QUASI-STEADY MODULATION OF NEAR-WALL TURBULENCE

Chernyshenko Sergei[1], Marusic Ivan[2] & Mathis Romain[3]
[1]*Department of Aeronautics, Imperial College London, London, UK*
[2]*Department of Mechanical Engineering, University of Melbourne, Melbourne, Australia*
[3]*Laboratoire de Mécanique de Lille, Université Lille Nord de France, UMR CNRS 8107, Villeneuve d'Ascq, France*

Abstract We investigate a hypothesis that the response of the near-wall turbulence to large-scale structures is quasi-steady. The most natural and compact form of expressing this is given by the usual Reynolds-number-independent representation of flow quantities scaled in wall variables, in which, however, in the definition of wall units the mean skin friction is replaced by large-scale low-pass-filtered fluctuating skin friction. We demonstrate that such a representation linearized assuming that the large-scale fluctuations are small as compared to the mean quantities leads to explicit theoretical expressions for the superposition and modulation coefficients of the empirical predictive models of the skin friction and streamwise fluctuating velocity found earlier. It is found that the theoretical predictions agree well with the coefficients determined experimentally.

INTRODUCTION

According to the classical view on the universality of near-wall turbulence, all flow variables, if expressed in wall units, are independent of the Reynolds number Re. For skin friction τ and the velocity u this means that

$$\tau = \bar{\tau}\tau^*\left(\frac{t\bar{u}_\tau^2}{\nu}, \frac{x\bar{u}_\tau}{\nu}, \frac{y\bar{u}_\tau}{\nu}\right), \quad u = \sqrt{\frac{\bar{\tau}}{\rho}}u^* = \bar{u}_\tau u^*\left(\frac{t\bar{u}_\tau^2}{\nu}, \frac{x\bar{u}_\tau}{\nu}, \frac{y\bar{u}_\tau}{\nu}, \frac{z\bar{u}_\tau}{\nu}\right), \tag{1}$$

where $\tau^*(t^+, x^+, y^+)$ and $u^*(t^+, x^+, y^+, z^+)$ are universal functions of their arguments in the sense that for sufficiently high Re all their statistics are independent of Re and the flow geometry, $\bar{u}_\tau \neq 0$ is the mean skin friction, and other symbols have the usual meaning. The influence of large-scale structures on the near-wall turbulence reported by many authors[1] and frequently called a modulation effect, is in contradiction with (1). According to the empirical model of this effect [2, 3], universal, that is having Re-independent statistics, fluctuation functions $\tau'^*(t^+, x^+, y^+)$ and $u'^*(t^+, x^+, y^+, z^+)$ are related to the observed flow parameters by the relations

$$\begin{aligned} \tau'^+ &= \alpha(z_{OL}^+)u_{OL}'^+ + (1 + \beta(z_{OL}^+)u_{OL}'^+)\tau'^*(t^+, x^+, y^+), \\ u'^+ &= \alpha_u(z_{OL}^+, z^+)u_{OL}'^+ + (1 + \beta_u(z_{OL}^+, z^+)u_{OL}'^+)u'^*(t^+, x^+, y^+, z^+), \end{aligned} \tag{2}$$

where primes denote fluctuations, superscript "+" denotes quantities in wall units, z^+ is the distance to the wall, $u_{OL}'^+$ is the fluctuating large-scale (that is low-pass-filtered) velocity signal from the log region at the distance z_{OL}^+ from the wall, $\alpha(z_{OL}^+)$, $\beta(z_{OL}^+)$, $\alpha_u(z_{OL}^+, z^+)$, and $\beta_u(z_{OL}^+, z^+)$ are universal empirical functions, obtained by fitting the data of experiments and direct numerical simulations. Equations (1) and (2) would be compatible for $\alpha = \beta = \alpha_u = \beta_u = 0$, but experiments gave $\alpha = 0.0898$, $\beta = 0.0867$, and α_u and β_u as nonzero functions of z^+ (for $z_{OL}^+ = \mathrm{Re}_\tau^{1/2}$). In [2, 3] the nonzero values of α and α_u were interpreted as a superposition effect of large-scale structures, and the nonzero values of β and β_u as a modulation effect of large-scale structures on the near-wall turbulence.

HYPOTHESIS AND DERIVATIONS

In the present work we demonstrate that both the superposition and the modulation effects (2) can be derived from a simple hypothesis that the effect of large-scale structures on the near-wall turbulence is quasi-steady. To be precise, we propose to replace the universality laws (1) with

$$\tau = \tau_L(t)\tau^*\left(\frac{tu_{\tau_L}^2(t)}{\nu}, \frac{xu_{\tau_L}(t)}{\nu}, \frac{yu_{\tau_L}(t)}{\nu}\right), \quad u = u_{\tau_L}(t)u^*\left(\frac{tu_{\tau_L}^2(t)}{\nu}, \frac{xu_{\tau_L}(t)}{\nu}, \frac{yu_{\tau_L}(t)}{\nu}, \frac{zu_{\tau_L}(t)}{\nu}\right), \tag{3}$$

where $\tau_L(t)$ is slowly-varying (low-pass-filtered) skin friction, $u_{\tau_L}(t) = \sqrt{\tau_L(t)/\rho}$, and τ^* and u^* are universal functions. Note that (2) is written in fluctuations and is linear with respect to fluctuations, while (3) is written in full quantities and is nonlinear. Rewriting (3) in fluctuations and linearizing it assuming that the fluctuations are small reduces (3) to the form of (2), with

$$\alpha = \beta = 2\left(U^* + z^+\frac{dU^*}{dz^+}\right)^{-1}_{z^+=z_{OL}^+}, \quad \alpha_u(z^+) = \frac{\alpha}{2}\left(U^* + \frac{dU^*}{dz^+}\right), \quad \beta_u(z^+) = \frac{\alpha}{2}\left(1 + \frac{z^+}{\tilde{u}_{rms}^*}\frac{d\tilde{u}_{rms}^*}{dz^+}\right), \tag{4}$$

[1]Please see references in [1], where this work is described in more detail.

where the universal mean profile $U^*(z^+)$ and the random mean square of the universal fluctuation function are

$$U^*(z^+) = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{+T} u^*(t^+, z^+)\, dt^+, \quad \tilde{u}^*_{rms}(z^+) = \sqrt{\lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{+T} \left(u^*(t^+, z^+) - U^*(z^+)\right)^2\, dt^+}.$$

COMPARISONS

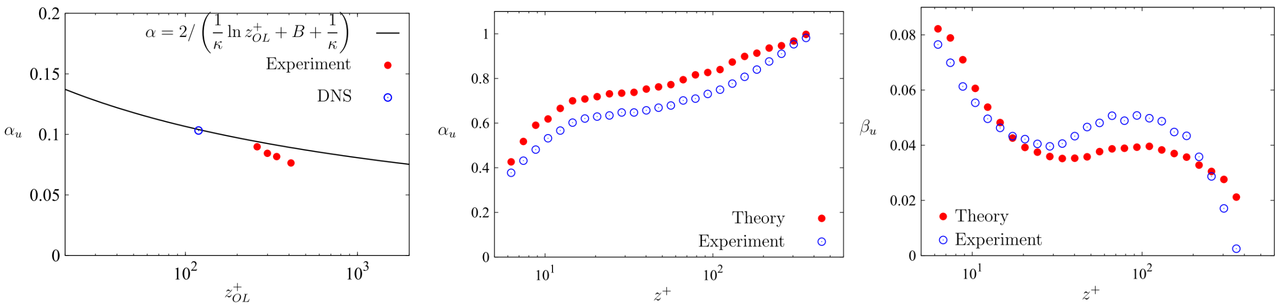

Figure 1. Comparisons between theoretical predictions (4) and experimental and numerical data of [3].

The experimentally found values of α and β are within 4%, which is close enough to agree with the theoretical prediction that $\alpha = \beta$. Comparisons for other predictions are shown in Figure 1. Since z^+_{OL} is in the logarithmic layer, the logarithmic law with $\kappa = 0.384$ and $B = 4.17$ was used for comparison of the prediction for α. For the comparisons of α_u and β_u the mean universal profile $U^*(z^+)$ and the universal random mean square fluctuation $u^*_{rms}(z^+)$ were taken to be equal to the mean profile in the calibration experiment and the random mean square of the universal signal of [3] respectively, and we used $\alpha = 0.0898$ from the same source[2]. Given the absence of any adjustable parameters in the theory, an approximate nature of linearization, and inevitable errors of experimental measurements, the comparisons appear to be satisfactory.

DISCUSSION AND CONCLUSION

The structure of (3) suggests an interpretation of the mechanism of the influence of large-scale structures on near-wall turbulence, which is somewhat different from the interpretation given in [2, 3] on the basis of (2). Unlike (2), (3) does not contain the superposition term. Instead, the amplitude modulation, in the form of a time-dependent factor in front of the universal function, applies to the total, rather than fluctuating quantity. In addition, in (3) the scales of the arguments of the universal function are also modulated. In the case of the time variable this effect is analogous to frequency modulation. However, when (3) is rewritten in fluctuations and linearized both the amplitude modulation and the wall-normal-scale modulation lead to the superposition term in (2).

The quasi-steady hypothesis (3) reveals a simple mechanism leading to reduction of the amplitude modulation of the turbulence kinetic energy of small-scale fluctuations as the distance from the wall increases. An increase in the large-scale velocity has two effects. First, it increases the skin friction, and this leads to an increase in the turbulence kinetic energy of small-scale fluctuations. Hence, the first effect creates a positive correlation between the large-scale velocity and small-scale turbulence intensity. Second, an increase in the skin friction leads to a decrease in the thickness of the inner region. As a result, at a fixed physical distance from the wall the value of z^+ increases. If this point happens to be in the region where the turbulence kinetic energy of small-scale fluctuations decreases with z^+, this second effect will decrease the degree of amplitude modulation, thus contributing negatively to the correlation.

The main conclusion is that the comparisons made so far support the hypothesis that the modulation of near-wall turbulence by large-scale structures is a quasi-steady phenomenon.

References

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[2]Note that the definition of α (and similarly for α_u) in the cited papers should be corrected to include the ratio of random mean squares and should be $\alpha = \max R(u'^+(z^+_{OL}), u'^+_{OL})(u'^+(z^+_{OL})_{r.m.s}/(u'^+_{OL})_{r.m.s}$. The plots were built with this correction.