SCALING OF THE TURBULENT BOUNDARY LAYER AT HIGH REYNOLDS NUMBERS

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Abstract Experimental results are presented for high Reynolds number turbulent boundary layers. A flat plate zero pressure gradient layer has been studied in a new high Reynolds number boundary layer tunnel. Measurements were made of the mean flow in the Reynolds number range of $3.6 \times 10^3 < R_\theta < 6.0 \times 10^4$ based on momentum thickness. The data supports the existence of a logarithmic law of the wall in the overlap region and constants $\kappa = 0.41$ and $A = 5.0$ are found to best fit the data.

Keywords:

1. Introduction

The classical approach to mean flow scaling is to seek similarity laws in two regions of the flow, a region adjacent to the wall and region adjacent to the edge of the boundary layer. These regions are often referred to as the *inner flow* region and the *outer flow* region. Applying dimensional analysis arguments in these regions leads to the classical laws

$$U/U_\tau = f(zU_\tau/\nu) \qquad (1)$$

$$(U_1 - U)/U_\tau = g(z/\delta_c) \qquad (2)$$

where U is the mean streamwise velocity, U_τ is the wall shear velocity, z is the wall normal coordinate, ν is the kinematic viscosity, U_1 is the local freestream velocity, and δ_c is the boundary layer thickness. Equation (1) (first derived by Prandtl, 1925) represents a similarity law for the inner flow and is known as *the law of the wall*, and (2) (first derived by von

Kármán, 1930) represents a similarity law for the outer flow and is known as the velocity defect law.

Using the Millikan, 1938 argument that there exists a region of overlap between (1) and (2) gives the classical results

$$\frac{U}{U_\tau} = \frac{1}{\kappa}\ln(z^+) + A \qquad \text{and} \tag{3}$$

$$\frac{U_1 - U}{U_\tau} = -\frac{1}{\kappa}\ln(\eta) + B \tag{4}$$

where $z^+ = zU_\tau/\nu$, $\eta = z/\delta_c$, κ is the Karman constant, A is the universal smooth wall constant and B depends on the large scale flow geometry. The overlap region will be referred to as the fully turbulent wall region (TWR) and it is often assumed to begin at $z^+ = 100$ and extend to $z/\delta_c = 0.15$. The Millikan, 1938 argument is one of several arguments that can be used to derive the logarithmic law of the wall. However all rely on the basic assumption that in the TWR the velocity gradient is independent of viscosity. Notable alternatives to the above theory have been proposed by George and Castillo, 1997 and Barenblatt et al., 2000.

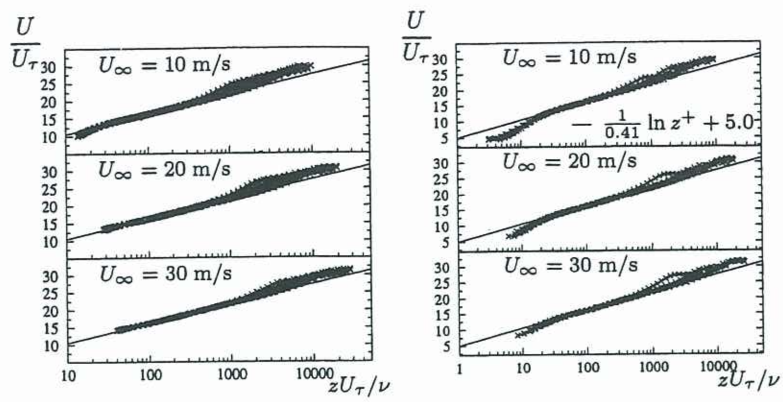
2. Experimental Method

Experiments were performed in an open return blower wind tunnel. The important feature of the tunnel is the working section length of 27 m. This allows high Reynolds numbers to be obtained through the long development length, thus avoiding many of the experimental difficulties associated with using the alternative methods of achieving high Reynolds numbers, such as the use of compressed air or high velocities. The tunnel was run at three reference Reynolds numbers corresponding to nominal reference freestream velocities of $U_\infty = 10\,\mathrm{m/s}$, $20\,\mathrm{m/s}$ and $30\,\mathrm{m/s}$.

In order to maintain a zero pressure gradient the ceiling incorporates adjustable spanwise slots which allow for the bleeding of air. In addition, the height of the ceiling can be varied. Through these mechanisms it was possible to maintain the C_p distribution to within ± 0.0050 for the $20\,\mathrm{m/s}$ and $30\,\mathrm{m/s}$ flow cases, where $C_p = 1 - (U_1/U_\infty)^2$. However for the $10\,\mathrm{m/s}$ flow case the the C_p distribution fell within ± 0.0065 .

Mean velocity profiles were measured with both a Pitot-static probe and a normal hot-wire. The Pitot tube readings were corrected for the effect of shear using the MacMillan, 1956 correction.

For the three reference Reynolds numbers measurements where made at different streamwise stations, varying from $x = 1\,\mathrm{m}$ to $x = 25\,\mathrm{m}$. This gave a Reynolds number range of $960 < K_\tau < 22400$, where $K_\tau = \delta_c U_\tau/\nu$. Transition to turbulence was initiated by a trip wire of diameter



(a) Measured with a Pitot tube (b) Measured with a hot-wire

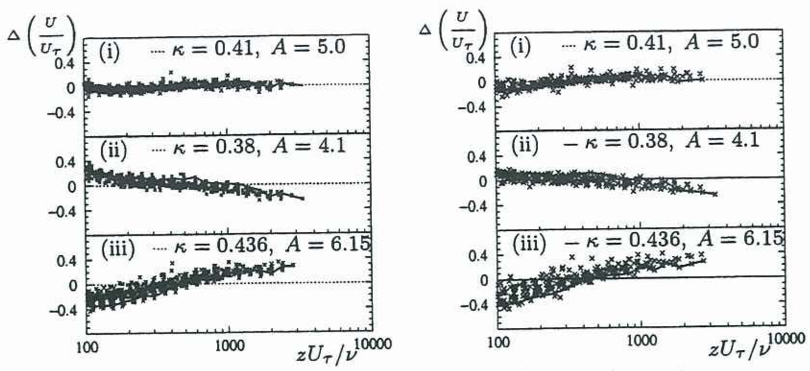
Figure 1. Mean velocity profiles for the complete range of streamwise stations.

0.4 mm placed at $x = 0$. The boundary layer studied developed on the smooth aluminium floor of the working section. The freestream turbulence level was found to be less than 0.05%.

For the results presented here the Clauser chart was used to determine the values of U_τ. It must be noted that for the Clauser chart to return correct values of U_τ (3) must be valid and constants A and κ specified. Hence care must be taken in interpreting the mean flow results since an a-priori assumption about the appropriate mean flow scaling has been made. However if the Clauser chart method is collapsing the data in the TWR, across the complete Reynolds number range of the experiments then at least a velocity scale has been found that is equal to U_τ to within a constant of proportionality and the only a-priori assumption is the that (1) is correct. The approach taken here is to analyse the data using the Clauser chart with different combinations of constants. If the data shows poor collapse onto (3) the constants can be discounted as being the correct ones. Three cases were considered: traditional constants of $\kappa = 0.41$ and $A = 5.0$; the Osterlund, 1999 constants of $\kappa = 0.38$ and $A = 4.1$; and the Zagarola and Smits, 1998 constants of $\kappa = 0.436$ and $A = 6.15$. To make any further comment on the validity of the constants in (3) requires an independent method of determining U_τ.

3. Inner-flow scaling

Mean velocity profiles measured with a Pitot are shown in figure 1(a) and measured with a normal hot-wire in figure 1(b). Here the data is normalised assuming $\kappa = 0.41$, $A = 5.0$ and it can be seen good collapse in the TWR is achieved across the full Reynolds number range. The



(a) Pitot tube data (b) Normal wire data

Figure 2. Deviation of data from log-law (3) when data is normalised to best fit; (i) $\kappa = 0.41$, $A = 5.0$, (ii) $\kappa = 0.38$, $A = 4.1$ and (iii) $\kappa = 0.436$, $A = 6.15$. Only data in the TWR ($100 < z^+ < 0.15K_\tau$) is shown.

quality of the collapse in the TWR is shown in figure 2 where (3) has been subtracted from the data (ie. $\Delta(U/U_\tau)$ is plotted). Also shown in figure 2 are the results of normalising the data assuming a priori that the correct constants are $\kappa = 0.38$, $A = 4.1$ and $\kappa = 0.436$, $A = 6.15$. It can be seen that the traditional constants (ie. $\kappa = 0.41$, $A = 5.0$) best collapse the data onto (3) with specified constants. When the other constants are used the data shows more scatter and this is most pronounced in the case of the Zagarola and Smits, 1998 constants. This scatter is a consequence of fitting the wrong log-law over a large Reynolds number range. Further, for the Osterlund, 1999 and Zagarola and Smits, 1998 constants the data does not appear to fit the required gradient $1/\kappa$ as a consequence the values of U_τ obtained from best fit (ie. Clauser chart) will be more sensitive to the limits defining the TWR.

The suitability of a logarithmic law in the overlap region can be investigated by plotting the non-dimensional velocity gradient pre-multiplied by z^+, ie.

$$D_1 = z^+ \frac{dU^+}{dz^+} , \tag{5}$$

where $U^+ = U/U_\tau$. If a log-law exists, equation (5) should equal a constant. Further, if the profiles have been scaled with the correct values of U_τ the constant should equal $1/\kappa$. Figure 3 shows the result of plotting equation (5) for all data in the range $100 < z^+ < 0.15K_\tau$. There is no smoothing or averaging of data, so figure 3 contains a degree of scatter. Nevertheless, the data does show agreement with the logarithmic law. The agreement is more clearly seen by considering the individual profile shown in figure 3, which corresponds to the highest R_θ profile. It should

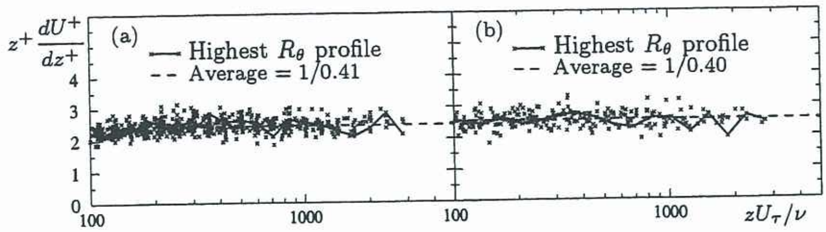


Figure 3. Logarithmic law diagnostic function (5) for all profiles, data from TWR, for (a) Pitot tube and (b) normal wire.

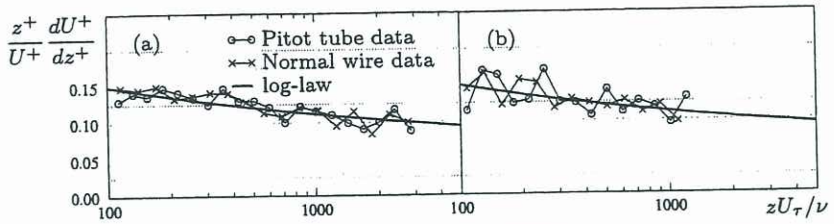


Figure 4. Power law diagnostic function (6) in TWR, for (a) $R_\theta = 62000$ profile and (b) for $R_\theta = 24000$ profile. Both Pitot and hot-wire results are shown.

be noted that the choice of U_τ serves to simply scale the values of D_1 for a given profile and the Clauser chart method does not force D_1 to a constant for a given profile.

Figure 4 shows the function

$$D_2 = \frac{z^+}{U^+} \frac{dU^+}{dz^+} \tag{6}$$

plotted in the range $100 < z^+ < 0.15K_\tau$, for the highest Reynolds number profile and a lower Reynolds number profile. If a power law is the correct form equation (6) should plot as a constant value equal to the power appearing in the power law. However the results indicate that D_2 has a preferred slope (consistent with a log law) which suggests the power law is not the correct functional form. A similar trend is observed for the other profiles at other Reynolds numbers.

4. Outer-flow scaling

Figure 5 shows the defect velocity obtained from Pitot measurements normalised using different velocity scales. Based on an analysis at infinite Reynolds number, George and Castillo, 1997 claims the correct choice for the velocity scale in (2) is U_1 and that the choice of U_τ is incompatible with similarity of the momentum equation in the outer region of the flow. However as can be seen from figure 5 the quality of the collapse is much better when U_τ is used as the scaling velocity. The quality of the collapse is further improved if we use $U_1\delta^*/\delta_c$ as the outer velocity scale where δ^* is the displacement thickness. Incorporating the integral scale δ^* in the definition of the velocity scale effectively forces this collapse. The velocity scale $U_1\delta^*/\delta_c$ is equivalent to the velocity scale $U_{CL} - \overline{U}$ that Zagarola and Smits, 1998 propose for pipe flow, where U_{CL} is the centreline velocity and $\overline{U}$ is the average pipe velocity. The question is whether the correct velocity scale is simply the velocity scale that best collapses the data. An alternative interpretation is given in Perry et al., 2002 where the zero pressure gradient layer is not assumed to necessarily be in a state of equilibrium (ie. defect self-similarity). In the analysis of Perry et al., 2002 U_τ is used as the velocity scale and the layer evolves from arbitrary initial conditions to a state very close to equilibrium. Hence as the layer is evolving defect self-similarity is not expected. Indeed if the stations upstream of $x = 5\,\mathrm{m}$ are removed from figure 5 the collapse when U_τ is used as the velocity scale is greatly improved.

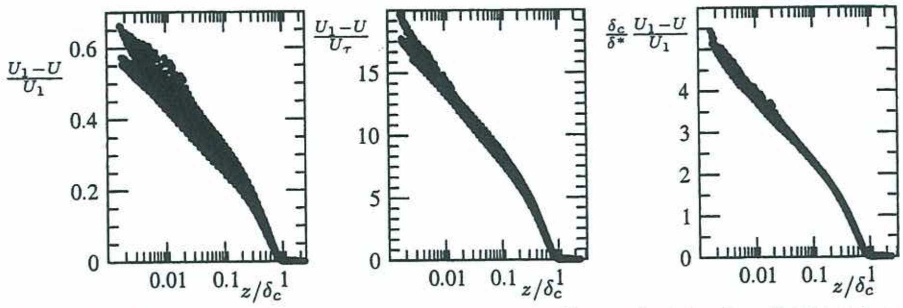


Figure 5. Velocity defect plots using different scaling velocities for all Pitot tube data ($994 < K_\tau < 22407$).

5. Conclusions

The mean velocity profiles are found to be well described by a logarithmic law of the wall in the fully turbulent wall region. The traditional values of $\kappa = 0.41$ and $A = 5.0$ are found to best collapse the data. Using the outer velocity scale $U_1\delta^*/\delta_c$ leads to the best collapse of the defect velocity. However using U_τ as the outer velocity scale may be the correct choice since this is consistent with the recent calculations of Perry et al., 2002 where the layer is expected to evolve to a self-similar state.

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