ON THE RELATIONSHIP BETWEEN LARGE- AND SMALL-SCALE MOTIONS IN TURBULENT BOUNDARY LAYERS

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Summary The relationship between large- and small-scale motions remains a poorly understood process in wall-bounded turbulence. Investigation performed by Hutchins and Marusic [3] in a high Reynolds number turbulent boundary layer has recently revealed a possible influence of large-scale log region motions on the small-scale near-wall cycle, akin to a pure amplitude modulation. For this study we build upon these observations, using the Hilbert transformation applied to the spectrally filtered small-scale component of fluctuating velocity signals, in order to determine the degree of amplitude modulation effect imparted by the large-scale structures onto the near-wall cycle.

INTRODUCTION

Since the insightful Direct Numerical Simulation (DNS) studies of Jiménez and Pinelli [4], there has been a tendency to view the near-wall cycle of streaks and quasi-streamwise vortices as self sustaining (capable of sustaining in the absent of external triggers or influences). However, autonomous view was based largely on understanding low Reynolds number studies. Latterly, advances in PIV measurements and higher Reynolds number facilities have revealed the presence of elongated regions of momentum deficit in the log-region, called 'superstructures' by [2]. The discovery of such very large-scale motions has in turn raised questions concerning their effect on the near-wall cycle. By studying fluctuating velocity signals from hot-wire sensors in the near-wall region, Hutchins and Marusic [3] recently observed that, in addition to the low-wavenumber mean shift, the largest scales appeared to be 'amplitude modulating' the small-scale fluctuations. Based on the pre-multiplied energy spectra map (Fig. 1), a scale decomposition around a carefully selected cut-off wavelength has revealed an interesting feature of the signals: A negative large-scale fluctuation (superstructure) was found to be phase-locked with the amplitude of the small-scale fluctuations (Fig. 2). This, seems to be similar to a pure amplitude modulation imparted to the large-scale log-region events onto the near-wall structures. For the present paper, we expand upon the initial observations of Hutchins and Marusic [3], using the Hilbert transformation in an attempt to quantify the relationship between large-scale fluctuations and any amplitude modulation of the small-scale energy in turbulent boundary layers.

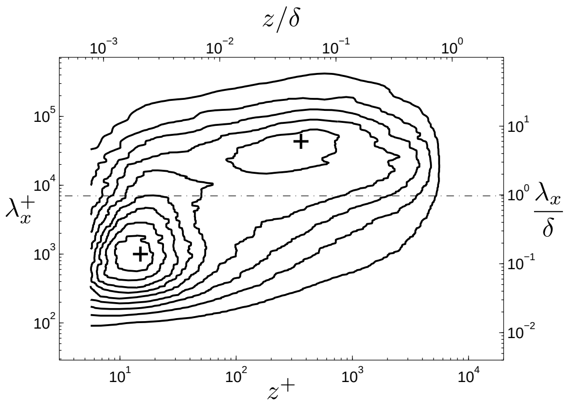

Figure 1. Iso-contours of the pre-multiplied energy spectra of streamwise velocity fluctuation $k_x\phi_{uu}/U_\tau^2$ ($Re_\tau = 7300$); Contour levels are from 0.2 to 2.0 in steps of 0.2. The " $+$ " symbols mark the inner peak ($z^+ = 15$, $\lambda_x^+ = 1000$) and the outer peak ($z/\delta = 0.05$, $\lambda_x/\delta = 6$); The horizontal dot-dashed line show the location of the spectral filter.

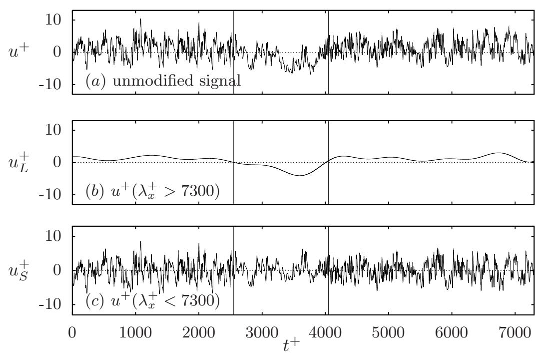

Figure 2. Example of fluctuating u signal in the near-wall region, $z^+ = 15$; (a) raw fluctuating component; (b) large-scale fluctuation $\lambda_x^+ > 7300$; (c) small-scale fluctuation $\lambda_x^+ < 7300$; Dashed vertical lines show region of negative large-scale fluctuation.

HILBERT TRANSFORMATION AND DEMODULATION METHOD

The Hilbert transformation has the interesting capacity to extract the envelope of any real-valued signal $x(t)$. In reality, the envelope corresponds to the modulus of the analytic signal $Z(t) = x(t) + i\mathcal{H}[x(t)]$, formed by the complex conjugated $x(t)$ and its transformation $\mathcal{H}[x(t)]$ [1]. In the particular case of an amplitude modulated signal, this has remarkable utility, since the envelope of the modulated signal is in fact the modulating signal shifted by a D.C. component. An example of this is given in Fig. 3. The modulated signal $u(t)$ is formed by using the modulating signal $m(t)$ applied to the carrier signal $c(t)$. It can be clearly seen that the envelope of $u(t)$ as calculated by the Hilbert transformation is strictly identical to $m(t)$.

This can be directly linked to the scale decomposition shown previously (Fig. 2). If we consider that the large-scale events have an amplitude modulation effect onto the small-scale structures, this implies that the envelope of the small-scale component should be correlated with the large-scale component. A meaningful proof of this can be obtained by calculating the coefficient correlation R between both, the large-scale component and the envelope of the small-scale fluctuations. Further details about this method are available in Mathis *et al.* [5]. An example of such results is presented for the signal previously decomposed in Fig 2. Fig. 4 shows the large-scale component of the signal (dashed) along with the envelope of the small-scale component obtained using the Hilbert transformation (solid). The overall correlation is calculated as 0.2 between these two signals, which clearly implies an amplitude modulation effect. The same process is repeated for all wall-normal locations across the boundary layer and the resulting correlation is shown in Fig. 5. A high level of correlation is observed in the viscous layer of the boundary layer, decreasing progressively towards the log-region. This is interpreted as strong evidence that the near-wall cycle associated with the viscous layer is strongly modulated by low wave-number motions associated with the log-region. In the log-region, the correlation decreases progressively to reach a zero value at about $z^+ = 300$, corresponding reasonably well to the position of the outer peak ($z/\delta \approx 0.05 \Leftrightarrow z^+ \approx 365$). This reversal in correlation behaviour is very much as predicted by Hutchins & Marusic [3] who found that the small-scale energy was smaller under negative large-scale fluctuations up to $z^+ \approx 300$, after which a reversal occurred (and the small-scale fluctuations were more energetic under negative large-scale excursions).

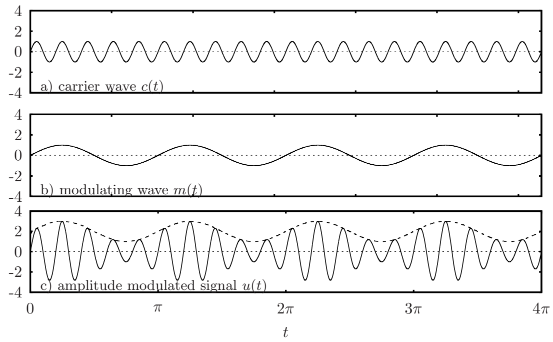

Figure 3. Example of amplitude modulation; (a) represents the carrier wave $c(t) = \sin(10t)$; (b) represents the modulating wave $m(t) = \sin(2t)$; (c) represents the modulated signal $u(t) = [2 + m(t)]c(t)$ (solid line) and its envelope calculated from the Hilbert transform (dashed line).

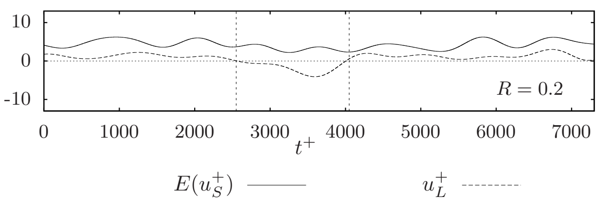

Figure 4. Example of Hilbert transform decomposition on the fluctuating velocity signal at $z^+ = 15$; - - - large-scale component u_L^+; — envelope $E(u_S^+)$ of the small-scale component calculated from the Hilbert transformation; Dashed vertical lines show region of negative large-scale fluctuation.

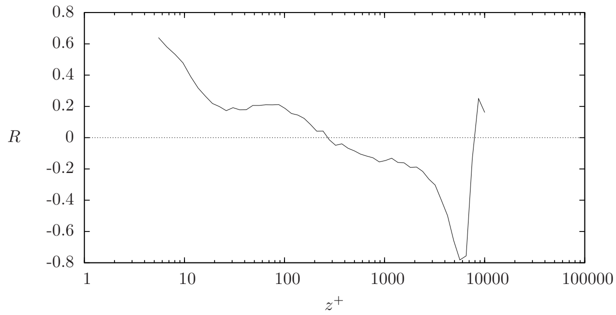

Figure 5. Wall-normal evolution of correlation coefficient $R(z^+)$, between the large-scale component and the envelope of the small-scale component, across the boundary layer ($Re_\tau = 7300$).

CONCLUSIONS

An analysis of scale-decomposed fluctuating velocity signals using the Hilbert transformation has revealed strong supporting evidence to confirm the initial assumptions proposed by Hutchins and Marusic [3]. It is shown that, in the viscous and buffer layers, the large-scale log region events influence the near-wall viscous-scaled structure in a manner akin to a pure amplitude modulation. This apparent modulation has numerous implications to our assumptions concerning turbulent boundary layers. The near-wall cycle, assumed for some time now to be an autonomous process, is shown here to reside under the modulating influence of the superstructures type events associated with the log region.

References

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