A COMPARATIVE STUDY OF THE SPECTRA OF TURBULENT JETS AND BOUNDARY LAYERS AT HIGH WAVE NUMBERS

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This paper describes measurements of spectra taken in a coflowing jet and in a boundary layer with zero streamwise pressure gradient. The spectra are compared to examine universality of the small scales and to examine the scaling laws of Kolmogorov. Comparisons are also used to examine differences in the distribution of the Reynolds shear-stress spectra in wavenumber space between the two flows and the experimental observations are explained in terms of models for the turbulence structure which have been developed by the authors for each of these flows.

1. INTRODUCTION

The measurement of spectra in turbulent shear flows has long been a useful tool in examining its underlying structure. However, comprehensive measurements in a range of different shear flows are still scarce. Work in the past has often concentrated on a single shear flow and this has limited the range of parameter space which can be investigated. One of the long-standing questions in the study of turbulence is whether the small scales are universal in different flows as suggested by the Kolmogorov universal equilibrium theory. This theory suggests that the smallest scales are locally isotropic in wavenumber space and depend only on viscosity (ν) and the dissipation of turbulent kinetic energy (ε). This leads to the definition of a length scale and a velocity scale

$$\eta = (\nu^3/\varepsilon)^{1/4} \quad \text{and} \quad u = (\nu \varepsilon)^{1/4}$$

respectively. Therefore if these scales are used correctly to non-dimensionalise the normal components of the spectra, each of these components should collapse onto a single curve in the high wavenumber range where the Kolmogorov arguments should apply. A good test of these ideas requires the comparison of different flows with as wide a range of η and ν as possible. Another consequence of the Kolmogorov theory is that if the Reynolds number is high enough it may be possible to have a range of wavenumbers which are locally isotropic but the eddies are large enough such that viscosity is not important (called the equilibrium range). This leads to a -5/3 power-law form for the normal spectra in this range.

If carefully interpreted, spectra can also give much insight into the underlying large scale structure of turbulent shear-flows. In particular a comparison of the spectra from two very different turbulent flows may be used to elucidate the fundamental differences in their structure. Recently the authors have had some success in physically modelling both jets and boundary layers using inviscid coherent structures. While the approach used in both cases is similar, the appropriate underlying structure used is quite different. Briefly, the major difference between the two is that the boundary layer model consists of a range of scales of eddies which are attached to the wall, whereas the jet model consists of a single scale of eddies which scale with the local jet width (Δ) and which are randomly jittered about a mean position.

In this paper, first the applicability of Kolmogorov scaling is examined for the two different flows and the existence of a -5/3 law examined. Then comparisons are made of the large scale structure of the two flows in the light of the existing coherent eddy models.

1Here attached is used in the sense of Townsend's attached eddy hypothesis i.e. the size of the eddies scales with their distance from the wall.
EXPERIMENTAL APPARATUS AND TECHNIQUES

The apparatus used for the jet flow and the boundary layer measurements are described in the respective theses of Nichols(2) and Marushi(3). Only a brief description will be given here.

In the case of the jet: the wind-tunnel has a cross-section of 1200x890 mm with a maximum velocity of 3.5 m/s and a turbulence intensity of less than 0.6%. The jet issues from a 4.4 mm nozzle aligned with the wind-tunnel centre-line. The nozzle has a contraction ratio of 9:1 which gives a turbulence intensity of less than 0.5% at the outlet. The exterior of the cone has perforations which allow for boundary layer suction to avoid separation of the flow on the outside surface and allow control of initial conditions. Flow in the jet is produced by a centrifugal blower through honeycomb and a series of screens. The jet flow could be varied from 0-35 m/s.

In the case of the boundary layer: the wind tunnel is of an open-return blower type. It consists of a contraction with area-ratio 8:9:1 leading to the inlet of a 4.3 m long working section with a cross-section measuring 940x388 mm. The free-stream velocity at the inlet of the working section can be varied between 2-35 m/s, while the free-stream turbulence intensity is of the order of about 1%. The working section consists of a smooth wall from a polished acrylic laminate and a series of louvres (or bleeding vanes) which are used to create the turbulence. The working section is operated above atmospheric pressure and the streamwise pressure gradient is adjusted by controlling the location and amount of air sucked in through the louvres. For the data presented in this paper, the streamwise pressure gradient was set to be nominally zero. Spanwise X-wire surveys of the spanwise velocity component confirmed the flow to be nominally two-dimensional in the span.

All turbulence measurements were made with 90° X-wires constructed with 5 μm diameter Pt-10% Rh Wollaston wire, etched approximately to 1.0 mm. Constant temperature i-wire anemometers were used and operated at a resistance ratio of 2.0.

Power spectral densities were calculated from dynamically matched but uncalibrated X-wire signals by using a FFT-algorithm. The signals were sampled at three different sampling rates to improve the frequency bandwidth of the spectrum at low frequencies and were low-pass filtered at less than half the digital sampling rate to avoid aliasing of the measured spectrum. The resulting spectral files were matched and joined to form a single spectral file which was then smoothed. To form the spectral file from the measurement frequency, \( f \), to form the streamwise wavenumber, \( k_x \), Taylor's hypothesis of frozen isotropy was used, i.e. \( k_x = fU_x \), where \( U_x \) is some local convection velocity assumed to be equal to the local mean velocity of the flow at the point of interest. The spectra were normalised so that

\[
\phi_i(k_1) = \int_{k_1}^{\infty} \phi_i(k_1) \, dk_1 = \frac{\phi_i(U_i)}{U_{ij}^2}
\]

\[
\epsilon = 15\nu \int_{k_1}^{\infty} k_1^4 \phi_i(k_1) \, dk_1,
\]

for want of a better method.

3. RESULTS AND DISCUSSION

3.1 Local isotropy and Kolmogorov scaling

In order to examine the universality of the fine-scales in the two flows, spectra were measured and compared. If the high wavenumbers are universal then it would be expected that the spectra from all flows should collapse when non-dimensionalised with the Kolmogorov length and velocity scales. Figure 1 shows a range of typical streamwise spectra from the two different flows scaled with this Kolmogorov scaling. It may be seen that all profiles shown from both flows seem to collapse onto a single curve in the high wavenumber region. There is however some peel-off from this universal form at the highest wavenumbers which is probably due to the spatial resolution of the hot-wire sensor. This explanation for the peel-off is supported by the fact that the spectra peel-off at values of \( k_H \) which correspond to \( k_H \sim \pi \) where \( \pi \approx 3.14 \) is the length of the hot-wire sensor. The limited length of the sensor acts as a spatial filter which attenuates all scales smaller than \( \pi \) (i.e. \( k_H > \pi \)).

The lower diagram in figure 1 shows the same streamwise spectra pre-multiplied by \( k_H^2 \). On this plot a -5/3 power-law in the spectrum appears as a plateau. The diagram shows a short region which appears to be of -5/3 power law form. It is also possible from this plot to estimate the Kolmogorov constant for the one-dimensional spectrum (Kc, where \( \phi_i(k_1)/k_1^5 = K_c \phi_i(k_1)^{5/3} \)). Here it appears to be approximately 0.5 < K_c < 0.6 which is close to the value found in various other flows (Townsend(4)) gives the value of Kc as 0.5 ± 0.03 but the data he presents varies more than this.

The arguments of Kolmogorov also suggest that the universal form for the spectra should correspond to a region of local isotropy. Since we have shown that the streamwise spectra appear to be universal it is worthwhile examining the existence of local isotropy in the flows. There is no definitive way to check for local isotropy experimentally but there are a number of options. Here we have chosen one of the simplest. We will examine the Reynolds shear-stress correlation coefficient defined as

\[
R_{ij}(k_H) = -\frac{\phi_{ij}(k_H)}{\sqrt{\phi_{i}(k_H)\phi_{j}(k_H)}}.
\]

Since there can be no shear-stress component in isotropic turbulence then \( R_{ij} \) should go to zero if the flow becomes locally isotropic. Figure 2 shows the pre-multiplied spectra and the correlation value at a single point of \( R_{ij} \) taken at a single value of \( k_H \) for both flows. Two points are worth noting. Firstly the plateaus occur, not where \( R_{ij} \) is zero, but in the region where it is dropping to zero. This is consistent with the findings of Saddoughi & Veeravalli(5) who examined a very high Reynolds number boundary layer flow. It should be pointed out that strictly it is possible that the fine scales may be locally isotropic even in a region where \( R_{ij} \) is non-zero since large scales may contribute something to the shear-stress spectrum at the high wavenumber end. This is due to the fact that if the velocity signature of an eddy is not simply sinusoidal it will contribute some energy to the spectra at all frequencies. Hence the flow may in fact be locally isotropic in the region of the -5/3 behaviour. More work would be needed to resolve this issue.

It is also worth noting that, the jet profile in figure 2 shows \( R_{ij} \) falling to negative values. The authors believe this to be a consequence of correlated high-frequency noise which becomes significant where the signal-to-noise ratio is very small, rather than being the consequence of any physical flow process.

The other point of interest is the difference in behaviour of the jet and the boundary layer flow. It would seem that \( R_{ij} \) in the jet drops off much more quickly at the same Taylor microscale Reynolds number than the boundary layer which indicates a difference in the
Figure 1: Collapse of data for jet and boundary layer with Kolmogorov scaling showing effect of limited spatial resolution. Dashed lines show median value of $k_1\eta$ for which $k_1\ell = 1$ where $\ell$ is length of hot-wire sensor. Lower diagram shows the extent of the $-5/3$-law for the different cases.

underlying structure of the flows. This point is taken up in the next section.

3.2 Structure

Before discussing the differences in structure of the two flows it is important to have a basis for comparison. It is necessary to define an appropriate Reynolds number and also to define an appropriate shear layer thickness so different levels through the layer may be compared. In boundary layer work a useful Reynolds number is the Kármán number $K_r = \delta U_\tau / \nu$ where $\delta_\tau$ is the boundary layer thickness (see Perry & Li(6) for details) and $U_\tau$ is wall shear velocity. For jets an equivalent Reynolds number based on local conditions can be defined as $R_s = \Delta U_\tau / \nu$ where $\Delta U_\tau$ is the local velocity excess (i.e. velocity on centre-line minus external velocity) and $\Delta$ is the standard deviation of the mean velocity profile. These definitions are based on measurements of the mean flow. In order to compare different cross-stream levels, a relationship between $\delta_\tau$ and $\Delta$ needs to be established. If we consider the edge of the turbulent zone to be the point where the Reynolds stress has fallen to zero (or some small value) and call this $\delta_{63}$ then $\delta_\tau / \delta_{63} \approx 1$ and $\delta_{63} / \Delta \approx 3$. Wherever possible, levels with similar $\delta_{63} / \delta_{63}$ and $r / \delta_{63}$ have been shown (although legends give $r / \Delta$ and $r / \delta_\tau$ values in all cases). Unfortunately, an equivalent factor relating $U_\tau$ and $\delta_\tau$ is unknown and hence, in the absence of a better criterion, similar values of $K_r$ and $R_s$ have been chosen for comparison. It should be noted that this choice does not seriously affect any of the conclusions to be presented.

Figure 2 shows profiles of $R_{13}$ for the two flow cases for similar Reynolds numbers and similar cross-stream levels. It can be seen that the behaviour is quite different. In particular it would seem that $R_{13}$ goes to zero much earlier in the jet case than in the boundary layer case. This suggests that the eddies which contribute to the Reynolds shear stress extend to much higher wavenumber in the boundary layer.

In order to examine the effect of changes of Reynolds number on the wavenumber extent of the Reynolds shear-stress spectra ($\phi_{13}$), figure 4 shows the premultiplied Reynolds shear-
stress spectra for several Reynolds number cases for each flow at a fixed cross-stream level. Since we are only interested in the wavenumber extent, or bandwidth of the spectra they have been divided by their maximum values. Hence we can see the change of shape without regard to the actual magnitudes.

It can be seen that the bandwidth does not seem to change significantly for either the jet or the boundary layer flow. A possible physical explanation for the result will be considered shortly.

Figure 5(a) shows the change in the bandwidth of the Reynolds shear-stress spectra at a fixed Reynolds number for varying cross-stream levels. It may be seen that as the wall is approached the bandwidth decreases for the boundary layer flow but remains substantially unchanged for the jet flow. This suggests that, as the wall is approached, the range of scales which contribute to the Reynolds shear-stress increases for the boundary layer. In particular the contribution from high-wavenumbers and hence smaller eddies increases.

Based on these (and other observations) it is possible to suggest physical models which mimic this behaviour. In the case of the boundary layer, the appropriate model is based on the attached eddy hypothesis of Townsend as developed by Perry & Chong. Briefly this model consists of random arrays of attached eddies of different scales which extend from the wall and scale with their distance from the wall and are distributed with an inverse power-

Figure 4: Comparison of effect of Reynolds number on form of premultiplied Reynolds shear-stress spectra at a fixed cross-stream position.

The results for the Reynolds shear-stress spectra from the models are shown in figure 5(b). It may be seen that they seem to agree very well at least qualitatively with the experimental behaviour shown in figure 5(a). Quantitative agreement depends on the eddy shapes assumed in the model. These results give further support to the basis for the structure of the models, namely: with an increase in \( z/\delta_0 \) in the boundary layer, the bandwidth in \( k_{12} \phi_{12}(k_{12}) \) increases whereas for the jet, the bandwidth remains essentially constant as \( r/\Delta \) is decreased. It should be stressed that these models also predict other trends and quantities not shown in this paper (for example all components of Reynolds stresses, mean flow and other components of spectra). Further details of these models may be found in Nickels and Perry & Marulidi (for the boundary layer).

It is also worth noting that the \( \phi_{13} \) component is not the strongest indicator to show the differences in structure between the two flows. However, it is the only component which is not affected by additional locally isotropic Kolmogorov contributions which will affect the bandwidth of the other components. A feasible way of subtracting off the Kolmogorov contribution is not known at this stage and therefore only \( \phi_{12} \) has been considered here.

4. CONCLUSIONS

Comparisons of spectra in a jet and a boundary layer suggest that the fine-scales may be universal and scale with the Kolmogorov scaling. There is also some evidence of the existence of a -5/3 law in the streamwise spectra. Measurements of Reynolds shear-stress correlation coefficient spectra suggest that the finest scales are locally isotropic, however, the region where the -5/3 law seems to apply may not be locally isotropic except at the highest of wavenumbers. More work needs to be done to resolve this issue.
Figure 5: Comparison of variation of bandwidth of premultiplied Reynolds shear-stress spectra at a fixed Reynolds number with cross-stream position. Note increase in bandwidth for boundary layer as wall is approached. (a) Experimental results, $K_e = 4412$ for BL; $R_{\Delta} = 2384$ for Jet, (b) Results from coherent structure models developed by the authors for same levels as in (a).

Further comparisons have illustrated the difference in structure between boundary layers and jets and suggest that the boundary layer has a much wider range of scales which contribute to the Reynolds shear-stress. This is in line with models for both flows which have been developed by the authors.

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4. REFERENCES