WIND-GENERATED ELECTRICAL ENERGY USING FLEXIBLE PIEZOELECTRIC MATERIALS

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ABSTRACT

Electrical energy can be harvested from the wind using flag-like membranes (termed piezoelements or bimorphs) composed of flexible piezoelectric materials. The flapping of the membrane caused by the wind induces bending stresses which generate a voltage across electrodes positioned on the surfaces of the material via the piezoelectric properties of the material. The best results to date using PVDF (polyvinylidene fluoride) as the piezoelectric material are 10 mW of dc power (50 Vdc across 250 kΩ) obtained from a single bimorph element (size about 8"X11"X 0.02") flapping a frequency of 5 Hz in a wind speed of 15 mph. The overall efficiency was low (estimated at 0.1%) and to improve this figure, three specific areas of improvement are targeted. The efficiency of the piezoelement in converting wind energy into stored elastic energy will be optimized by determining the best geometry and dimensions for the element in conjunction with upwind bluff bodies using wind tunnel tests. A novel circuit, termed a quasi-resonant rectifier, for extracting the electric energy from the piezoelectric has been developed which can extract several times more power from the piezoelement than a conventional full-wave rectifier. A number of piezoelectric and electrostrictive materials including PZT fibers embedded in an epoxy matrix (termed active fiber composites or macro fiber composites) which may have higher generation efficiencies than PVDF will be examined for this application.

I. INTRODUCTION

Devices and systems such as wireless sensors, wireless networks, wearable computers, numerous portable personal electronics such as cell phones, MP3 players, etc. are being developed that require electrical power without being connected to the electrical grid. This requires either batteries or connection to electrical power sources that obtain energy from the ambient (usually termed energy harvesting or energy scavenging) for either direct operation of the systems or for charging the batteries used by the systems.

These off-grid applications often require only small amounts of power ranging from tens of microwatts to a few tens of watts. Additionally these sources need to be as small and as portable as the devices/systems being powered. One potential source of energy is the wind. Wind-to-electric energy conversion is presently done exclusively with wind turbines coupled to conventional magnetostatic-based generators such as permanent magnet generators and ac induction generators rated at several hundred kW to several MW. Smaller rated turbines for farm and residential applications are available and even smaller rated turbines (kW or less) are sometimes used for battery charging applications. Ongoing research efforts around the world in government labs, industry, and universities are centered exclusively on wind turbines utilizing magnetostatic-based generators. Very little attention has been given to alternative methods of harnessing wind for electrical generation.

One such alternative is to use piezoelectric materials. Conceptually mechanical structures convert the kinetic energy of the wind into periodic strains in the piezoelectric material and the induced strains immediately generate electrical energy via the piezoelectric coupling in the material. The concept has an appealing simplicity but very few studies have been carried
out so its feasibility is uncertain. Most of the studies to date involving piezoelectric materials have utilized small windmills that have cantilever bimorph blades that flex at the rotation rate of the windmill [1,2] or the windmill shaft drives a camshaft which deflect the free ends of bimorph cantilevers. [3,4] In either case only a few milliwatts of power have been realized.

In another study [5,6,7] germane to this project, flexible PVDF bimorphs, (termed eels because the bimorph was very long compared to its width and thickness), were positioned in the trail or wake of traveling vortices (vortex street) generated from a bluff body placed in a channel of flowing water. The vortices caused periodic flexing or undulation of the bimorph (overall movement very similar to that of a swimming eel) and hence generation of electric energy via the piezoelectric properties of the PVDF. The experimental arrangement was similar to the proposed wind generation system shown in Fig. 1. The conversion from the kinetic energy of the flowing water to electric energy process was most efficient when the mechanical resonant frequency of the eel was approximately the same as the vortex shedding frequency. The claimed attributes of the system were mechanical simplicity, easily scalable in size from milliwatts to many watts, and relatively low cost. [5]

II. FLEXIBLE PIEZOELECTRIC WIND-ELECTRIC GENERATION SYSTEM CONCEPT

The basic concept of generating electricity using flexible piezoelectrics is shown in Fig. 1. The generator is a piezoelectric bimorph element (termed a piezelement) affixed at one end to an anchor rod with the opposite end free, equivalent to a flag or a cantilever plate. The rod is positioned cross-wise to the wind as indicated in the figure and the wind direction is parallel to the length L of the piezelement. When the wind speed exceeds a critical value, which is a function of the dimensions and stiffness of the piezelement, the bimorph begins to undulate or flap as diagrammed. The flapping, which is the same as for a conventional flag, is caused by the Kelvin-Helmholtz instability, a well-known phenomena in fluid mechanics. [8,9]

The periodic bending stresses in the bimorph generate time varying charge on the outer surfaces of the bimorph because of the piezoelectric properties of the material. The charge is collected by electrodes which cover the surfaces of the PVDF layers and the charge induces a voltage between the upper and lower electrodes. If an external load is connected to the bimorph electrodes as shown, electrical power is delivered to the load. The strains induced in the piezelement are largest at the two outer surfaces of the piezelement and are zero at the neutral axis. Thus piezoelectric material is positioned on these outer surfaces as indicated to maximize charge generated by the strain caused by the flapping of the element.

A conceptual wind generation system based on these piezoelectric elements is shown in the block diagram Fig. 2. The power output would be scalable by adding additional piezoelements to the system. The power conditioning electronics would be modular so that as additional piezoelements are added to the system, additional power conditioning units could be added as needed.
matched to the load requirements such as either DC (for example to charge batteries) or 60 Hz AC.

III. INITIAL EXPERIMENTS AND EFFICIENCY ESTIMATES

A simple proof-of-concept experiment was carried out utilizing a simple homemade suction wind tunnel and a unimorph (a bimorph structure with one of the two piezoelectric layers removed). The nonpiezoelectric core was formed from multiple layers of 8.5”X11” overhead transparency sheets (approximately 50 microns thick). A single PVDF layer or unimorph 10 inches long, 7 inches wide and 0.002 inches was affixed to the core. The layers were glued together with spray-on adhesive. Insulated wire normally used for wire wrap applications was used for electrical connections to the outside world. These connections are visible in the photographs of Fig. 3.

At a wind speed of 5 m/s, the piezoelement generated an open circuit voltage of 40 Vrms at a frequency of 5 Hz. Two sequential pictures of the flapping unimorph are shown in Fig. 3. The electrical power delivered to a resistive load matched to the capacitive source impedance of the unimorph was approximately 1 mW.

The overall efficiency of the system can be expressed as [5]

\[ E_f = \frac{P}{\rho_a U^3 A/2} = E_{f1}E_{f2}E_{f3} \]  

where \( P \) is the electrical power out, \( U \) is the wind velocity, \( \rho_a \) is the air density, \( A \) is the effective wind energy capture area of the piezoelement, \( E_{f1} \) is the aerodynamic efficiency of the piezoelement (mechanical energy stored in the piezoelement divided by the input wind energy), \( E_{f2} \) is the efficiency of the piezoelectric material in converting stored mechanical energy into stored electrical energy, and \( E_{f3} \) is the efficiency of extracting electrical energy from the piezoelement and delivering it to a load. For our preliminary results we estimate, \( E_{f1} = 1-10\% \) and \( E_{f2} \approx E_{f3} \approx 10\% \). The uncertainty in the \( E_{f1} \) estimate is due to the difficulty in estimating the effective wind energy capture area of the piezoelement. If the physical area of the unimorph (70 sq. in. or about 0.05 \( \text{m}^2 \)) is used, the overall efficiency of this initial experiment is about 0.03%.

Although the proof-of-concept experiment indicates the proposed concept is scientifically sound, the practicality of the system is questionable unless the efficiency can be improved. The following sections describe methods to substantially increase each of the efficiency factors.

IV. INCREASING THE FLAPPING AMPLITUDE

A. Piezoelement Element Geometry

The first step towards improving the overall conversion efficiency is to convert the kinetic energy of the wind as efficiently as possible into elastic energy (flapping) in the piezoelement and thus increase the efficiency factor \( E_{f1} \).

Simple considerations dictate that there will be an optimum thickness for the piezoelement. For a given amplitude of flapping, the stresses generated in the flapping element become larger as the distance from the neutral axis of the element increases. [5] Hence placing piezoelectric materials on the bottom and top surfaces of relatively thick flapping elements such as illustrated in Fig. 1 will generate the most electrical energy. However at a specific wind speed, overly thick elements will be too stiff to flap with any appreciable amplitude resulting in small strains being developed in the piezoelectric materials and thus low electrical power generation.

The optimum thickness for the piezoelement will depend on the elastic properties (Young's modulus) of the material from which the piezoelement is made and its dimensions (length, width, and thickness) and the wind speed. A simple
approach for a baseline design for the piezoelement is summarized below which can be then modified based on experimental results. A linear analysis of the on-set of the Kelvin-Helmholtz instability (the threshold of strong flapping) indicates that the piezoelement will begin to flap strongly at a wind velocity given by [8]

$$U_{crit} = (2.5 \, L) \, (2\pi f_1^2)$$

(2)

where $L$ is the length of piezoelement in the direction parallel to the wind and [10]

$$(2\pi f_1) = \frac{3.5}{L^2} \frac{Y h^2}{12 \rho_{pz}}$$

(3)

The frequency $f_1$ is the mechanical resonant frequency of the cantilevered piezoelement. In the equation for $f_1$, $Y$ is Young’s modulus for the material from which the element is made, $h$ is the thickness, and $\rho_{pz}$ is the mass density (kg/m$^3$) of the material.

The speed of the wind is random and highly variable both with respect to time and location. A logical choice for the wind speed for baseline design purposes is the average speed $U_{av}$ for the site where the system is to be installed. The average wind speed in Minneapolis, MN where this project will be done is typically 4.5 meters/sec at typical building top heights. [11] For purposes of illustration, we will consider a length $L$ of 0.25 meters for a baseline design. For example sheets of PVDF are available in maximum lengths of 10 inches or 0.25 meters. Using published values [12] for PVDF of $Y = 3 \times 10^9$ J/m$^2$, $\rho_{pz} = 1800$ kg/m$^3$, and setting $U_{crit} = 4.5$ m/sec, the thickness of the piezoelement should be $h = 0.7$ mm. The frequency of flapping for these dimensions and wind speed would be $5f_1 = 3.1$ Hz. This example piezoelement is nearly identical to the one used in the proof-of-concept experiment discussed earlier and the calculated flapping frequency of 3 Hz is comparable to 5 Hz observed in the experiment.

Figure 4. Use of bluff bodies to generate vortices to enhance the flapping amplitude of the piezoelement. Both a standard bluff body (solid line) and an arrangement (dotted lines) to implement a slit or jet similar to the arrangement of Fig. 3 are shown.

B. Use of Bluff Bodies

A measure to employ for increasing the amplitude of the flapping and thus the electrical generation efficiency is the use of an upwind bluff body as shown in Fig. 4. The bluff body will form vortices as indicated in the figure and these vortices may increase the flapping amplitude if the vortex shedding frequency ($f_s = 0.2 \, U/D$) of the bluff body approximately matches the flapping frequency of the piezoelement. This was done in experiments with PVDF bimorphs in flowing water and found to be effective in enhancing the flapping amplitude. [5,6,7]

In order to study the efficacy of using a bluff body, we have developed a high resolution imaging system to quantitatively measure the flapping motions of a piezoelement in a wind tunnel. Figure 5 shows the basic experimental setup and Fig. 6 shows examples of some preliminary measurements. Detailed studies of the use of bluff bodies are currently in progress.

Figure 5. Wind tunnel arrangements to monitor the flapping piezoelement. The hot wire probe interfaced to a laptop computer using LabView software records wind speeds and vortex frequencies. A digital high speed camera (Photron, not shown) records the detailed movements of the piezoelement.

C. Adding Mass to Free End of Piezoelement

Adding mass to the free end of the piezoelement is also being explored. A preliminary experiment was performed in which the wind speed was held constant (15 mph) and increasing amounts of mass, in the form of small diameter wood dowels, were affixed to the free end of the piezoelement. With no mass on the free end, the piezoelement flapped weakly and generated only a modest voltage. Adding...
mass to the free end increased the open-circuit voltage as shown in Fig. 7. The flapping frequency gradually decreased with increasing mass as is also shown in Fig. 7. The maximum dc power that the piezoelement could deliver to a resistive load via a full-wave rectifier is estimated and plotted in the same figure. The estimate is based on a simple equivalent circuit model developed in the next section. This study is continuing.

![Figure 6](image)

**Figure 6.** Output voltage (upper row) of a flapping piezoelement as a function of time. Spatial variations (lower row) of the piezoelement’s flapping amplitude as a function of distance from the anchor rod at the time indicated by the dot in the voltage versus time graph directly above.

![Figure 7](image)

**Figure 7.** Effect of adding mass to the free end of a flapping piezoelement. The wind speed for the measurements shown was 15 mph.

V. INCREASING THE ELECTRICAL ENERGY EXTRACTION

A. Full-Wave Vs Quasi-Resonant Extraction

The standard approach for extracting and using the electrical energy generated by the piezoelement for dc power is the use of a full-wave rectifier and capacitive filter shown in Fig. 8a. However under the same input conditions (wind speed, etc), several times more energy can be extracted by the nonlinear processing technique shown in Fig. 8b. [5,13] In this scheme, the switch S_w is closed at the positive and negative peaks of the piezoelement voltage as shown by the waveforms in Fig. 8c. The switch closure time T_{cl} is one half the resonant period of the L-C_s combination (the filter capacitor C_s is much larger than C_s) and is given by

\[
T_{cl} = \frac{1}{2f_o} = \pi \sqrt{LC_s}
\]

(4)

In Fig. 8, the resistor R represents the resistance of the inductor windings and electrical losses in the switch (on-state resistance) and piezoelement and R_L is the load resistance.
A comparison between experimental measurements and the theoretical models for both circuits shown in Fig. 9 for a PVDF bimorph flapping at a frequency of 5 Hz (wind speed of 15 mph) is shown in Fig. 10. The two PVDF layers of the piezoelement were 7” wide, 10” long, and 0.002” thick. The nonpiezoelectric core was 0.016” thick. The capacitance of the two PVDF layers connected was 0.19 µF. The inductance in the circuit of Fig. 8b was 0.5 H and the resistor R was 470 ohms. In this experiment the quasi-resonant rectifier delivers about 2.4 times more power to an optimum load (7.8 mW to a 220 kΩ load) compared to the conventional full-wave rectifier (3.3 mW to a 220 kΩ load). The theoretical models are in reasonable agreement with the experimental results.

The power that can be extracted via the quasi-resonant rectifier is sensitive to the $Q_s$ of the inductor. For the measurements shown in Fig. 10, $Q_s$ was about 3.8, which is a comparatively low value. For infinite $Q_s$, the $R_s$ value of the quasi-resonant rectifier is four times smaller than that of the full-wave rectifier and thus could extract four times more power from the piezoelement. $Q_s$ values of 10 to 20 can be readily obtained by careful choice of inductors.

![Figure 8](image_url)  
*Figure 8. Electrical energy extraction from piezoelement using (a) standard full-wave rectifier and (b) using a synchronized switched inductor or quasi-resonant rectifier. Trigger signal timing for closing the switch is shown in (c).*

From a dc output voltage perspective, the circuits of Fig. 8a and 7b can be modeled by the equivalent circuit shown in Fig. 9. For the standard full-wave rectifier of Fig. 8a, the source resistance of Fig. 8a is given by [13]

$$R_s = \frac{T_s}{4C_s}$$  \hspace{1cm} (5)

where $T_s$ is the period of the ac waveform $V_{pz}(t)$. $R_s$ of the quasi-resonant rectifier of Fig. 8b is given approximately by (see Appendix B)

$$R_s = \frac{T_s}{16C_s} \left[ 1 + \frac{(1 + 4Q_s^2)(1 - e^{-\phi})}{Q_s^2(1 + e^{-\phi})} \right]$$  \hspace{1cm} (6)

where $\phi = \pi/(2Q_s)$ and $Q_s = \omega_o L/R$ with $\omega_o = (L/C_s)^{-1/2}$. The dc power $P_o$ delivered to the load is given by

$$P_o = \frac{V_s^2 R_s}{(R_s + R_L)^2}$$  \hspace{1cm} (7)

![Figure 9](image_url)  
*Figure 9. DC equivalent circuit for the standard full-wave rectifier of Fig. 8a and the quasi-resonant rectifier of Fig. 8b.*

B. Quasi-Resonant Rectifier Implementation.

Figure 11 shows the functional implementation of the quasi-resonant rectifier. A PNP transistor implements the series switch and the transistor is turned on whenever the lower NPN transistor is activated by the one-shot which outputs a pulse of duration $T_{cl}$ given by Eq. (4). The one-shot is configured to output a pulse whenever the output from the comparator goes from high (3V) to ground. The input to the comparator is the output from the differentiator signal which triggers the comparator at every zero crossing of the differentiator output. The input to the differentiator is a scaled version (scaled by the voltage divider) of the output of the full
wave rectifier. The various waveforms described above are shown in Fig. 12.

![Functional diagram of the quasi-resonant rectifier showing the switch implementation and generation of the switch trigger signal.](image)

**Figure 11.** Functional diagram of the quasi-resonant rectifier showing the switch implementation and generation of the switch trigger signal.

![Waveforms in various parts of the quasi-resonant circuit of Fig. 12 showing how the trigger signal is derived.](image)

**Figure 12.** Waveforms in various parts of the quasi-resonant circuit of Fig. 12 showing how the trigger signal is derived.

Low current op amps are used to implement the various parts of the trigger circuit of Fig. 11. The total current draw from the 3 V dc bus is about 20 microamps so that only about 60 microwatts is needed for the trigger circuit which can be supplied for many hours from a small button battery.

Alternatively, the dc voltage developed across the energy storage capacitor C can used as the input to a switching regulator to produce the needed 3V directly from the piezoelement. We have implemented such a step-down regulator and the total power required for both the regulator and the trigger circuit is about 200 microwatts. The regulator includes provisions for a soft-start in which the load capacitor is charged up through a full-wave rectifier until a pre-set voltage is reached which is sufficient to operate the regulator and trigger circuit. At that point quasi-resonant operation commences.

VI. Improved Piezoelectric Materials

PVDF has been used in the initial stages of this project because it is quite flexible and readily available. However PVDF is not a strong piezoelectric material and stronger piezoelectric materials are potentially available that could substantially increase the piezoelement power output compared to a PVDF-based element. A material of particular interest is the so-called AFC (active fiber composite) or MFC (macro fiber composite). [14] This is a composite material is in a thin sheet format and is composed of very thin PZT fibers (250 µm diameter) embedded in an epoxy film matrix with electrodes on each side of the sheet.

One way to compare the performance of PVDF versus the MFC is to consider a two piezoelements, one made from PVDF and the other from MFC. Each piezoelement would have the same geometry, which is shown in Fig. 13 and would flap at the same frequency \( f_s \).

![Rectangular piezoelement under peak tensile stress (strain \( \delta \)) caused by ac stress from wind-induced bending.](image)

**Figure 13.** Rectangular piezoelement under peak tensile stress (strain \( \delta \)) caused by ac stress from wind-induced bending.

It is shown in Appendix A that the maximum dc power obtainable from the piezoelement by full-wave rectification is given by

\[
P_{\text{max}} = (d_{13}Y)^2 f_s(LWT)\delta^2 = \gamma f_s(LWT)\delta^2. \tag{8}
\]

In Eq. (8), \( d_{13} \) is the piezoelectric strain constant [m/V], \( Y \) is Youngs modulus [N/m\(^2\)], \( \varepsilon_{33} \) is the dielectric constant, and \( \delta \) is the strain caused by the bending. The dimensions \( L, W, \) and \( T \) are defined in Fig. 13. For identical piezoelements flapping at the same frequency \( f_s \), the maximum power is proportional to the constant \( \gamma \). This constant is evaluated in Table I for identical piezoelements made from PVDF and a MFC. A MFC piezoelement produces about 25 times more power than a comparable PVDF piezoelement. We are currently acquiring sample MFCs to test as piezoelements.

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>( d_{13} ) [C/N]</td>
<td>2.3x10^{-11}</td>
<td>1.70x10^{-10}</td>
</tr>
<tr>
<td>( Y ) [N/m(^2)]</td>
<td>3x10^9</td>
<td>3x10^{10}</td>
</tr>
<tr>
<td>( \varepsilon_{33}/\varepsilon_0 )</td>
<td>12</td>
<td>3000</td>
</tr>
<tr>
<td>( \gamma ) [J/m(^3)]</td>
<td>4x10^7</td>
<td>10^9</td>
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**Table 1.** Performance comparison between PVDF-based piezoelements and MFC(PZT)-based piezoelements.
Piezoelectric materials can become depoled under high ac stresses such as encountered in the proposed piezoelements. [15] The depoling occurs because high electric fields, exceeding the coercive field, will be generated in the material every half cycle which is in the opposite direction of the poling field. To avoid this problem, a dc bias voltage needs to be applied across the element electrodes which has the same polarity as the poling voltage. The dc bias voltage will require negligible power since it is being maintained across a capacitor and thus could be supplied for an extensive period of time from a small battery. However a more ideal situation would be to obtain the bias voltage from the power supplied by the piezoelement itself. Such a self-bias scheme is shown in Fig. 14. This scheme has been applied to a PVDF piezoelement and had no short term adverse effect on the operation of the piezoelement or the energy extraction circuit. We will be testing the circuit for an extended time period to determine how well it prevents depoling in the near future.

**VII. CONCLUSIONS**

A novel way of harvesting electrical energy from the wind, the use of flexible piezoelectric elements (bimorphs) flapping in the wind analogous to flags, has been demonstrated. The scheme has an appealing simplicity and may have several applications such as wireless sensors and wireless networks if the generation efficiency can be significantly improved. Several methods for improving the efficiency are currently being explored. The use of bluff bodies and mass on the free end of the piezoelement offer improvements in the conversion of the kinetic energy of the wind into stored elastic energy in the piezoelement. The use of stronger-coupling flexible piezoelectric materials such as AFCs (active fiber composites) could theoretically improve the efficiency of converting stored elastic energy into stored electrical energy by a factor of 25. The use of the quasi-resonant rectifier to extract electrical energy from piezoelement instead of standard full-wave rectifier has already demonstrated an increase in the efficiency by a factor of 2.3 and could be as large as four with the use of higher-Q inductors.

**ACKNOWLEDGEMENTS**

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14. G.A. Rossetti, Jr., A. Pizzochero, and A.A. Bent, “Recent Advances in Active Fiber Composites Technology”,

**Figure 14.** Self-bias circuit to minimizing depoling of the piezoelectric material. The capacitor $C_{\text{block}}$ prevents the dc bias from affecting the energy extraction circuit. $V_{oc}$ is the base-to-peak ac voltage generated by the piezoelement.


APPENDIX A – MAXIMUM OUTPUT POWER FROM PIEZOELEMENT

The electric displacement \( D_3 \) for the piezoelement shown in Fig. 13 is given by

\[
D_3 = \varepsilon_{33} E + d_{31} \sigma_1 \quad (A1)
\]

where \( \sigma_1 \) is the tensile stress indicated in Fig. 13. The stress is related to the strain \( \delta \) by Young’s modulus \( Y \) so that

\[
\sigma_1 = Y \delta \quad (A2)
\]

Under open-circuit conditions \( D_3 = 0 \) and \( V_{oc} = E_3 T \). Substituting this information into Eq. (A1) yields

\[
V_{oc} = \frac{d_{31} Y T \delta}{\varepsilon_{33}} \quad (A3)
\]

If the voltage generated by a flapping piezoelement is full-wave rectified, the maximum dc power delivered to the load is, using Eq. (7), given by

\[
P_{\text{max}} = \frac{V^2}{4R_s} \quad (A4)
\]

with \( R_s \) given by Eq. (5). The source capacitance \( C_s \) is given by

\[
C_s = \frac{\varepsilon_{33} L W}{T} \quad (A5)
\]

The source resistance \( R_s \), using Eq. (A5) and using \( T_s = 1/f_s \), can be expressed as

\[
R_s = \frac{T}{4f_s \varepsilon_{33} L W} \quad (A6)
\]

Equating \( V_s \) to \( V_{oc} \), the expression for \( P_{\text{max}} \) becomes

\[
P_{\text{max}} = \frac{(d_{31} Y)^2}{\varepsilon_{33}} f_s (LWT) \delta^2 \quad (A7)
\]

APPENDIX B – EFFECTIVE SOURCE RESISTANCE OF QUASIC-RESONANT RECTIFIER

During the interval when the switch \( S_w \) in the quasi-resonant rectifier (see Fig. 8b) is closed and energy is being extracted from the piezoelement, the diode bridge is conducting and current is flowing from the piezoelement through the switch and inductor to the energy storage capacitor \( C \). The energy storage capacitor \( C \) is large and the change in the output voltage \( V_0 \) even in the intervals between recharging is small (small ripple voltage). Hence the storage capacitor is modeled as a constant voltage source \( V_0 \) during the energy extraction. Interval. Under these conditions, the circuit of Fig. 8b can be simplified to the equivalent circuit shown in Fig. A-1. Note that this circuit is valid twice during each cycle of the input voltage, once during the positive peak voltage and once during the negative peak voltage.

![Figure A-1. Equivalent circuit of the quasi-resonant rectifier valid during the time intervals when energy is being extracted from the piezoelement.](image)

Under these conditions, the current \( i(t) \) is given by

\[
i(t) = A \exp(-t/\tau) \sin(\omega_\text{p}t) \quad (A-1)
\]

where \( \tau = L/(2R) \), \( \omega_\text{p} = \omega_o \sqrt{1 - 4/Q_s^2} \), \( \omega_o = \sqrt{LC} \) and \( Q_s = \omega_o L/R \). The charge \( Q_{ex} \) extracted from the piezoelement during this interval is

\[
Q_{ex} = \int_0^\tau i(t) \, dt = \frac{A}{\omega_o(1 + 4/Q_s^2)} (1 + e^{-\phi}) \quad (A-2)
\]

where \( \phi = \pi/(2Q_s) \). The charge \( Q_{ex} \) replaces the charge drained from the storage capacitor \( C \) during the time interval \( T_s/2 (T_s = 1/f_s \) where \( f_s \) is the frequency of the flapping of the piezoelement). The charge drawn from the storage capacitor is approximately given by

\[
Q_{ex} = \frac{V_s T_s}{2R_L} \quad (A-3)
\]

and the constant \( A \) in Eqs. (A-1) and (A-2) becomes

\[
A = \frac{V_0 T_s (4Q_s^2 + 1) \omega_o}{8R_L Q_s^2 (1 + e^{-\phi})} \quad (A-4)
\]
During the time interval $T_{cl} = \pi/\omega_o$, the energy $E_s$ extracted from the piezoelement is given by

$$E_s = V_s Q_{ex} = \frac{V_s V_o T_s}{2R_L}$$

(A-5)

This extracted energy is balanced by energy dissipation and energy storage in the rest of the circuit. Conservation of energy considerations yields

$$E_s = E_{Cs} + E_R + E_C$$

(A-6)

with $E_{Cs}$ being the energy stored in the capacitor $C_s$ at the end of the extraction interval, $E_R$ is the energy dissipated in the inductor parasitic series resistance $R$ (and on-state resistance of the switch) during the $T_{cl}$ and $E_C$ is the energy stored in the capacitor $C$ and then dissipated in the load between recharging intervals.

The energy $E_C$ is given by

$$E_C = \frac{V_o^2 T_s}{2R_L}$$

(A-7)

The energy $E_R$ dissipated in the resistor $R$ is given by

$$E_R = \frac{\pi}{\omega_o} \int_0^{\omega_o} R i^2(t) dt = \frac{V_o^2 T_s^2 (1 + 4Q_s^2)(1 - e^{-\phi})}{32C_s R_s Q_s^2 (1 + e^{-\phi})}$$

(A-8)

At the start of the energy extraction interval, there is a charge stored on the capacitance $C_s$ of the piezoelement equal to $C_s V_{cap}$. At the end of the interval, the voltage on $C_s$ has the same magnitude but opposite polarity. The current $i(t)$ is the cause of the change in voltage polarity and stored charge. The total change in charge on $C_s$ is $Q_{ex}$ given by Eq. (A-3) and the initial value $C_s V_{cap} = Q_{ex}/2$. The energy stored on $C_s$ by this charge is given by

$$E_{Cs} = \frac{(Q_{ex}/2)^2}{2C_s} = \frac{V_o^2 T_s^2}{32C_s R_s^2}$$

(A-9)

This initial energy is partially dissipated in $R$ and the rest is stored on the capacitor $C$ as $C_s$ is discharged by $i(t)$. The current then replaces this charge and effects the change in voltage polarity across $C_s$. Thus every extraction interval an amount of energy given Eq. (A-9) is supplied by the source $V_s$.

Using Eqs. (A-5), (A-7),(A-8) and (A-9), the conservation of energy equation, Eq. (A-6) becomes

$$\frac{V_o V_s T_s}{2R_L} = \frac{V_o^2 T_s^2}{32C_s R_s^2} + \frac{V_o^2 T_s^2 (1 + 4Q_s^2)(1 - e^{-\phi})}{32C_s R_s^2 Q_s^2 (1 + e^{-\phi})} + \frac{V_o^2 T_s}{2R_L}$$

(A-10)

Simplifying Eq. (A-10) yields

$$V_o = \frac{V_s}{(1 + R_s/R_L)}$$

(A-11)

$R_s$ is given by

$$R_s = \frac{T_s}{16C_s} \left[ 1 + \frac{1 + 4Q_s^2}{Q_s^2 (1 + e^{-\phi})} \right]$$

(A-12)

which is the same as Eq. (6) in the main text.