SIMILARITY PREDICTIONS BASED ON THE ATTACHED EDDY HYPOTHESIS IN TURBULENT BOUNDARY LAYERS

A. K. Mesbah Uddin, A. E. Perry and I. Marušić

Dept. of Mechanical and Manufacturing Engineering, University of Melbourne, Parkville, Victoria 3052, AUSTRALIA.

ABSTRACT This paper presents a similarity formulation for the streamwise component of the fluctuating velocity \( u_1 \) in a turbulent boundary layer based on the attached eddy model of wall turbulence being developed at the University of Melbourne by Perry and co-workers. The consequences of this formulation for increasing Reynolds numbers is tested against recent high Reynolds number data. The model is based on the assumption that there exist eddies of different length scales in a turbulent boundary layer and the probability density function (p.d.f.) of the eddy length scale distribution follows an inverse power law for eddies in the turbulent wall region. Such a distribution is necessary to obtain the log-law of the mean velocity: the spectral scaling laws provide indirect evidence for this. In this paper the results from a template matching technique will be presented which gives further support for the proposition of an inverse power law p.d.f. of attached eddy length scales.

1. Introduction

Based on the Townsend attached eddy hypothesis [1] and the flow visualization experiments of Head and Bandyopadhyay [2], Perry and Chong [3] postulated that wall turbulence consists of a forest of hairpin, horseshoe or 'A' vortices. These vortices originate from the viscous buffer zone and are inclined at approximately 45° with the downstream direction. Perry and Chong [3] found that in order to obtain a logarithmic mean velocity distribution, a region of constant Reynolds shear stress and correct \( u_1 \) spectral behaviour in the turbulent wall region, it is necessary to assume that a range of scales of geometrically similar representative eddies exist. The simplest assumption is that all the representative eddies have the same characteristic velocity scale \( \sim U_r \), wall shear velocity) and their length scale \( \delta \) (eg. eddy height) varies from the smallest scale \( \delta_1 \) which is assumed to be proportional to Kline et al. [4] scaling (i.e. \( \sim \nu/U_r \)) to the the largest scale equal to the boundary layer thickness \( \delta_B \). The probability density function (p.d.f.) of eddy length scale \( \delta \) follows an inverse power law distribution if a continuous distribution is assumed. Perry et al. [5] extended this model to the whole turbulent boundary layer beyond the viscous region and found that the p.d.f. of the eddy length scales must be modified by a
weighting function \( \omega(\delta/\delta_H) \). Thus the p.d.f. \( P(\delta/\delta_H) \) is given by

\[
P(\frac{\delta}{\delta_H}) = \frac{\delta_H}{\delta} \omega(\frac{\delta}{\delta_H}).
\]

Perry et al. [6] introduced another weighting function \( T(\delta/\delta_H) \) to account for the variation of characteristic velocity with the representative eddy length scale.

All of the above weightings are used in the following equations for mean flow and stress distributions

\[
\frac{du/U_r}{dz/\delta_H} = \int_{\delta_1}^{\delta_H} f(z/\delta_H) P(\frac{\delta}{\delta_H}) T(\frac{\delta}{\delta_H}) \frac{d\delta}{\delta},
\]

\[
\frac{\overline{u_i u_j}}{U_r^2} = \int_{\delta_1}^{\delta_H} I_{ij}(z/\delta_H) P(\frac{\delta}{\delta_H}) T^2(\frac{\delta}{\delta_H}) \frac{d\delta}{\delta},
\]

where \( f \) and \( I_{ij} \) are functions depending only on the assumed eddy geometry. It can be seen that \( \omega(\delta/\delta_H) \) puts different weight on the large eddies so as to produce a mean velocity law similar to that given by Coles [7] beyond the logarithmic wall region. There is support for the existence of an inverse power law p.d.f. for eddy length scales \( \delta/\delta_H \ll 1 \) in the fully turbulent wall region and this comes from the spectral scaling laws and Reynolds stress scaling laws, eg. see Perry et al. [5]. In this paper further support is given using a more explicit approach based on template matching. In this region \( \omega = \text{const.} \) and \( T = 1 \). In fact here we will assume \( T = 1 \) for all \( \delta/\delta_H \) for the time being (this is a fairly good assumption for zero pressure gradient flow) and variations of \( T \) can be accounted for by minor modifications in later work.

Perry et al. [5] proposed similarity hypotheses for all three components of the velocity fluctuations by spectral dimensional similarity arguments and also the same relationships were arrived at by using an extended version of the Townsend [1] attached eddy hypothesis. Perry et al. [5,8] and Perry and Li [9] modified these relations to take into account the finite Reynolds number effect. By combining the attached eddy model of Perry and Chong [3], the Reynolds number similarity hypothesis of Townsend [10] and using an analogy to that of Coles [7] for mean flow, we present here a similarity relation for the streamwise component of the velocity fluctuation \( (u_1) \) in a zero-pressure gradient turbulent boundary layer with sufficiently large Reynolds number.

2. The p.d.f. measurements

The experiments were carried out in a wind tunnel with a working section of 940×388 mm. The boundary layer was tripped at a distance 3160 mm from the measuring station with a 1 mm diameter trip rod giving Reynolds numbers \( R_\theta \) of 4140 and 11928 at nominal free-stream velocities of 10 m/s and 30 m/s respectively. Here \( R_\theta = U_1 \theta/\nu \) where \( U_1 \) is the freestream velocity, \( \theta \) is the momentum thickness and \( \nu \) is the kinematic viscosity. All the measurements were made with a dynamically-calibrated normal wire.

The estimates for the p.d.f. of the eddy length scale distributions were made using a template matching technique where the velocity template for an eddy of a particular length scale was generated assuming a \( \Pi \)-shaped eddy at an angle of 45° with the flow direction. This assumed shape is somewhat arbitrary and has been adopted for its simplicity for this preliminary work. The other relevant geometric properties of the assumed eddy shape are: a core-diameter \( d_0 = 0.1 \delta \), an aspect ratio of 1 (for the ratio of eddy height to spanwise width) and a Gaussian distribution of vorticity within the core and \( d_0 \) is
Figure 1: Probability density function (p.d.f.) of eddy length scale distribution in a turbulent boundary layer as obtained by template-matching technique. Authors' data — see Uddin [11]. The $z/\delta_H$ and $z_+$ values refer to the position of the probe.

twice the standard deviation. The velocity field of the eddy was calculated from Biot-Savart law calculations and the convection velocity of an eddy with a length scale $\delta$ was assumed to be equal to the local mean velocity at $z = 0.5\delta$, where $z$ is the distance normal to the wall. It was found that variations to the convection velocity assumptions have only a minor effect on the conclusions for $\delta/\delta_H$ small but an appreciable effect for $\delta/\delta_H$ large and needs to be clarified. The Biot-Savart velocity signature was correlated with the measured temporal velocity signal and a match, representing an eddy, is expected to be characterized by a peak in the template-signal correlation. A threshold for this peak needed to be chosen but conclusions are insensitive to this choice.

The p.d.f.'s of the eddy length scale distribution as obtained by the template matching technique for various probe positions are shown in figure 1. It can be seen that the expected p.d.f. is obtained. Full details of the template matching technique and given by Uddin [11].
3. Streamwise velocity fluctuations

The attached eddy model of wall turbulence suggest that, like mean velocity, there is a log-law region for $\overline{u'_3^2}/U'_2^2$ for $z_+ \gg U_// \nu$ and $z \ll \delta_H$. Experimental data of $\overline{u'_3^2}/U'_2^2$ from various sources suggests that, like the mean velocity, $\overline{u'_3^2}/U'_2^2$ exhibits a similar trend with two deviations from the log-line: one near the wall and the other in the outer layer. This leads us to propose that

$$
\frac{\overline{u'_3^2}}{U'_2^2} = B_1 - A_1 \ln \left[ \frac{z}{\delta_H} \right] - V_{g1}[z_+] - W_{g1}[\frac{z}{\delta_H}]
$$

(4)

where the deviation $V_{g1}[z_+]$ is termed as the viscous deviation function due to finite Reynolds number and $W_{g1}[z/\delta_H]$ is termed as the wake deviation function, which according to Townsend’s Reynolds number hypothesis should be a function of $z/\delta_H$ alone. In (4) $A_1$ is a universal constant and $B_1$ is a characteristic constant which is related to the Coley wake factor which is assumed fixed for high Reynolds number zero-pressure-gradient turbulent boundary layers. The function $V_{g1}[z_+]$ is probably controlled by many effects. One due to the Kolmogoroff cut-off at high wavenumbers (which is an isotropic effect) and another is due in part to the cut-off of geometrically similar attached eddies, i.e. $\delta < \delta_1$. There are no doubt other additional anisotropic viscous effects entering as we approach the buffer zone. The viscous deviation function $V_{g1}[z_+]$ is such that

$$
V_{g1}[z_+] = C z^{-\frac{1}{2}} \quad \text{as} \quad z_+ \to \infty
$$

from the theory of Kolmogoroff, e.g. see Perry and Li [10], and from work conducted here it appears that

$$
V_{g1}[z_+] \to E_1 - A_1 \ln [z_+] \quad \text{for approx.} \quad z_+ < 100
$$

if it were not for a sudden peak which occurs in $\overline{u'_3^2}/U'_2^2$ for $z_+ \to 20$ and then $\overline{u'_3^2}/U'_2^2 \to 0$ as $z_+ \to 0$. The scaling for this viscous buffer zone region of $0 < z_+ < 100$ is not yet understood. The universality of the functions $V_{g1}[z_+]$ and $W_{g1}[z/\delta_H]$ for $z_+ > 100$ imply that if these are deduced by a curve-fitting procedure from one set of data, then they must be applicable for other sets of data. These functions were determined with the experimental data from the authors’ data shown in figure 2(a) and its validity was checked by comparing the experimental data of Smith [12] and Petrie et al. [13] as in shown in figure 2(b). These are the highest Reynolds numbers so far recorded for such smooth wall turbulence measurements. Note that values of $A_1=1.1$ and $B_1=2.0$ were used and are only slightly different to the values 1.03 and 2.39 used by Perry and Li [9]. The observed good agreement between the experimental data of Smith [12] and Petrie et al. [13] seem to support our conjecture. As seen in the figure, the $\overline{u'_3^2}/U'_2^2$ versus $z_+$ profile has an approximate plateau at $z_+ = O(100)$ which continues to rise with Reynolds number. We need to go to higher and higher Reynolds numbers to see how far this scaling remains valid. According to this formulation the plateau will rise indefinitely with Reynolds number. From these results, it can clearly be seen that conventional wall functions for $\overline{u'_3^2}/U'_2^2$ cannot work at least in the turbulent wall region.
Figure 2: Validity of the similarity formulation of $u_1$. (a) Determination of the functional form for $V_{g1}[z_2]$ and $W_{g1}[z/\delta_H]$ by curve-fitting the authors’ data at $Re=11928$. (b) Comparison of similarity predictions and the experimental data of Smith [12] (marked with $s$) and Petrie et al. [13] (marked with $p$).

4. Conclusions

The p.d.f.'s of attached eddy length scales in a zero pressure gradient turbulent boundary layer have been measured experimentally using a template matching technique. The results seem to agree with an expected inverse power law in the turbulent wall region of the flow. Turbulence intensity scaling which is based on inverse power law p.d.f.'s predict that the turbulence intensity at the outer edge of the buffer zone increases without limit with increasing Reynolds number.

References