Use of dual plane PIV to assess scale-by-scale energy budgets in wall turbulence

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Introduction

Turbulence in wall-bounded flows can be characterized by variations in physical space or in scale space. In physical space, the near wall region can be classified into a viscous sub-layer, the buffer region, the logarithmic region and the outer region. In the space of scales, turbulent energy is produced at the large scales and transferred to smaller scales, finally dissipating in the form of heat in a mechanism commonly known as the Richardson cascade. These are seemingly parallel approaches to describing turbulence, and in order to describe the scale-dependent dynamics in the presence of spatial inhomogeneities, a more general approach combining these two ideas is required.

For homogeneous shear flow, the generalized Kàrmàn-Howarth equation can be used in anisotropic conditions. By extending this equation to inhomogeneous flows, the relation between spatial fluxes and the energy cascade can be studied in detail. This modified equation is averaged in the space of scales on two-dimensional square domains in wall-parallel planes to yield

$$T_r(r, Y_c) + \Pi_e(r, Y_c) = E_e(r, Y_c) \tag{1}$$

where r represents the scale and Y_c represents the wall-normal coordinate. In other words, this expression implies that the transfer across scales plus the effective production must equal the effective dissipation at every scale and wall-normal location. Each of the terms shown in the above equation is a combination of individual terms in the derived equation. The derivation of this equation is described in detail in [1]. This relation will hold through the various regions of the boundary layer; however the relative importance of each term varies with the location. While the production and transfer terms are most important in the buffer layer, in the outer region, the dissipation and transfer are the most important terms. The log layer is nominally an equilibrium layer, where local production and local dissipation match each other and thus ensures a constant flux of energy across scales.

Results from DNS and PIV Data

Previously, data from a Direct Numerical Simulation (DNS) of a channel flow at a friction Reynolds number $Re_\tau = 180$ were analyzed at representative locations within the boundary layer to evaluate the terms of Eqn. 1 [1]. The present work extends the earlier study to larger Reynolds number flows. Data were examined from a DNS of a channel flow at $Re_\tau = 934$, which had a computational domain of size $8\pi \times 3\pi$ in the streamwise and spanwise directions, with a resolution of 11.46×5.73 viscous wall units respectively [2]. A second dataset was obtained experimentally in a zero pressure gradient boundary layer flow at $Re_\tau = 1160$ using dual plane Particle Image Velocimetry (PIV) [3]. The dual plane setup allows for the determination of the full velocity gradient tensor in a plane. The data were acquired in streamwise-spanwise planes in the logarithmic region ($Y_c^+ = 110$) with fields of size $1.1\delta \times 1.1\delta$ and velocities were resolved to 25 viscous wall units.

From Fig. 1(a), it is observed that at small scales, there is very little production and the transfer is the dominant term, thus reducing the scale dynamics to the classical Richardson cascade. As the scale increases, the production term increases, which is consistent with the idea of production of energy at the larger scales. Simultaneously, the transfer term reaches a maximum at the transfer scale $l_t^+ = 50$, and then starts to reduce. At the crossover scale $l_c^+ = 80$, the transfer and production terms are equal, and at scales larger than this, the production becomes more dominant as compared to the transfer. Although similar trends of production and transfer are observed with the PIV data (Fig. 1(b)), the value of $l_c^+ = 60$ is lower compared to the DNS value. Further, the rate of reduction of transfer with increasing scale is much less, which would have implications in the energy balance. This is clearly seen in the inset, where a mismatch between the terms of Eqn. 1 is seen for the PIV data, while a very good balance is seen for the DNS data.

This mismatch is likely due to errors in calculating the dissipation term, which is a result of the limited spatial resolution of the experiments. The resolution is constrained by the size of the interrogation window in the PIV processing. A study by [4] showed the effect of averaging and reduced spatial resolution in the computation of velocity and velocity gradients using PIV in a turbulent boundary layer. With this knowledge, additional PIV experiments were conducted at $Re_\tau = 1100$ and $Y_c^+ = 100$ with a smaller field of view and finer spatial resolution. These high resolution data were acquired in streamwise-spanwise planes in fields of size $0.45\delta \times 0.45\delta$, and velocities were resolved to 10 viscous wall units.

As can be seen from Fig. 1(c), the better resolution provides a more accurate value of the dissipation term at this wall-normal location, and thus the balance between the terms of Eqn. 1 is much better. It must be noted that the production term does not change substantially compared with Fig. 1(b) since it does not involve local gradients of fluctuating quantities. On the other

hand, the transfer term is computed more accurately and hence the value of $l_c^+ = 80$ matches well with the value predicted by the DNS.

Conclusions and Future work

The balance between the various terms of the turbulent kinetic energy budget was verified for DNS and PIV data and it was shown that high resolution PIV experiments are required to accurately resolve the terms. The results from the DNS and high resolution PIV data matched each other very well, both in terms of trends observed and values of scales computed. [1] argued on the basis of the classical equilibrium theory that $l_c \simeq \kappa Y_c$, where κ is the Kármán constant. However, the value of l_c computed from the DNS and PIV data at a higher Reynolds number are larger than these predicted values. This suggests a dependence of this value on the Reynolds number of the flow. Further experiments at different wall normal locations and higher Reynolds numbers will enable a better understanding of these trends.

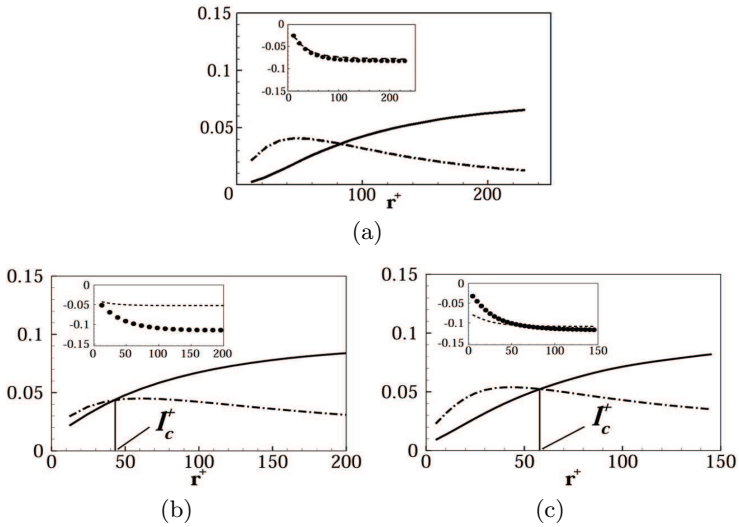

Fig. 1. Detailed balance (1) in the log-layer $Y_c^+ = 110$ for (a) DNS (b) PIV (c) High resolution PIV. The solid line is $-\Pi_e$ and the dash-dotted line is $-T_r$. (Inset) The sum $(T_r + \Pi_e)$ is represented by filled symbols and E_e is given by the dashed line. All terms are normalized by $(u^*)^4/\nu$.

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