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Turbulence intensity similarity formulations for wall-bounded flows

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1 Introduction

Recently Marusic and Kunkel [1] extended the Marusic, Uddin and Perry [2] streamwise turbulence intensity similarity formulation to be applicable across the entire smooth-wall, zero-pressure-gradient, turbulent boundary layer. The formulation is based on the attached-eddy model of Perry and Marusic [3] and is in excellent agreement with datasets spanning Reynolds numbers from laboratory to atmospheric scales. Here we present extended formulations for the wall-normal and spanwise turbulence intensities, which are also applicable across the entire smooth-wall turbulent boundary layer. The formulations are compared to computational and experimental data, both laboratory and atmospheric, covering a Reynolds number range of $O(10^2)$–$O(10^6)$.

The new formulations are similar to the streamwise formulation in that they incorporate wake and wall deviations to the high Reynolds number asymptotic form that were originally proposed by Townsend [4] for the log region. Following Townsend's Reynolds number similarity hypothesis, the wake deviation in the outer part of the flow is Reynolds number independent when scaled with outer-flow variables. The turbulence intensities in the outer portion of the flow are also seen to be independent of surface roughness, further agreeing with Townsend's hypotheses. In addition we consider a modification to the streamwise formulation to make it applicable to turbulent pipe flow.

2 Discussion

Similar to the extended streamwise formulation, the extended wall-normal and spanwise formulations consist of an inner- and outer-region which are blended with a cubic curve. For example, the spanwise formulation is given by

$$\overline{u_2^2}^+ = \begin{cases} f_{I2}[z^+]f_{T2}[z^+, Re_\tau] & \text{for} \quad z^+ \leq z^+_{inner} \\ \\ f_{O2}[z^+, Re_\tau] & \text{for} \quad z^+ \geq z^+_{outer}. \end{cases} \tag{1}$$

Following the streamwise formulation z^+_{inner} and z^+_{outer} are taken to be 30 and 150 respectively, the exact values of which are of secondary importance. The inner portion is comprised of an empirical curve fit of high resolution experimental or computational data (f_{I2}) that is scaled with f_{T2} so that the outer region of the inner portion of the formulation follows the inner region of the outer portion [1]. Note also that f_{I2} scales with inner-flow variables and has the correct near-wall behavior. The outer portion of the formulation is obtained by incorporating a viscous and wake deviation to the existing asymptotic log-region form

$$f_{O2}[z^+, Re_\tau] = B_2 - A_2 \ln\left[\frac{z}{\delta}\right] - V_{g2}\left[z^+, \frac{z}{\delta}\right] - W_{g2}\left[\frac{z}{\delta}\right]. \tag{2}$$

Similar to the streamwise formulation, $A_2(= 0.475)$ is a universal constant and $B_2(= 1.20)$ is a large scale characteristic constant, V_{g2} is the viscous deviation and W_{g2} is the wake deviation. Here the viscous term is simply comprised of the same isotropic attached eddy cutoff as the streamwise formulation. It is expected that there is also an anisotropic component; however, due to the lack of high resolution experimental data, it is not currently known. For consistency the wake deviation is based on a polynomial expression for the wake deviation of the mean flow ([2]) and is obtained by forcing the boundary conditions in the inner- and outer-regions, $\overline{u_2^2}^+ = 0$; $\partial(\overline{u_2^2}^+)/\partial(z/\delta) = 0$ at $z/\delta = 1$ and $W_{g2} = 0$ as $z/\delta \to 0$.

The extended wall-normal formulation is functionally the same except that the outer region is described by

$$f_{O3}[z^+, Re_\tau] = A_3 - V_{g3}[z^+, \frac{z}{\delta}] - W_{g3}[\frac{z}{\delta}] \tag{3}$$

and, therefore, since the asymptotic log-region form is a constant ($A_3 = 1.78$), no scaling function is needed for the inner region. The functional form looks exactly like Eqn. 1 without the scaling function (f_{T2}).

Figures 1 and 2 show the extended spanwise and wall-normal formulations respectively. The formulations agree well with the data. Note the outer region of all formulations are valid for rough walls and that they correctly describe the the rough wall atmospheric data. There is some disagreement between the formulations and the data in the log region. This can be attributed to the lack of the anisotropic portion of the viscous deviation.

The extended streamwise turbulence intensity similarity formulation is also modified to describe pipe flow by adjusting the wake deviation from the asymptotic form. The implicit assumption made is that the turbulence intensities in inner-wall and mean-velocity log regions, similar to the mean-velocity profiles, are comparable in internal and external flows. Therefore, in accordance with the formulations, the only modification required to the asymptotic log-region form of the similarity formulation is an adjustment of the large scale characteristic constant ($B_{1pipe} = 2.8$). Also, it is expected that the wake will need to be adjusted for internal pipe flow so that $\overline{u_1^2}^+ = \overline{u_{1C}^2}^+$ at $z/\delta = 1$, where $\overline{u_{1C}^2}^+$ $(= 0.8)$ is the

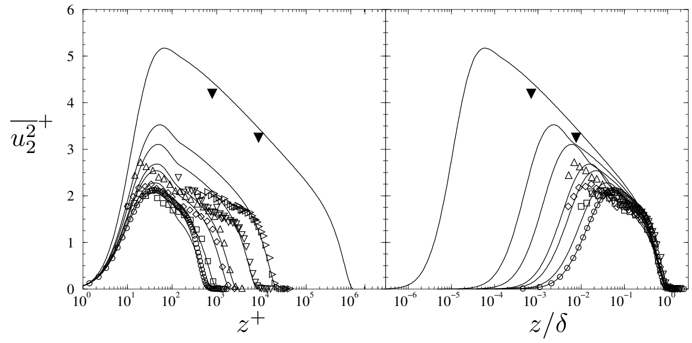

Figure 1: Extended spanwise similarity formulation. Open symbols are laboratory data;Re_τ = 689–Spalart [5]; 1028, 1988, 2835–Bruns *et al.* [9]; 8080, 22510–Fernholz *et al.* [7]. Solid symbols are atmospheric data; $Re_\tau \approx 1 \times 10^6$ (Unpublished data from Nick Hutchins, University of Minnesota).

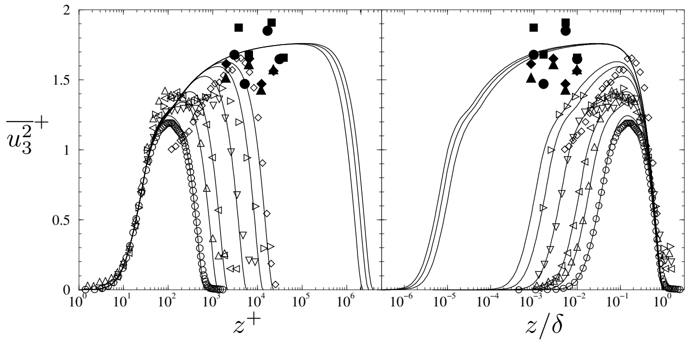

Figure 2: Extended wall-normal similarity formulation. Open symbols are laboratory data; Re_τ = 689–Spalart [5]; 1335, 2217, 5813, 13490–DeGraaff *et al.* [6]; 23010–Fernholz *et al.* [7]. Solid symbols are atmospheric data; $Re_\tau \approx 3 \times 10^6$–Kunkel [8].

pipe centerline turbulence intensity. A comparison of streamwise formulation adjusted for pipe flow with high Reynolds number internal flow data from the Princeton/DARPA/ONR Superpipe is shown in figure 3 [10]. It is important to note that B_{1pipe} and $\overline{u_{1C}^2}^+$ were obtained from one data set and held constant for all of the others. Again, the formulations appear to describe the data well.

3 Acknowledgments

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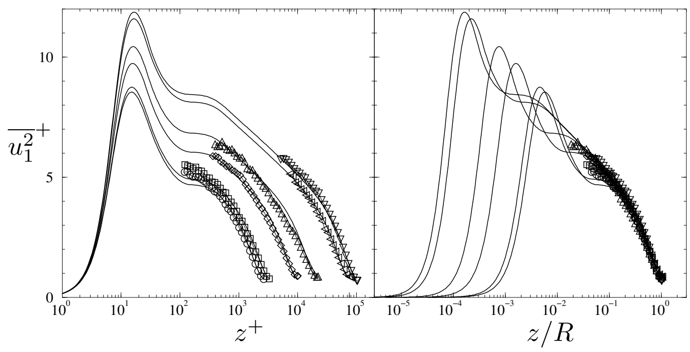

Figure 3: Extended streamwise similarity formulation adjusted for pipe flow. Symbols are Superpipe data; $R^+ = 2639$, 3312, 9867, 21330, 74920, 101400–Zhao *et al.* [10].

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