# SPATIAL RESOLUTION AND REYNOLDS NUMBER EFFECTS IN WALL-BOUNDED TURBULENCE

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### Abstract

In this paper we consider the contributions to the streamwise broadband turbulence intensity from large and small scale motions in turbulent boundary layers, where here small-scale refers to motions with wavelengths less than one boundary layer thickness long. How these contributions change with increasing Reynolds number is discussed together with the attenuation effects due to insufficient spatial resolution, particularly in the near-wall region. These considerations are combined with the attached eddy hypothesis to guide a new correction formula that accounts for spatial filtering effects at all wall-normal positions. The new formulation is validated against existing data and is shown to work very well.

# **1** Introduction

The past decade or so has seen a heightened interest in high Reynolds number wall-bounded turbulent flows. This has largely been driven by the commissioning of new facilities that provide detailed access to high Reynolds number wall turbulence, under incompressible conditions. These include the Princeton Superpipe (Zagarola & Smits 1998), the Stanford pressurized wind tunnel (DeGraaff & Eaton 2000), the MTL wind tunnel at KTH (Osterlund et al. 2000), the National Diagnostic Facility at IIT (Nagib et al. 2007), and the High Reynolds Number Boundary Layer Wind Tunnel at the University of Melbourne (Nickels et al. 2007). In addition, the Surface Layer Turbulence and Environmental Science Test (SLTEST) facility in Utah (Metzger & Klewicki 2001) has provided high quality data in the atmospheric boundary layer, which has been invaluable for studying the behavior at Reynolds numbers one or two orders of magnitude larger than what is possible in the laboratory. Other, more general purpose facilities, have also been employed, including the DNW, the German-Dutch wind tunnel (Fernholz et al. 1995), and the US Navy's William B. Morgan Large Cavitation Channel (Etter et al. 2005, Winkel et al. 2010), where suspended flat pates have been installed into these largescale facilities.

The interest in high Reynolds number conditions has also been spurred by other lower Reynolds number experimental and numerical simulation studies (Adrian et al. 2000, del Alamo et al. 2004, Abe et al. 2004, Hoyas & Jimenez 2006, Schlatter et al. 2009, Elsinga et al. 2010) where the availability of planar and volumetric information has provided new insights into the structure and interactions across the boundary layer. The collective data have raised questions about how things change at high Reynolds number.

#### **High Reynolds number effects**

One of the main questions that has arisen is how the mean flow and turbulence intensities scale with increasing Reynolds number, and this has prompted new scaling arguments and new criteria for independence from initial conditions to be proposed (George & Castillo 1997, DeGraaff & Eaton 2000, Wei et al. 2005, Monkewitz et al. 2007 and others). In this paper we will not focus on the mean velocity as it has been extensively discussed elsewhere (Monkewitz et al. 2008, Marusic et al. 2010). Rather, here we focus on the streamwise turbulence intensities  $(\overline{u^2})$ and their corresponding spectra. One important new finding resulting from such data relates to the verylarge scale motions (VLSM) or superstructures (Kim & Adrian 1999, Hutchins & Marusic 2007), which are found to be much larger in the streamwise direction than previously believed, and questions remain open concerning their scaling and behaviour as Reynolds number increases. Recently, Mathis et al. (2009) showed clear evidence that these very large scale motions interact and modulate the nearwall motions, including the skin friction, and this nonlinear interaction increases in magnitude and strength with increasing Reynolds number.

Figure 1 shows a clear indication of the differences in the u-spectra for zero-pressure-gradient boundary layers at low and high Reynolds numbers. Here, the



Figure 1: Contour maps showing variation of one-dimensional pre-multiplied *u*-spectra with wall-normal position for two Reynolds numbers. An inner and an outer peak are noted at the higher Reynolds number. From Hutchins & Marusic (2007).

spectra are premultiplied and are shown across the entire boundary layer, where  $\delta$  is boundary layer thickness, versus dimensionless streamwise wavelength  $\lambda_x^+$ . The friction Reynolds numbers,  $Re_\tau = \delta U_\tau / \nu$ , are indicated on the plots. Here, z denotes wall-normal position, and the superscript "+" denotes normalization with either the velocity scale  $U_{\tau}$  (=  $\tau_0/\rho$ , where  $\tau_0$  is the mean wall shear stress and  $\rho$  is fluid density), or the length scale  $\nu/U_{\tau}$ , where  $\nu$  is the fluid kinematic viscosity. Hutchins & Marusic (2007) found plotting spectrograms in such a manner revealed two prominent peaks: an "inner peak" located nominally at  $z^+ = 15; \lambda_x^+ = 1000$ , and an "outer peak" at  $\lambda_x \approx 6\delta$ . The wall-normal position of the outer peak was found by Mathis et al. (2009) to be located within the logarithmic layer at  $z^+ \approx 3.9 R e_{\tau}^{1/2}$ . The inner peak is associated with the well-documented near-wall coherent streaks, and scales with wall variables. The outer peak is associated with superstructures (Hutchins & Marusic 2007) or the very large scale motions which are seen in figure 1 to have a very weak signature at the low Reynolds number, but a major presence at the higher Reynolds number. As indicated above, the precise scale for the superstructures remains uncertain. What is clear is that the signatures from these long wavelengths extend across the majority of the boundary layer, right down towards the wall. In studying these two scales it is useful to partition the velocity signals into large and small length scale contributions. This is shown in figure 2 where a sharp-spectral filter was used to separate large-scales motions (with  $\lambda_x > \delta$ ) from small-scale motions (the remainder of the signal corresponding to  $\lambda_x < \delta$ ). Here, Taylor's hypothesis of frozen turbulence has been applied, such that  $\lambda_x = U/f$ , where U is the local mean velocity and f is frequency. Figure 2(b) shows how the decomposed components change with increasing Reynolds number. The small-scale component is seen to be invariant with Reynolds number across most of the boundary layer, while the outer-component clearly increases in magnitude across all wall-normal positions with increasing Reynolds number. The increasing influence in the near-wall region (around  $z^+ = 15$ ) is noted. The result in figure 2 shows that the  $\overline{u^2}^+$  profile can be considered to be the sum of two competing modes: a small viscous-scaled component primarily located in the near-wall region, and a larger outerscaled component peaking in the log region. There is considerable overlap between these modes, with the large-scale extending down to the wall, and a diminishing small-scale influence penetrating to the edge of the boundary layer.

Figure 3 shows profiles of  $\overline{u^2}$  in a boundary layer, scaled with wall variables, for five different Reynolds numbers. It is noted that the peak turbulence intensity (at  $z^+ \approx 15$ ) is seen to slowly increase with increasing Reynolds number, and from the results in figure 2 it is clear that this increase is due to increasing large-scale contributions near the wall.

#### **Spatial resolution effects**

The scaling behaviour of  $\overline{u^2}$  and other turbulence statistics has been clouded in the past due to a number of issues, one of the most significant being the effect of insufficient measurement spatial resolution. This is understandable as the smallest scales in the flow become very small in most facilities in the near-wall region, and spatially resolving them becomes a major challenge. A prototypical issue related to this challenge has been the question of whether  $\overline{u^2}^+$  is invariant at a fixed near-wall  $z^+$  (especially the peak location  $z^+ = 15$ ) and this has been considered extensively in the literature. Classical wall-scaling arguments suggest that it should be invariant as supported by the surveys by Mochizuki & Nieuwstadt (1996)



Figure 2: Scale decomposition of the streamwise turbulence intensity profile  $\overline{u^2}/U_{\tau}^2$ . (a) For  $Re_{\tau} = 7300$ , together with total (summed) contribution; (b)  $Re_{\tau} = 3900, 7300, 19000$ . From Marusic et al. (2010).



Figure 3: Streamwise turbulence intensity profiles for hot-wires measurements with  $l^+ \approx 22$  for Reynolds numbers  $Re_{\tau} = 2820, 3910, 7340, 13620, 18830$ . Figure adapted from Hutchins et al. (2009).

and others, while more recent studies, with well resolved measurements, such as those of DeGraaff & Eaton (2000) and Metzger & Klewicki (2001) show a clear increase in  $\overline{u^2}^+_{PEAK}$  with increasing Reynolds number. The data shown in figure 3 are taken from Hutchins et al. (2009) who used hot-wire anemometry with different sized wires for each Reynolds number, such that the length of the sensing element, l, was matched in dimensionless wall variables. That is, for each case  $l^+$  was matched at a value of 22, while maintaining an appropriate length-to-diameter ratio of the wire (> 200) as recommended by Ligrani & Bradshaw (1987). While some (small) attenuation will occur even for  $l^+ = 22$  it was argued that this level of attenuation will be the same for all Reynolds numbers. (It is interesting to note that recent studies by Hultmark et al. 2010 in pipes show no such increase in  $\overline{u^2}_{PEAK}^+$  even with matched  $l^+$  values, suggesting that there may be fundamental differences between pipe and boundary layer flows. Here, we will confine our discussion to boundary layers.)

The effect of insufficient spatial resolution is shown in figure 4. Here, three different sized hot-wires with  $l^+ = 22$ , 79 and 153 are used to measure the same profile at a given Reynolds number ( $Re_{\tau} = 7300$ ), and clear attenuation is noted for wall-normal positions considerably larger than the size of the sensing element. The corresponding contributions from the large and small scales, as was done in figure 2, are also shown in the figure. As expected, the attenuation observed for the larger values of  $l^+$  are confined to under-resolving the small-scale contributions to the broadband intensity.

It is noted that values of  $l^+$  considered in figure 4 are not atypical of high Reynolds number experiments. For example,  $l^+ = 100$  in the Princeton Superpipe for  $Re_{\tau} = 10^5$  corresponds to a wire length of  $l = 65\mu$ m, which is well below that obtainable by conventional hot-wire anemometry. For this reason, considerable effort is currently being devoted to the design and development of new sub-miniature hot-wires using micro- and nano-fabrication techniques. An example of which is the NanoScale Thermal Anemometry Probe (NSTAP) developed at Princeton (Bailey et al. 2010).

Notwithstanding these micro-probe advances, there is a continuing need to better understand and quantify the effects of limited spatial resolution, particularly in the near-wall region. It is in this region that the flow is strongly non-homogeneous in the wall-normal direction and existing spatial-resolution schemes, such as that of Wyngaard (1968) that are based on assumptions of small-scale isotropy, are clearly inadequate. Alternative methods have been proposed including that of Chin et al. (2010), which appear in this symposium's proceedings. Here a



Figure 4: Turbulence intensity profiles decomposed into small-scale ( $\lambda_x < \delta$ , solid symbols) and largescale components ( $\lambda_x > \delta$ , open symbols). The symbols show results for the different  $l^+$  values. The lines show sum of small- and large-scale components for  $l^+ = 22$  (solid line),  $l^+ = 79$  (dashed line) and  $l^+ = 153$  (dotted line). Taken from Hutchins et al. (2009).

detailed account of the missing energy is modeled using the spanwise-streamwise information in twodimensional spectra from a direct numerical simulation (DNS) of turbulent channel flow. The approach is seen to be promising, despite being rather complicated. In the following we will consider the above described effects of changing Reynolds number on the flow, and use this to guide a new simple formulation for modeling the effects of spatial resolution across the boundary layer.

## 2 Near-wall eddy scales

The above results show that modeling the effects of spatial filtering requires knowledge of the local small-scale eddies that contribute to the turbulence intensities. The systematic study by Hutchins et al. (2009) using a series of wires with varying  $l^+$ over a range of Revnolds number, reaffirmed the previous low Reynolds number study of Ligrani & Bradshaw (1987). These studies contend that the criterion for sufficiently resolved measurements are based on a value of  $l^+$ . This is consistent with the findings of Kline et al. (1967) and many others based on physical observation that the near-wall coherent streaks scale with wall units, with a spacing of nominally  $100^+$  $(=100\nu/U_{\tau})$  over a large range of Reynolds numbers. Furthermore, there is strong evidence to suggest that the near-wall streaks are a manifestation, or at least are associated with, the legs of near-wall attached eddies as described by Adrian and coworkers (Adrian 2007). Perry & Chong (1982) proposed a physical model for wall turbulence based on an assemblage of such eddying motions, distributed in scale and strength according to the Townsend (1976) attached eddy hypothesis. The essential feature here is that the eddies scale with their distance from the wall, and Perry & Chong proposed that the smallest attached eddies are on the order of the Kline et al. streak spacing  $(O(100^+))$ .

One issue that often arises when discussing spatial filtering effects relates to the fact that the smallest motions in any turbulent flow scale with the Kolmogorov length scale,  $\eta$ , and consequently the appropriate criterion for spatial resolution should be stated as a limiting value of  $l/\eta$  and not  $l^+$ . However, the issue here is whether or not the Kolmogorov scale motions contribute to the broadband turbulence intensities. Recent studies by Stanislas et al. (2008) and others have shown that the core diameter of the smallest filamentary vortex structures (and likely the segments of the energy-containing eddies) in boundary layers are of order of  $12\eta$ . The only way that these different observations can be reconciled is if  $n^+$  is approximately a constant in this near-wall region, meaning that the two scalings are effectively the same (that is,  $\eta \propto \nu/U_{\tau}$ ). Figure 5 shows estimates for  $\eta^+$  across the boundary layer using the same data that Hutchins et al. (2009) used to obtain figure 3. Indeed, the results show that in the near-wall region (say,  $z^+ < 20$ )  $\eta^+$  effectively can be taken as a constant, and across the entire inner-region, including the log region,  $\eta^+$ versus  $z^+$  is invariant with Reynolds number. Similar results were also found by Carlier et al. (2005) and recently by Yakhot et al. (2010) across a large range of Reynolds number in pipe flows.



Figure 5: Kolmogorov length scale with wall scaling across turbulent boundary layers. Data as in Hutchins et al. (2009). Here,  $\eta = (\nu^3/\epsilon)^{1/4}$ , where the isotropic assumption is used to estimate the dissipation rate:  $\epsilon = 15\nu \int_0^\infty k_x^2 \phi_{uu} dk_x$ . Symbols indicated  $Re_{\tau}$  values:  $\circ$  2800,  $\Box$  3900,  $\diamond$  7300,  $\triangle$  13600,  $\triangleright$  19000.

Given that far from the wall the eddy core sizes continue to scale with  $\eta$  (Stanislas et al. 2008), this implies that the Kolmogorov length should remain a relevant scale for unresolved energy so long as it is less than the sensor size (which likely is the case). When such eddies inevitably 'die', their filaments are effectively broken up leading to motions scaling exclusively with  $\eta$ . Therefore, there will be some measure of unresolved energy even at large wall-distances (where l/zis negligible) and this should strictly follow  $l/\eta$  scaling. However, the results in figure 5 indicate that for modeling purposes, it does not likely matter whether we choose  $l^+$  as a parameter or  $l/\eta$ . Therefore, for consistency with Hutchins et al. (2009), in the remainder of the paper we will stay with the parameter  $l^+$ , even though  $l/\eta$  may be more physically appropriate. In addition, the use of  $l^+$  is more straight-forward to use as a parameter, as for unresolved measurements  $\eta$ is not known with certainty and would require some modeling to estimate the missing contribution to the dissipation rate.

#### Attached eddies

The results in figure 4 highlight that the attenuation due to inadequate spatial resolution relates to the small-scale motions (with  $\lambda_x < \delta$ ), and it is interesting to note in the classical logarithmic region (as determined from the mean flow) these small-scale contributions to  $\overline{u^2}^+$  also show a nominally logarithmic profile (as indicated by the solid lines in figure 2b). This is as predicted by Townsend's (1976) attached eddy hypothesis. Here, the energy-containing scales of turbulence increase in size with distance from the wall, and the schematic shown in figure 6 illustrates this concept using idealized hairpin shaped eddies as the representative eddies as proposed by Perry & Chong (1982). Essentially, the attached eddy model involves the idea that turbulence may be described by a random array of eddies attached to the wall. The distribution of eddies is such that there are hierarchies of eddies, the smallest scaling in viscous units and thereafter following a geometric progression in size, with the largest being roughly the height of the boundary layer. Therefore, these can be taken as the major contributors to the small-scale energy, which we are interested in modeling. In figure 6 the idea of hierarchies of eddies is illustrated by lining up eddies of each scale. The distribution of hierarchies is such that there are many more small eddies than large eddies; Townsend (1976) proposed that the probability density function of eddy sizes should be inversely proportional to the eddy size to give a region of constant Reynolds shear stress. In figure 6, a typical hot-wire probe is shown, having sensor width approximately the size of the fourth largest hierarchy of eddies (blue). At the wall-normal distance of the probe shown, the sensor captures most of the contributions from the eddies. However, as the sensor moves down, it is clear that it will begin to lose contributions from eddies at an increasing rate. It can be shown that the attenuation in energy is a function of l/z if a roughly inverse probability density function of eddies,  $p_H(\lambda)$  is chosen. For example, if

$$p_H(\lambda) = \frac{1}{\lambda},\tag{1}$$

and eddies smaller than  $\alpha l$  are not resolved at all and eddies larger than  $\alpha l$  are fully resolved, then

$$\left(\overline{u^2}^+\right)_{\text{unresolved}} = \Delta(\overline{u^2})_L^+ = C_1 \log\left(\frac{\alpha l}{z}\right).$$
 (2)

While this might have relevance to very high Reynolds number flows at distances far from the wall, in the region of most interest here (i.e.,  $15 \leq z^+ \leq 3.9 R e_{\tau}^{1/2}$ ), there are many characteristic contributors that should be accounted for in addition to these idealised attached eddies. Accounting for such contributors is a difficult task and is beyond the scope of this paper. However, the attached eddy model provides a conceptual framework that suggests the lost contributions due to finite, fixed sensor size should be a function of l/z, simply because eddy size increases rapidly with wall-distance and the number of large eddies is much less than the population of the small eddies.

#### 3 Formulation for the unresolved energy

Recently, Smits et al. (2010) revisited the work of Hutchins et al. (2009) and proposed that their formulation for missing  $\overline{u^2}^+$  due to spatial resolution at  $z^+ = 15$  could be extended to higher wall-normal positions (i.e.,  $z^+ \gg 15$ ) with the inclusion of an additive l/z term. This is consistent with the arguments presented above concerning attached eddies. In the following we will make use of this proposed result and adopt a linear scaling in l/z of missing energy,  $\Delta(\overline{u^2})_L^+$ , for  $z^+ \gg 15$ . At  $\underline{z^+} = 15$ , the Hutchins et al. (2009) formulation

for  $\Delta(\overline{u^2})^+_L$  is:

$$\Delta(\overline{u^2})_L^+ = Bl^+ + C \frac{l^+}{Re_\tau}.$$
 (3)

Nearer to the wall, for simplicity it will be assumed that the lost energy will be a linear function of  $z^+$ . This leads to the following conceptual formulations for lost energy due to spatial resolution:

$$\Delta(\overline{u^2})_L^+ = \begin{cases} Az^+ & z^+ \ll 15\\ Bl^+ + Cl^+/Re_\tau & z^+ \approx 15\\ Dl^+/z^+ & z^+ \gg 15 \end{cases}$$

This can be recast, since  $M = Bl^+ + Cl^+/Re_\tau$  is a constant for a given experiment. Therefore,

$$\Delta(u^2)_L^+ = Mf(z^+),$$
  
where  $f(z^+) \to \begin{cases} k_1 z^+ & z^+ \ll 15\\ 1 & z^+ \approx 15\\ k_2/z^+ & z^+ \gg 15 \end{cases}$ 



Figure 6: Scale drawing of a hot-wire sensing an artificially arrayed sample of idealized attached eddies. It is observed that the wire will resolve exponentially less eddies as the wall is approached. Top and front views are also shown.



Figure 7: The function  $f(z^+)$ .

Figure 7 shows these three functions (as broken lines). A functional form is required that blends between these three curves, and an ideal tool for achieving this is a logistic dose response curve, as described by Joseph & Yang (2009). A modified version is adopted here of the form

$$f(z^{+}) = f_3 + y_b \left[ f_2 + y_a \left( f_1 - f_2 \right) - f_3 \right]$$

where

$$f_1 = k_1 z^+, \qquad f_2 = 1, \qquad f_3 = k_2 / z^+.$$

The following constants and smoothing functions are used in the curve fit

$$y_a = \left[1 + (z^+/a)^{m_1}\right]^{-m_1^{-1}}$$
$$y_b = \left[1 + (z^+/b)^{m_2}\right]^{-m_2^{-1}}$$
$$k_1 = 1/11.3 \quad k_2 = 11.3$$
$$a = 9 \qquad b = 8$$
$$m_1 = 8 \qquad m_2 = 10$$

The formulation for the missing energy is obviously heavily dependent on the value of M, the peak missing energy. Hutchins et al. (2009) determined the functional form for  $M = Bl^+ + Cl^+/Re_{\tau}$  by curve-fitting experimental data in the Reynolds number range  $2800 < Re_{\tau} < 20000$  and  $11 < l^+ < 150$ . It is linear in  $l^+$  and it is not certain whether the function will apply for either very large  $l^+$  or significantly higher  $Re_{\tau}$ , and this will require further verification.

# 4 Validation of the missing energy formulation

Here we present turbulence intensity data acquired in the Melbourne high Reynolds number boundary layer facility, (see Hutchins et al. 2009 for details). The experiments were performed with matched Reynolds number, but varying sized probes so that the spatial resolution effects were different for each measurement. Figures 8a & 9a show turbulence intensity results for  $Re_{\tau} = 7300 \& 13600$  respectively. In figures 8b and 9b, the unresolved energy modeled by equation (4) has been added back with the constant M chosen such that the peak energy is that expected with a wire length of 22 wall units (as for the most resolved measurement). That is,

$$M = B\left(l^{+} - 22\right) + C\left(\frac{l^{+} - 22}{Re_{\tau}}\right).$$
 (4)

The function  $f(z^+)$  is unchanged. The anticipated result is that the larger sensor data will collapse onto the  $l^+ = 22$  data following the application of the above correction. This is the only method of validation possible, since it is not possible to measure/simulate the fully resolved ( $l^+ \approx 0$ ) turbulence intensity at this Reynolds number.

The corrected turbulence intensity profiles agree very well with the  $l^+ = 22$  profile for both higher and lower Reynolds numbers. The greatest differences in corrected data occur at the peak location. This could be partly due to the inaccuracy and/or simplicity of the curve-fit by Hutchins et al. (2009), however, the differences are more likely indicative of the extent of experimental repeatability.

A final validation is provided from the channel flow DNS data of del Alamo et al. (2004) at  $Re_{\tau} =$ 934. Chin et al. (2009) simulated the spatial resolution effects of finite sensor sizes by spatially filtering the DNS data. The results of that study are shown in figure 10*a*. In this case, *M* is chosen so as to correct the turbulence intensity toward the profile expected with a wire length of  $l^+ = 19$  (for consistency with the preceding experimental data analysis). That is,

$$M = B\left(l^{+} - 19\right) + C\left(\frac{l^{+} - 19}{Re_{\tau}}\right).$$
 (5)

Again, the function  $f(z^+)$  does not change. Note that choosing M in this way means that the effect of the correction to the  $l^+ = 3.8$  profile (corresponding to the DNS spanwise grid resolution) is an attenuation toward the  $l^+ = 19$  profile. The corrected intensity profiles are shown in figure 10b. As with the experimental data, the agreement between corrected profiles is very good for all but the smallest  $z^+$  values. In fact, the overall agreement is somewhat better than for the experiments and this is likely due to the fact that the DNS data are calculated from the same velocity field, while the experimental data are from six independently measured experiments. The small deviations at  $z^+ < 5$  seen for the DNS data corrections are due to the assumption of the correction formula having a simple linear drop off near the wall. It should strictly follow  $(z^+)^2$ , as determined by a Taylor's series expansion at the wall, and this would need to be modified in any future fine tuning if this very near-wall region was required.



Figure 8: Left: Turbulence intensity profiles measured with various wire lengths at  $Re_{\tau} = 7300$ . Right: Turbulence intensity profiles corrected using equation (4).



Figure 9: Left: Turbulence intensity profiles measured with various wire lengths at  $Re_{\tau} = 13600$ . Right: Turbulence intensity profiles corrected using equation (4).



Figure 10: Left: Turbulence intensity profiles from the DNS of del Alamo *et al.* at  $Re_{\tau} = 934$  with simulated spatial resolution effects (see Chin et al. , 2009). Right: DNS turbulence intensity profiles corrected using equation (4).

# 5 Conclusions

A functional form for the unresolved contribution to the streamwise turbulence intensity due to finite sensor size has been proposed. The formulation was chosen to coincide with the peak formulation by Hutchins et al. (2009) at  $z^+ = 15$  and to fall off with l/z as  $z^+$  becomes large as suggested by Smits et al. (2010). These trends are guided by consideration of the attenuation due to unresolved attached eddies. The new formulation is found to work very well for turbulent boundary layer data and from simulated results using DNS channel flow data. The utility of the formulation is its simplicity and ability to correct for all wallnormal positions.

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