1. INTRODUCTION

ABSTRACT

The evolution of turbulent boundary layers is a fundamental problem in fluid mechanics and heat transfer. Recent theoretical and experimental works have suggested that the attached eddy model of Perry is not sufficient to describe the turbulent flow. A new approach, based on the classic close-to-wall problem, has been proposed by Perry and others. This approach allows for a more accurate prediction of the turbulent boundary layer profile, which is crucial for engineering applications.

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THE EVOLUTION OF TURBULENT BOUNDARY LAYERS

APPLICATION OF THE ATTACHED EDDY HYPOTHESIS FOR

FLUID MECHANICS AND HEAT TRANSFER

SECOND INTERNATIONAL SYMPOSIUM


\[ 0 = \left[ \begin{array}{c} \mathbf{G} \\ \mathbf{H} \end{array} \right] \left( \begin{array}{c} \mathbf{X} \\ \mathbf{Y} \end{array} \right) \]

A these have a known function of \( \mathbf{G} \), \( \mathbf{H} \), \( \mathbf{X} \), and \( \mathbf{Y} \). The following equation defines the problem of finding \( \mathbf{G} \) and \( \mathbf{H} \) such that

\[ \mathbf{G} \mathbf{X} + \mathbf{H} \mathbf{Y} = 0 \]

where \( \mathbf{X} \) and \( \mathbf{Y} \) are known functions. Here \( \mathbf{G} \) and \( \mathbf{H} \) are unknown functions. These unknown functions are related to an acceleration parameter (\( \alpha \)), where

\[ \frac{\left[ \begin{array}{c} \mathbf{G} \\ \mathbf{H} \end{array} \right] \mathbf{X} \mathbf{Y}}{\mathbf{H}^T \mathbf{Y} \mathbf{X}} = \frac{\mathbf{Y}^T \mathbf{H} \mathbf{X}}{\mathbf{Y}^T \mathbf{H} \mathbf{X}} \]

In order to describe the stress of the element, we use the following equations:

\[ \mathbf{X} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad \mathbf{Y} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \]

By applying these equations, we can determine the stresses of \( \mathbf{G} \) and \( \mathbf{H} \).
\[(91)\]
\[
\frac{d}{dt} \rho \nabla \phi = \frac{\partial}{\partial x} \left( \frac{\rho}{c_0^2} \right) \phi
\]

From (91) and (90) we have

\[
\frac{d}{dt} \rho \nabla \phi = \frac{\partial}{\partial x} \left( \frac{\rho}{c_0^2} \right) \phi = \frac{\partial}{\partial x} \left( \frac{\rho}{c_0^2} \right) \phi = \frac{\partial}{\partial x} \left( \frac{\rho}{c_0^2} \right) \phi
\]

where \( c_0 \) is a universal number and \( \rho \) is a universal constant.

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and so

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\]

where \( \phi \) is the Townsend shear stress function.

\[
\frac{d}{dt} \rho \nabla \phi = \frac{\partial}{\partial x} \left( \frac{\rho}{c_0^2} \right) \phi = \frac{\partial}{\partial x} \left( \frac{\rho}{c_0^2} \right) \phi
\]

Now the Townsend shear stress function is given by

\[
\frac{d}{dt} \rho \nabla \phi = \frac{\partial}{\partial x} \left( \frac{\rho}{c_0^2} \right) \phi = \frac{\partial}{\partial x} \left( \frac{\rho}{c_0^2} \right) \phi
\]

so

\[
\frac{d}{dt} \rho \nabla \phi = \frac{\partial}{\partial x} \left( \frac{\rho}{c_0^2} \right) \phi = \frac{\partial}{\partial x} \left( \frac{\rho}{c_0^2} \right) \phi
\]

The double integral is a universal number and so

\[
\frac{d}{dt} \rho \nabla \phi = \frac{\partial}{\partial x} \left( \frac{\rho}{c_0^2} \right) \phi = \frac{\partial}{\partial x} \left( \frac{\rho}{c_0^2} \right) \phi
\]

Integrate (9) with respect to \( \rho \) to give

\[
\frac{d}{dt} \rho \nabla \phi = \frac{\partial}{\partial x} \left( \frac{\rho}{c_0^2} \right) \phi = \frac{\partial}{\partial x} \left( \frac{\rho}{c_0^2} \right) \phi
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\]
3. CONCLUSIONS AND DISCUSSION

If we make \( M = M' \) this is indistinguishable from (22), as seen in Fig. 2.

![Figure 2](image-url)

Although it is still early days, it seems that a very useful application of the analysis

\begin{align}
\frac{\partial}{\partial t} \left[ (\sqrt{\Pi}) \frac{\partial}{\partial x} \right] & = \left( \frac{\partial}{\partial x} \right)^2, \\
\end{align}

where \( \Pi \) is a universal constant. Figure 2 shows experimental data compared to

\begin{align}
\frac{\partial}{\partial t} \left[ (\sqrt{\Pi}) \frac{\partial}{\partial x} \right] & = \left( \frac{\partial}{\partial x} \right)^2, \\
\end{align}

then from (17) and (18) we obtain \( a = 1/2 \) and \( b = 1/2 \), i.e.,

![Graph](image-url)


