and other sonics.

Sonic number	1	2	3	4
$(R_{uu_5})_{\max}$	0.43	0.49	0.57	0.70
θ_{uu_5}	11.8	15.6	22.2	32.6

This work was supported by the David and Lucile Packard Foundation and the Australian Research Council. We wish to thank Dr. Gary Kunkel and Dr. Nicholas Hutchins for assisting with the experimental studies and Professor Joseph Klewicki for hosting the SLTEST field tests.

*imarusic@unimelb.edu.au

[1] T. B. Nickels, I. Marusic, S. M. Hafez, and M. S. Chong, Phys. Rev. Lett. **95**, 074501 (2005).
[2] M. V. Zagarola and A. J. Smits, J. Fluid Mech. **373**, 33 (1998).
[3] C. J. Swanson, B. Julian, G. G. Ihas, and R. J. Donnelly, J. Fluid Mech. **461**, 51 (2002).
[4] J. C. Klewicki, J. F. Foss, and J. M. Wallace, in *Flow at Ultra-High Reynolds and Rayleigh Numbers*, edited by R. J. Donnelly and K. R. Sreenivasan (Springer, New York, 1998).
[5] G. J. Kunkel and I. Marusic, J. Fluid Mech. **548**, 375 (2006).
[6] M. A. Carper and F. Porte-Agel, J. Turbul. **5**, 1 (2004).
[7] S. E. Hommema and R. Adrian, Bound.-Lay. Meteorol. **106**, 147 (2003).
[8] S. C. Morris, S. R. Stolpa, P. E. Slaboch, and J. Klewicki, J. Fluid Mech. **580**, 319 (2007).
[9] N. Hutchins and I. Marusic, J. Fluid Mech. **579**, 1 (2007).
[10] M. Metzger, B. J. McKeon, and H. Holmes, Phil. Trans. R. Soc. A **365**, 859 (2007).
[11] G. R. Brown and A. S. W. Thomas, Phys. Fluids **20**, S243 (1977).
[12] U. Piomelli, J. Ferziger, P. Moin, and J. Kim, Phys. Fluids A **1**, 1061 (1989).
[13] I. Marusic, G. J. Kunkel, and F. Porte-Agel, J. Fluid Mech. **446**, 309 (2001).
[14] G. J. Kunkel and I. Marusic, Exp. Fluids **34**, 285 (2003).
[15] W. D. C. Heuer and I. Marusic, Meas. Sci. Technol. **16**, 1644 (2005).
[16] R. B. Stull, *An Introduction to Boundary Layer Meteorology* (Kluwer, Dordrecht, 1988).
[17] S. K. Robinson, Chem. Eng. Commun. **43**, 347 (1986).
[18] R. Stoll and F. Porte-Agel, Bound.-Lay. Meteorol. **118**, 169 (2006).
[19] U. Piomelli and E. Balaras, Annu. Rev. Fluid Mech. **34**, 349 (2002).
[20] R. S. Boppe, W. L. Neu, and H. Shuai, Bound.-Lay. Meteorol. **92**, 165 (1999).
[21] I. Marusic, Phys. Fluids **13**, 735 (2001).
[22] R. J. Adrian, C. D. Meinhart, and C. D. Tomkins, J. Fluid Mech. **422**, 1 (2000).
[23] B. Ganapathisubramani, N. Hutchins, W. T. Hambleton, E. K. Longmire, and I. Marusic, J. Fluid Mech. **524**, 57 (2005).
[24] N. Hutchins, W. T. Hambleton, and I. Marusic, J. Fluid Mech. **541**, 21 (2005).