The logarithmic region of wall turbulence: Universality, structure and interactions

Ivan Marusic
Department of Mechanical Engineering
University of Melbourne, Victoria 3010, Australia

Abstract
The logarithmic region, also referred to as the inertial subrange in physical space, or the turbulent wall region, is arguably the most important region of wall-bounded turbulent flows due to the multi-scale processes that take place. In this paper we review recent studies that report on the universality of this region in terms of the scaling laws for the mean velocities and turbulence intensities. New high Reynolds number experimental data from a variety of flows are shown to be consistent with universal von Kármán and Townsend-Perry constants. This is in support of classical theory but contrary to the prevailing view that has emerged over the past decade or so. The nature of the logarithmic region is further discussed in terms of large and very large superstructures, and their role in interacting across the boundary layer is considered, with particular emphasis on their role in modulating and altering the fluctuating wall-shear stress.

Introduction
Canonical wall-bounded turbulent flows, that is the flat-plate turbulent boundary layer with zero streamwise pressure gradient, and fully developed pipe and channel flows, are traditionally described in terms of layers or regions based on normal distance from the surface, z. Following Prandtl (1925) and von Kármán (1930), classically there is an inner-region and an outer-region. In the inner-region the turbulence statistics are assumed to scale with the friction velocity, $u_\tau = \sqrt{\tau_w/\rho}$, and the viscous length scale, $\nu/\tau_w$. (Here, $\tau_w$ is the average, or mean, wall shear stress, $\rho$ is the fluid density, and $\nu$ is the kinematic viscosity of the fluid.) Throughout this paper we use the superscript + to refer to normalisation with inner scaling. For example, $z^+ = z \nu/\tau_w$ and $U^+ = u/\tau_w$. In the outer region it is assumed that the appropriate length scale is the boundary layer thickness, $\delta$, and the velocity scale continues to be $U_\tau$, as $U_\tau$ sets up the inner boundary condition for the outer flow.

In this paper we will focus attention on the portion of the inner region that is sufficiently far from the wall that the influence of viscosity is negligible and does not affect the mean relative motions (following Townsend 1956). By referring to mean relative motions this includes the local mean velocity gradient, $\partial U^+/\partial z^+$, and the energy-containing terms $\langle u^2 \rangle$, $\langle v^2 \rangle$ and $\langle w^2 \rangle$, where u, v and w denote the fluctuating velocity components in the streamwise, spanwise and wall-normal directions, respectively. Classically, this region is also taken to be the overlap between the inner and outer regions and is therefore referred to as the inertial sublayer in physical space. By a number of arguments this region leads to a logarithmic profile for the mean velocity, of the form

$$U^+ = \frac{1}{\kappa} \log(z^+)+A,$$

where $\kappa$ is the von Kármán constant, and with the additive constant depending on the geometry (pipe, channel, or boundary layer) and the wall roughness. The applicability of equation (1) leads to the terminology of the logarithmic region, which is used in this paper. While the logarithmic region physically only occupies typically 10-15% of the boundary layer thickness, the dynamics of this region are crucial, and they become more important as the friction Reynolds number, $Re_{f} = \delta U_\tau/\nu$, increases. Asymptotically, it is the logarithmic region that dominates the contribution to net turbulent kinetic energy production.

Over recent years there has been extensive discussion and controversy concerning the universality, and even the existence, of the logarithmic region. It is not the author’s intention to thoroughly review these issues here. For that and related matters on wall turbulence, the reader is referred to recent reviews by Klewicki (2010), Marusic et al. (2010), Smits et al. (2011), and Jimenez (2012). Rather, in the limited space available here we focus on some recent findings related to the logarithmic region concerning the universality of the constants in the scaling laws for the mean flow and turbulence intensities, the nature of the large and very large structures that have become evident in the past decade, and the role of these large-scale motions in interacting with the flow across the boundary layer, in particular at the wall via the wall-shear stress.

Universality of the logarithmic region
In recent years there has been vigorous debate as to whether the von Kármán constant, $\kappa$, is indeed a universal constant, and whether it is different in different flow geometries (Zagorola & Smits 1998), or a function of pressure gradient or Reynolds number (Nagib & Chauhan 2008, Wosnik et al. 2000, amongst others). Testing this with experiments is complicated by a number of challenges. A major factor is the need to conduct high accuracy measurements physically close to the wall at high Reynolds numbers. All the competing views recognise that the extent of the logarithmic region grows with increasing $Re_t$. In the case of pipes and channels, the outer edge of the logarithmic region is taken to be a fixed fraction of the pipe radius, or channel half-height, typically around 0.1-0.2. Therefore, in these flows the growth of the logarithmic region with increasing Reynolds number is clearly towards the wall, and accessing this region at very high Reynolds numbers requires extremely small sensors. Extensive advances towards this have been made recently by the Princeton group with the development of NSTAP (Nano-Scale Thermal Anemometry Probe; Valkièvi et al. 2011).

Recently Marusic et al. (2012) considered the issue of the universality of the logarithmic region by using four very high Reynolds number datasets, which included flows from flat plate boundary layers in different facilities, pipe flow data (from the Princeton Superpipe) and data measured in the atmospheric surface layer. They found that, contrary to the prevailing view over the past decade, within the experimental uncertainty all the results were consistent with a universal value of the von Kármán constant, with $\kappa \approx 0.39$. The four datasets are all recent and are nominally in the Reynolds number range $2 \times 10^5 < Re_t < 6 \times 10^5$. The datasets included $U^+$ and $u'^+\tau^+$ profiles from Ku-
landiaivelu (2012) at $Re_C = 18000$ measured in the large Melbourne wind tunnel (HRNBLWT) with a 2.5 μm diameter hot-wire, from Winkel et al. (2012) at $Re_T = 68780$ measured in the US Navy’s William D. Morgan Large Cavitation Channel (LCC) using laser-Doppler velocimetry, from the Princeton Superpipe at $Re_T = 98190$ reported by Hultmark et al. (2012) using NSTAP sensors, and from Hutchins et al. (2012) for measurements at the SLTEST site in Utah’s Western desert using a wall-normal array of nine sonic anemometers under nominally neutrally buoyant conditions. The Superpipe results are those using NSTAP and the observed differences between these results and the previous Princeton Superpipe data of Zagarola & Smits (1998) and McKeon et al. (2004), where it was determined that $\kappa = 0.421$, are the subject of an ongoing study by Bailey et al. (2012, in preparation). The preliminary findings suggest that the most likely cause for the differences are due to errors in estimating the wall distance in the earlier work.

The data analysed by Marusic et al. (2012) are shown in figure 1. Figure 1(a) shows the SLTEST, LCC and Melbourne datasets in inner-scaled variables (where the SLTEST results have been vertically shifted by $\Delta U' = 1.2$ to account for the roughness), together with two lower Reynolds number cases from the Melbourne tunnel. The solid red line, with $\kappa = 0.39; A = 4.3$, is seen to fit the data well. Figure 1(b) shows the Superpipe, LCC and Melbourne high Reynolds number cases where the logarithmic function is subtracted to highlight the deviations from the logarithmic region. The departure at the outer edge of the logarithmic region is found to scale as a fixed fraction of $\delta$, and Marusic et al. (2012) used $\delta/\delta \leq 0.15$ for curve-fitting purposes. Determining the lower bound of the logarithmic region is not so clear-cut, and controversy regarding this remains. The classical description takes the lower bound to be at a fixed value of $z^+$, although a range of estimates exist. For example, Tennekes & Lumley (1972) used $z^+ > 30$, while Nagib et al. (2007) suggested $z^+ > 200$ and Zagarola & Smits (1998) adopted $z^+ > 600$ for pipe flows. An alternative lower bound for the inertial subrange is for it be a fixed value of $z^+/Re_1^{1/2}$, as argued by Klewicki et al. (2009) who estimated that the mean viscous force loses leading order influence for $z^+/Re_1^{1/2} > 2.6$, in all turbulent wallflows. Figure 1(b) shows the data versus $z^+/Re_1^{1/2}$ and the vertical dashed lines, corresponding to $z^+/Re_1^{1/2} > 3$, appear to be a reasonable estimate for the lower bound given the experimental uncertainties. The error bars shown on figure 1(b) are for $U'$ at nominal wall-normal positions of interest, and the reader is left to interpret the implications for themselves given that the separation between bias and random errors is unknown. However, it is clear that the deviations of the data from the logarithmic law at the lower bound are very slow and weak, making firm conclusions difficult.

In order to move beyond the difficulties of discerning the range of the logarithmic region from the mean velocity alone, Marusic et al. (2012) adopted the broader consideration of the inertial subrange description in terms of Townsend’s (1976) attached eddy hypothesis. We extend that discussion here. Townsend (1976) and Perry & Chong (1982) considered the inertial subrange to consist of a hierarchy of geometrically-similar attached eddies, whose geometric lengths scale with $z$ and with population densities per characteristic eddy height that scale inversely with $z$. Townsend showed that this argument leads to a logarithmic profile for the mean velocity, as well as for the streamwise and spanwise turbulence intensities. Asymptotically, the (kinematic) Reynolds stresses take the form

$$\overline{u'^2} = B_1 - A_1 \log(z/\delta), \quad \overline{w'^2} = B_2 - A_2 \log(z/\delta), \quad \overline{u'w'} = A_3, \quad \overline{\nu'^2} = 1.$$ (4)

The results for the wall-normal turbulence intensities and Reynolds shear-stress have been generally well supported by experimental data (Kinkel & Marusic 2006, Jimenez & Hoyas 2008, Smits et al. 2011), however support for a logarithmic profile for $\overline{u'^2}$ has convincingly only come recently with very high Reynolds number data (Hultmark et al. 2012). Figure 2(a) shows the $\overline{u'^2}$ profiles corresponding to those for the mean flow in figure 1(a). Figure 2(b) shows the same data versus $z^+/Re_1^{1/2}$, and here the data are seen to deviate from the logarithmic profile more clearly at the lower bound, suggesting $z^+ > 3Re_1^{1/2}$ is a reasonable estimate. The blue solid lines in figure 2(b) are a modified version of the formulation of Marusic et al. (1997), which accounts for departures from equation (2) by a wake deviation in the outer region and a deviation at the near-wall region due to viscous effects and by restricting the range of scales of attached eddies for a given Reynolds number. Here, the Townsend-Perry constant $A_1 = 1.26$, while $B_1$ is a characteristic constant which is allowed to vary for each profile. In figure 2(b) $B_1$ varies from 2.0-2.4, and the attached eddy length scales are restricted to the range $3Re_1^{1/2} < \ell < Re_T$. At the Reynolds numbers considered here the viscous contributions are small, and overall the formulation is seen to fit the data well for the logarithmic region and beyond.

Very few (well-resolved) spanwise turbulence intensity data are available in the literature, and no spanwise velocity statistics are available from the experiments shown in figures 1-2, apart from the sonic anemometer measurements of Hutchins et al. (2012). Recently, some $\overline{v'^2}$ measurements have been carried out in the Melbourne wind tunnel by Baidya et al. (2012) at moderately high Reynolds number using custom-built miniature $\times$-wires, and results from this study are reported in these 18AFMC proceedings. Figure 3(a) shows these data together with the Utah measurements (which would be subject to significant measurement uncertainties). In general, these $\overline{v'^2}$ results are seen to be consistent with Townsend’s formulation. Figure 3(b) shows the wind tunnel measurements with a logarithmic formulation subtracted, and as in figures 1(b) and 2(b) the vertical dashed line indicates $z^+ = 3Re_1^{1/2}$. Here, it is not clear whether there is a small deviation from this lower bound or whether the log region indeed is closer to the wall. The differences cannot be discerned within the experimental uncertainty.

### Attached eddy model considerations

When considering the attached eddy description for the inertial region, it is important to note that nothing in Townsend’s description excludes the possibility that the bounds of the logarithmic region are Reynolds number dependent. As discussed in Marusic et al. (2012), Townsend (1976) does not specify the bounds and simply states that for equations (2)-(5) to hold $\ell_1 < z < \delta$, where $\ell_1$ scales with the size of the smallest attached eddy. This implies that the Reynolds number must be sufficiently high, and asymptotically the condition holds for $\ell_1^+ \sim Re_1^{1/2}$, provided $n < 1$. From Perry & Marusic (1995), in the inertial region we can express the contributions of the attached eddies as

$$\frac{dU}{U_z} \frac{dz}{dz_0} = \int_{z_0}^{z} f(z/\delta) T(\ell/\delta) \omega(\ell/\delta) \frac{1}{\ell^2} dl \quad (6)$$

and

$$\frac{\overline{u'w'}}{U_z^2} = \int_{z_0}^{z} l_1(z/\delta) T^2(\ell/\delta) \omega(\ell/\delta) \frac{1}{\ell} dl. \quad (7)$$
Figure 1: (a) Profiles of mean velocity: open symbols correspond to the Melbourne wind tunnel turbulent boundary layer data of Kulandaivelu (2012) for $Re_\tau = 3510, 8160, 18000$. Asterisk symbols correspond to the LCC data of Winkel et al. (2012) at $Re_\tau = 68780$, and the solid squares correspond to the SLTEST atmospheric surface layer data of Hutchins et al. (2012) at $Re_\tau \approx 628000$. The SLTEST mean flow data have a shift of $\Delta U^+ = 1.2$ added, corresponding to the transitionally rough conditions. (b) Adapted from Marusic et al. (2012), showing the mean velocity with log law function, where $\kappa = 0.39$, subtracted versus $z^+/Re^{1/2}$ for LCC and a Melbourne dataset (symbols as in (a)), together with Superpipe data at $Re_\tau = 98190$ (solid circles). The horizontal lines indicate the best fit for this range highlighting the log region plateau. Error bars of $U^+$ are shown at the indicated locations.

Figure 2: Profiles of streamwise turbulence intensities for the same datasets (as corresponding symbols) as in figure 1(a).

Figure 3: Spanwise turbulence intensity data of Baidya et al. (2012) and Hutchins et al. (2012). The solid red lines correspond to equation (3) with $A_2 = 0.21$. 

Here, \( f(z/\ell) \) and \( I_{ij}(z/\ell) \) are the Townsend eddy cross-stream vorticity and eddy intensity functions, respectively, where \( \ell \) is the representative eddy length scale. In the integration across all eddy scales (from \( \ell_1 \) to \( \delta \)) an inverse power law population density function is applied, and \( \omega/(\delta/\ell) \) represents any deviation away from this inverse power law, and \( T(\ell/\delta) \) represents the variation of the velocity scale for each attached eddy scale away from \( U_\ell \). In Townsend’s formulation both \( \ell_0 \) and \( T \) are equal to 1, and thus the velocity scale is always \( U_\ell \) and the population density distribution is always \( \ell^{-1} \). Deviations of \( T \) and \( \ell_0 \) from 1 become relevant in the wake region (that is, beyond the logarithmic region). Perry et al. (1991) showed the above equations can be expressed as convolution integrals by applying a transformation, where

\[
\lambda = \log(\ell/z), \quad \lambda_1 = \log(\ell_1/z), \text{ and } \lambda_E = \log(\delta/z)
\]

and therefore, equations (6) and (7) may be rewritten as

\[
\frac{dU_{ij}^D}{dt} = \int_{\lambda_1}^{\lambda_E} f(\lambda)e^{-\lambda} T(\lambda - \lambda_E) \omega(\lambda - \lambda_E) \, d\lambda,
\]

\[
\frac{\partial U_{ij}}{\partial t} = \int_{\lambda_1}^{\lambda_E} f(\lambda) T(\lambda - \lambda_E) \omega(\lambda - \lambda_E) \, d\lambda.
\]

Figure 4 shows graphically how these convolution integrations can be interpreted. (Note that \( dU_{ij}^D/dt = z^+dU^+_{ij}/dz^+ \).) On the left of figure 4 are sketches of typical eddy functions. These eddy functions can be computed by Biot-Savart integral computations once the representative eddy shape is chosen, where the wall is treated as a slip surface and image vortices in the wall ensure that there is no flow through the wall. The key features are the boundary conditions where for \( z/\ell \to 0 \); \( I_{11} \sim \text{constant} \), \( f_{ij} \sim \text{constant} \), \( -I_{13} \sim z/\ell \) and \( I_{13} \sim (z/\ell)^2 \), and the eddy cross-stream vorticity contribution must decay above the eddy (which is always the case for any physical eddy in a boundary layer). Figure 4 shows that these conditions alone lead to equations (1-5), indicated by the red lines in the figure, provided the weighting functions become constant. That is, provided the velocity scale is \( U_\ell \) for all eddy scales and the population density distribution follows \( \ell^{-1} \).

Figure 4 also illustrates the consequence of a finite range of eddy scales, consistent with a finite Reynolds number effect, indicated by the \( \lambda_1 \) cut-off. (In the asymptotically high Reynolds number case: \( \lambda_1 \to -\infty \) and \( \lambda_E \to \infty \).) The larger \( Re_\ell \), the larger \( \lambda_E - \lambda_1 = \log(\delta/\ell_1) \), and hence the longer the extent of the log region for \( U^+, \rho U^+ \), and \( \frac{\partial U^+}{\partial z^+} \), and the closer \( w^{+2} \) and \( -\pi_m^+ \) are to constants as one moves towards the wall in the logarithmic region.

An important thing to note from figure 4 is that if one adopts a finite smallest attached eddy scale \( \ell_1 \), then the lower bound deviations from the logarithmic region for \( z^+ < \ell^+ \) will impact on all the components, although the rate of the deviation for each component is not known. Despite the difficulty in locating the \( z^+ \); Extensive evidence now exists, both empirical (Sreenivasan & Sahay 1997) and from theory (Klewicki et al. 2009), that the peak Reynolds shear stress occurs at a \( z^+ \) location that scales with \( Re_\ell^{1/2} \), and for the attached eddy model to reproduce this result thus requires \( \ell_1^+ \sim Re_\ell^{1/2} \). (This is despite the diffi-
culty of locating this position from what is nominally a plateau on a log-linear plot.) As indicated above, since the attached eddy model kinematically accounts for the entire inertial field the mean flow and turbulence cannot be regarded separately. Therefore, if \( l_1^+ \sim Re_1^{1/2} \) is adopted, then the attached eddy model leads to a \( z^+ \sim Re_1^{1/2} \) lower bound for all components of the Reynolds stresses and the mean flow. The results in figures 1-3 would seem to be consistent with this, although the evidence for the spanwise component is particularly weak and further experiments at high Reynolds numbers, which obtain all the components of the Reynolds stresses and the mean flow, are required.

**Logarithmic region structure**

Recent reviews (e.g., Adrian & Marusic 2012, Jimenez 2012, Marusic et al. 2010) have extensively discussed the role of coherent structures in wall turbulence, and associated with this the corresponding signatures that come from two-point correlation statistics and spectra. Since the 1950s it has been noted that correlation data show long tails that persist for many boundary layer thicknesses in the streamwise direction. Townsend (1976) discusses this to a certain degree and from this infers that the characteristic attached eddies would be elongated in the streamwise direction. However, there was no account given to explain the very long tails in the correlations. This issue was further considered by a number of authors, notably by Brown & Thomas (1977). Flow visualizations of boundary layers had long highlighted that in the outer layer, the edge of the turbulent zone has bulges that are about 2-3δ long (Kovasznay et al. 1970) separated by deep crevasses between the back of one bulge and the front of another (Cantwell 1981). The backs have stagnation points formed by high-speed fluid sweeping downward, and the shear between the high-speed sweep and the lower speed bulge creates an inclined, δ-scale shear layer, and the bulges propagate at about 80-85% of the free stream velocity.

These issues only really became prominent again with the studies of Kim & Adrian (1999) and the Adrian et al. (2000) where particular emphasis was given to interpreting the results in terms of organised coherent motions. Kim & Adrian (1999) considered streamwise velocity spectra in pipe flows and documented that in the logarithmic region two prominent large streamwise length scales appear. These were designated as large-scale motions (LSM) of typical length 2-3δ, and very-large scale motions (VLSM) that extend to approximately 12-14δ. Adrian et al. (2000), using PIV data in streamwise–wall-normal planes, inferred that the boundary layer was populated by hairpin-type vortices that are spatially organised into packets and that the LSM are the largest packets in the boundary layer. Kim & Adrian (1999) speculated that the VLSM could be a concatenation of packets or LSM, and the question of the origin and scaling of the VLSM remains open and is the focus of ongoing studies by a number of groups (see McKeon & Sharma 2010; Jimenez 2012).

Hutchins & Marusic (2007) investigated the length of the large scale motions in boundary layers by using a spanwise array of hot-wires (and sonic anemometers in the atmospheric surface layer) and found evidence of very long organized motions in the logarithmic layer, often in excess of 108. Hutchins & Marusic (2007) refer to these motions as “superstructures” because of their very large extent (both in the wall-normal and streamwise direction), their clear signature as an outer peak in the u-spectrogram across the layer, and because they account for a major proportion of the Reynolds shear stress (Ganapathiraman et al. 2003; Balakumar & Adrian 2007). The shorter lengths noted in the spectra (6δ) were believed to be caused by the spanwise meandering nature of the motions. Monty et al. (2008) performed similar measurements to Hutchins & Marusic in pipe flow, and found organized motions up to 308 in length, consistent with the VLSM of Kim & Adrian (1999) in pipes. The issue of whether the superstructures found in boundary layers by Hutchins & Marusic (2007) were the same as the VLSM found in pipes by Kim & Adrian (1999) and Guala et al. (2006) was investigated by Monty et al. (2009) who made detailed comparisons of the spectra in a pipe, channel and ZPG boundary layer with matched Re and measurement probe resolution. Their results indicate that the largest energetic scales in pipes and channels are distinctly different to those found in a boundary layer, although the large-scale phenomena have been shown to be qualitatively similar. Differences in the geometrical confinements between these flows are likely a factor for the quantitative differences. Dennis & Nickels (2011a,b), who used high-frame rate PIV in a boundary layer showed results that agree well with the interpretation of Hutchins & Marusic’s hot-wire data. The Dennis & Nickels results also provide invaluable information on the three-dimensional structure of the large motions, and while not conclusive, strongly support the suggestion by Kim & Adrian (1999) that the very-large superstructures are a result of a concatenation of packets. Support for this also comes from atmospheric surface layer and laboratory measurements as described in Hambleton et al. (2006) and Hutchins et al. (2012), and some representative results of this are shown in figure 5. Here simultaneous streamwise/spanwise and streamwise/wall-normal plane three-component velocity measurements reveal signatures entirely consistent with the superstructure events consisting of an organized array of packet structures. The lower schematics in figure 5 indicate comparisons with the Adrian et al. (2000) packet paradigm with Biot-Savart calculations of an idealized packet of hairpin vortices to infer what the corresponding spanwise velocity signatures would be in the relevant orthogonal planes. Here, vortex line segments are arranged into hairpin shapes, which are then repeated to generate the packet feature. The induced velocity field due to these vortex segments, together with their image vortices in the wall (Perry & Marusic 1995), is calculated using the Biot-Savart law. The streamwise/wall-normal plane is chosen such that it cuts through the inclined legs of the hairpin packet, and it is found that alternating regions of positive and negative spanwise velocity are formed in the wall-normal plane that look qualitatively similar to the inclined regions observed in the experimental data. The figure also indicates that it is likely that the stripiness observed in the instantaneous field is due to a packet of vortices moving together rather than an individual hairpin vortex.

**Interactions of logarithmic region structures at the wall**

In the preceding section it was seen that the dominant signature of the very-large superstructures was found in the logarithmic region. However, an important dynamical consequence of such large motions is that they are not confined to the logarithmic region, and in fact considerably influence the turbulence all the way to the wall. This is in contradiction to the classical viewpoint in which the inner and outer regions are assumed not to interact. While extensive experimental evidence to the contrary exists (going back to Rao et al. 1971), recent DNS results (Hoyas & Jimenez 2006, Orlu & Schlatter 2011) show clear and definitive evidence that the interactions are significant. Hutchins & Marusic (2007b) inferred that the large outer region motions extend down to the wall, and modulate the flow in the inner layer, including the buffer layer. This interaction was quantified by Mathis et al. (2009), and formed the basis of an algebraic model by Marusic et al. (2010b), wherein the
Figure 5: Adapted from Adrian & Marusic (2012). Top panel: Instantaneous velocity fluctuations in the streamwise-wall-normal \((x-z)\) plane and instantaneous streamwise velocity fluctuations in the streamwise/spanwise \((x-y)\) planes for data from laboratory PIV (Hambleton et al. 2006) and for the atmospheric surface layer using an array of sonic anemometers (Hutchins et al. 2012). High positive \(v\) regions are indicated by red while blue denotes highly negative \(v\) regions. High negative \(u\) regions are indicated by dark gray while light gray shade denotes highly positive \(u\) regions. Bottom panel shows the Biot Savart law calculations for an idealized packet of hairpin vortices with their image vortices in the wall, as per the schematic of Adrian et al. (2000) shown on the left side.

statistics of the streamwise fluctuating velocity in the near-wall region could be predicted given only the large-scale velocity signature in the outer logarithmic region of a given flow. Chung & McKeon (2010) and Guala et al. (2011) have also proposed methodologies to describe this effect, with the latter identifying that the modulation can also be described in terms of the spatial phase relationship between large- and small-scale turbulent activity (see also Hutchins et al. 2011).

Hutchins et al. (2011) directly considered the interaction of the large outer structures at the wall by using a spanwise array of surface-mounted hot-film shear-stress sensors together with a traversing hot-wire probe. With this they were able to identify the conditional structure associated with a large-scale skin friction event in a high-Reynolds-number turbulent boundary layer. It was found that the large-scale skin-friction events convect at a velocity that is much faster than the local mean in the near-wall region, and that the instantaneous shear-stress data indicate the presence of large-scale structures at the wall that are comparable in scale and arrangement to the superstructure events. Recently Talluru et al. (2012) repeated similar measurements but using a multiple hot-wire probe and the array of skin-friction sensors at a friction Reynolds number of \(Re_{\tau} = 7000\). Figure 6 shows some results from these measurements, where the statistics have been conditionally averaged on a low skin-friction event. Figure 6(a) shows the isocontours of fluctuating friction velocity, \(u_{\tau}^{+}\), on the wall based on the condition that \(u_{\tau} < 0\) at the location \((x,y) = (0,0)\). The isocontours in this figure show that the local low skin-friction event is characterised on average by a large scale signal, which extends over several boundary layer thicknesses in the streamwise direction. The two-dimensional signature is similar to the two-point correlation for large-scale filtered skin friction, as reported by Hutchins et al. (2011) where an elongated positive correlation is flanked on either side by anti-correlated behaviour with the spanwise width separating the anti-correlated lobes. This is very similar to the signature for the two-point correlation for the streamwise fluctuating velocity reported by Hutchins & Marusic (2007) in the logarithmic region, suggesting that the two regions are influenced by the same large-scale structure. This interaction is confirmed in figures 6(b,c), which show the iso-contours of the streamwise fluctuating velocity, \(u_{\tau}^{+}\), and the instantaneous Reynolds shear-stress, \(-uw^{+}\), respectively conditioned on the low skin-friction event at the wall.

Figure 6(b) shows an elongated forward-leaning large low-speed feature, extending to a distance of nominally 5 \(\delta\) in the streamwise direction. The spanwise distribution indicates the low-speed region flanked by high speed regions on either side. These results are further considered by Talluru et al. (2012) where it is noted that there is a streamwise growth of the conditional structure, with the strong, negatively correlated region moving away from the wall as we move downstream of the conditioning point. Talluru et al. (2012) also present an analysis of the small-scale energy associated with these large-scale events revealing that the small-scale velocity fluctuations are attenuated near the wall and upstream of a low skin-friction event, while downstream and above the low skin friction event the fluc-
tations are significantly amplified. A similar trend is noticed in the conditional average plot of Reynolds shear stress shown in figure 6(c). These results are consistent with what one would obtain from a coherent structure as highlighted in figure 5.

The strong interaction between the large scale motions in the logarithmic region and the wall, as highlighted in figure 6, was analyzed by Mathis et al. (2009) in terms of the superposition of the large-scale at the wall together with a modulation of the large scale on the small-scale near-wall motions. Marusic et al. (2010b) extended these observations to a predictive model, whereby a statistically representative fluctuating streamwise velocity signal near the wall could be predicted given only a large-scale velocity signature from the logarithmic region of the flow. The model was shown to work well over a large Reynolds number range for various statistics, including higher order moments. The formulation involves a universal signal and universal parameters, which are determined from a once-off calibration experiment at an arbitrarily chosen (but sufficiently high) Reynolds number. Mathis et al. (2012) extended the model to predict the fluctuating wall-shear stress given only a large-scale streamwise velocity signal from the logarithmic region. The model is of the form

\[ \tau_{wp}^+(t^+) = \tau_w^+(t^+) \{ 1 + \alpha u_{ol}^+(t^+) \} + \alpha u_{ol}^+(t^+), \]

where \( \tau_{wp}^+ \) is the predicted time-series normalised by wall variables, \( \tau_w^+ = \tau_w / (\rho U_2^2) \) and \( t^+ = t U_2 / \nu \). The time-series \( \tau_w^+ \), which is normalised in wall units, represents the statistically “universal” wall shear-stress signal that would exist in the absence of any inner-outer interactions, and \( \alpha \) is a constant. The parameters \( \tau_w^+ \) and \( \alpha \) were determined by Mathis et al. (2012) from a DNS calibration dataset, and are hypothesized to be Reynolds number independent. Once these parameters are known, the only user input required for the model is the characteristic signal of the large-scales from the log-region, \( u_{ol}^+ \). It is noted that the model consists of two parts. The first part in equation 10 models the amplitude modulation of the small-scales (\( \tau_w^+ \)) by the large-scale log-region motions, \( u_{ol}^+ \). The second term, \( \alpha u_{ol}^+ \), models the superposition of the large-scales felt at the wall.

Figure 7 shows the results of the predictive model (equation 10) where, given only the large-scale signal \( u_{ol}^+ \), it is used to reconstruct a statistically representative wall shear-stress signal \( \tau_{wp}^+ \) at a given Reynolds number. Pre-multiplied energy spectra of the predicted wall shear-stress signals, \( f^+ \Phi_{wp} \), (where \( f \) is frequency) are shown in figure 7(a) for a range of Reynolds numbers. Though no experimental data are available for comparison, it can be seen that the model captures well the overall Reynolds number trend, i.e. a slight increase with \( Re_z \) of the large-scale energy content. This behaviour supports recent findings that the strength of the large scales increases with \( Re_z \) (Metzger & Klewicki 2001, Hoyas & Jimenez 2006, Hutchins & Marusic 2007). The increase in the energy contained in the long wavelengths also suggests an increase in the r.m.s of \( \tau_w^+ \). This is confirmed by figure 7(b) which shows the fluctuation magnitude of the predicted wall shear-stress signal, \( \tau_{wp}^+ \), as a function of \( Re_z \). Also included in this figure are available data for DNS of flat-plate zero-pressure-gradient turbulent boundary layer from Schlatter et al. (2010). The overall Reynolds number trend of \( \tau_{wp}^+ \) appears to be correctly captured by the model, and it is in good agreement with the results of Orlu & Schlatter (2011) (see also Inoune et al. 2012).

**Concluding remarks**

We have seen that the classical description of the logarithmic region in terms of having a universal von Kármán constant is
supported by recent high Reynolds number experiments. Moreover, we show that the logarithmic region also admits logarithmic profiles for the streamwise and spanwise turbulence intensities, despite open questions as to the precise bounds of the logarithmic region. Existing data have been shown to be reasonably consistent with the lower bound occurring at a fixed value of $z^+ / Re_{	au}^{1/2}$ as argued by Klewicki et al. (2009). The overall findings are shown to be consistent with the Townsend (1976) attached eddy hypothesis.

A clearer kinematic description of the logarithmic region has emerged in recent years in terms of coherent structures in the form of packets of hairpin-type vortices, large and very-large scale motions, and these have been discussed. However, further work is needed to fully understand their origins and dynamics. A key feature of the largest scale motions has been their role in interacting with the near-wall region, and a mathematical description for the fluctuating wall-shear stress has been reviewed that describes this interaction in terms of a superposition and modulation by the large scales on the small-scale motions near the wall.

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