9th AUSTRALASIAN
FLUID MECHANICS CONFERENCE,
AUCKLAND, 8-12 DECEMBER 1986

An Equation for Predicting Bedload Transport in Steep Mountain Step-Pool Stream

J. G. WHITTAKER

Central Laboratories, Ministry of Works and Development.

ABSTRACT

A sediment transport equation is modified by the inclusion of a loss coefficient to predict transport rates in a step-pool mountain stream. A predictive equation is developed for the loss coefficient, which accounts for energy lost to the transport process through turbulent mixing in the pools.

1 INTRODUCTION

Longitudinal stream profiles do not usually exhibit a smoothly varying slope, but take the form of reaches of alternating flatter and steeper slope, associated respectively with deeper and shallower flow sections. This tendency has frequently been reported in steep mountain streams (Whittaker, 1982, gives a list of relevant articles), particularly those with relatively low throughput rates of sediment. In these steep mountain streams, the alternating sections are often called steps and pools because of their stair-case like appearance. Steps are formed from large bed elements, the size of which can be of the same order as the depth of flow or even comparable to the width of the channel. The whole stream bed is typically armoured, meaning that the step-pool pattern is particularly stable. Flow in step-pool streams has been designated tumbling flow because of the visual impression given as the flow accelerates over steps and plunges into the pools below. A significant amount of energy dissipation occurs in the pools due to turbulent mixing.

Sediment transport in steep mountain step-pool streams is characterised by several features. First, sediment for transport is derived from limited sites within the catchment. Secondly, while in transit through the stream, this sediment is stored in the pools. Once the bulk of this stored material has moved through a channel reach, the transport rate drops considerably until there is another input from one of the sediment production sites. Consequently, observed bed load transport rates vary both spatially and temporally. The characteristic tumbling flow of step-pool streams, and the aspects of sediment transport described above, present major difficulties in predicting bed load transport rates in steep mountain streams. Most theoretical sediment transport relationships are confined to cohesionless granular materials, and to steady uniform flows which can only be treated as two-dimensional. To overcome the difficulty in predicting sediment transport rates in step-pool streams, Ashida et al (1976, 1981) suggested that an empirical adjustment (through a loss coefficient) be made to a conventional sediment transport formula to allow for the flow energy dissipated in the pools and thus unavailable for transporting sediment. Ashida et al did not, however, determine a suitable adjustment.

The intention of this paper is to present an adjustment to the Smart/Jaeggi sediment transport formula (a formula derived specifically to predict sediment transport in very steep, but plane, channels) to enable it to be used for predicting sediment transport rates in step-pool streams.

2 ENERGY LOSS ADJUSTMENTS

Most sediment transport equations can be expressed as a relation between a transport parameter ϕ and a flow parameter θ. These are defined as, respectively:

$$\phi = q_b/[g(s-1)d^3]^{0.5} \quad (1)$$

and

$$\theta = RJ\xi/(s-1)d \quad (2)$$

in which ξ = form loss coefficient

The inclusion of an energy loss coefficient, usually called a form loss coefficient, is based on the assumption that only the energy dissipated on the granular roughness of the bed is expended in transporting sediment. The corollary of this is that part of the flow energy dissipated due to large scale bed forms is assumed lost to the transport process. For example, the Meyer-Peter and Müller (1948) bed load transport formula is

$$\phi = 8(\theta-\theta_{cr})^{1.5} \quad (3)$$

in which $\theta_{cr} = 0.047$

$$\xi = [k_s/k_r]^{1.5} \quad (4)$$

and $k_r = 26/d_{90}^{0.167}$ (5)

Here k_s is defined as a Strickler roughness coefficient indicative of the total resistance to flow created by the bed, while k_r is the Strickler value for grain resistance only. The exponent 1.5 in eqn.4 was determined empirically. Smart and Jaeggi (1983) showed that in fact form drag had little or no effect on Meyer-Peter and Müller's test results.

The Smart/Jaeggi formula is a recently developed bed load transport relation for steep essentially plane channels (Smart and Jaeggi, 1983; Smart, 1984). This formula can be expressed:

$$\phi = 4\,(d_{90}/d_{30})^{0.2}\,J^{0.6}\,c\,\theta^{0.5}\,(\theta-\theta_{cr}) \quad (6)$$

or

$$q_b = [4/(s-1)]\,(d_{90}/d_{30})^{0.2}\,q\,J^{0.6}\,(J-J_{cr}) \quad (7)$$

The factor $cJ^{0.6}$ in eqn.6 is almost equal to the Froude number ($Fr = cJ^{0.5}$), and the effect of this factor can be compared to that of the form factor $[k_s/k_r]$ in the Meyer-Peter/Müller formula (Smart and Jaeggi, 1983). The Smart/Jaeggi formula was evaluated empirically using the Meyer-Peter/Müller data as well as the results from the tests in which slopes ranged from 3% to 20%.

In reviewing the energy loss adjustments made to the above sediment transport relations, it is clear that the energy loss coefficient has been empirically derived. The derivation has involved the calibration of basic sediment transport formulae to measurements. This is also true for the loss coefficients derived for other sediment transport equations, despite the fact that a pseudo-theoretical derivation is typically developed.

3 PHYSICAL MODELLING OF STEP-POOL STREAMS

A loss coefficient α has been evaluated for the Smart/Jaeggi transport formula to enable sediment transport rates to be predicted for step-pool mountain streams. This loss coefficient acts in addition to Smart and Jaeggi's $\xi = cJ^{0.6}$. The derivation of α was based on the results of a number of laboratory tests. Only the test results for channel slopes in excess of about 7% were considered. Below this slope stream channels lose their step-pool aspect, and for $J <$ 2% exhibit riffles and pools. The irregularity of step-pool streams can be modelled by a succession of discrete weirs (Rouse, 1965). The author performed a number of investigations with such models, three of which are reported here. The first two of these test series (A and B) were performed in the Department of Agricultural Engineering, Lincoln College, New Zealand. The third series (C) was undertaken at the Laboratory of Hydraulics, Hydrology and Glaciology (VAW), ETH, Zürich, Switzerland.

The laboratory channel used for series A and B is shown in Whittaker (1982, 1985). This 10 m long tilting, re-circulating channel was 0.132 m wide and 0.6 m deep, and was able to be adjusted to slopes of up to 0.248. Steps dimensioned 0.285 m by 0.132 m by 0.033 m were placed at 0.5 m intervals to represent the steps in a step-pool system. The fields between the steps were filled with gravel (d_{90} = 0.0049 m) whose size distribution is shown in Figure 1.

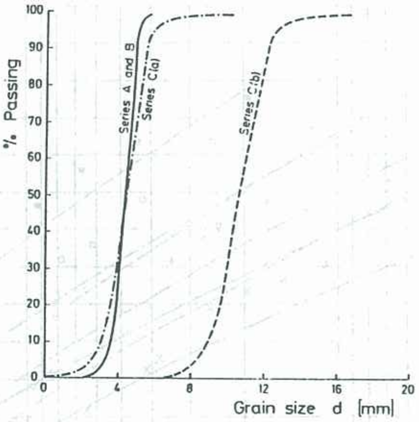

Fig. 1 Grain Size Distribution Curves

In test series A clear water scour was investigated for various combinations of slope and flow rate. A definition sketch of the variables measured is presented in Figure 2. In the analysis of the results for the derivation of α, interest was limited to those tests in which the scour hole did not develop to the extent of being strongly distorted by the step at the downstream end of the scour field.

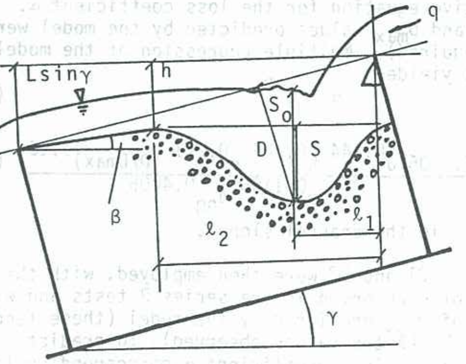

Fig. 2 Scour Variables Measured: Series A and B

(This consideration was also followed for the results of test series B and C). Where such strong distortion did occur, a flow instability was noted; these features of step-pool behaviour are described in Whittaker (1982, 1985, 1987).

For test series B, the scour dimensions defined in Figure 2 were measured for various combinations of flow rate, slope and sediment transport rate. The measurements were made when it had been determined that equilibrium transport conditions had been established. The results of test series B are plotted in Figure 3.

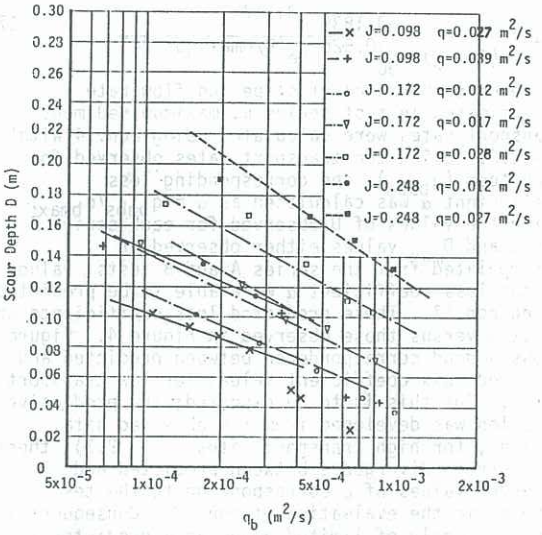

Fig. 3 Scour Depth D vs Sediment Transport Rate

A regression analysis was performed on the results shown in Figure 3, although it was necessary to exclude a number of the high transport rate results, giving:

$$\frac{D}{D_{max}} = \ln\left[\frac{0.8662\ J^{0.1097}\ q^{0.2164}}{d_{90}^{\ 0.0579}\ q_b^{\ 0.1778}}\right] \tag{8}$$

The laboratory channel used for the series C tests is shown in Whittaker (1985), and Smart and Jaeggi (1983). This channel was 0.235 m deep, 0.2 m wide and approximately 6 m long. Steps dimensioned 0.14 m by 0.2 m by 0.15 m were placed at 0.25 m intervals to simulate a step-pool system. Clear water scour tests were performed at slopes up to 0.254 for a variety of flow rates, and with the two sediment mixtures described in Figure 1. The scour dimensions defined in Figure 2 again were measured. The results of test series A and C were then analysed, using multiple regression to evaluate a number of descriptive equations, viz:

$$S_o = [1.4115\ (q-q_{cr})^{0.5034}\ h^{0.476}/d_{90}^{\ 0.2311}] \tag{9}$$

where $$q_{cr} = [0.047\ d\ (s-1)]^{1.667}/n\ J^{1.167} \tag{10}$$

in which 0.047 = critical Shields parameter and Manning's $n = 0.04168\ d^{0.167}$ (11)

$$S = [0.9121\ (q-q_{cr})^{0.4526}\ h^{0.5877}/d_{90}^{\ 0.2666}] \tag{12}$$

$$S_o/\ell_2 = 0.3404\ J^{0.7}\ (L/d_{90})^{0.399} \tag{13}$$

4 EVALUATION OF LOSS COEFFICIENT EQUATION

It then remains to use the results described above to evaluate a predictor for the loss coefficient α. It can be assumed that the case of sediment transport through a system of steps and pools is a special case of the more general bed load transport on a steep, plane slope. This means that the Smart/Jaeggi transport formula can be manipulated, with the loss term α, to describe the step-pool transport phenomenon. Equation 7 can then be re-expressed:

$$q_b = [4/(s-1)]\ (d_{90}/d_{30})^{0.2}\ \alpha\ q\ J^{0.6}\ (J-J_{cr}) \quad (14)$$

$$J_{cr} = \{[0.047\ d\ (s-1)]^{1.667}/nq\}^{0.857} \quad (15)$$

Now, eqn.8 can be re-arranged to yield:

$$q_b = \frac{0.4458\ J^{0.617}\ q^{1.217}}{d_{90}^{0.326}\ [e^{(D/Dmax)}]^{5.6243}} \quad (16)$$

Combining Eqns.14 and 16, and solving for the loss coefficient α, gives (after setting $J^{0.017} \cong 1.0$, and assuming $d_{30} \cong d_{90}$)

$$\alpha = \frac{0.1839\ q^{0.217}}{(J-J_{cr})\ d_{90}^{0.326}\ [e^{(D/Dmax)}]^{5.6243}} \quad (17)$$

For each combination of slope and flow rate investigated in test series B, maximum sediment transport rates were calculated using eqn.14 with $\alpha = 1.0$. Using the transport rates observed for each test (q_{bobs}), the corresponding loss coefficient α was calculated as $\alpha = q_{bobs}/q_{bmax}$. Using the values of D observed for each series B test, and D_{max} values either observed or interpolated from the series A and B tests, values of the loss coefficient α were able to be predicted using eqn.17. These predicted loss coefficients are plotted versus those observed in Figure 4. Figure 4 shows a good correspondence between predicted and observed loss coefficient values for low transport rates. But this is to be expected; the predictive equation was developed from the observed data. However, for high transport rates ($\alpha > 0.3$), there is a strong divergence between predicted and observed values of α corresponding to the tests ignored in the evaluation of eqn. 8. Consequently, eqn.17 is only of limited value as a predictor for the loss coefficient α.

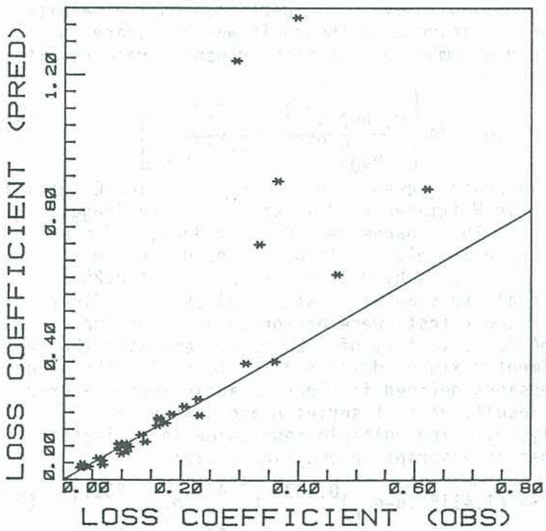

Fig. 4 Loss Coefficient: Predicted (eqn.17) vs Observed

To circumvent this problem, a clear water scour model was developed using the results from test series A and C, specifically eqns 9, 12, and 13 (see Fig. 2). The model is based on the assumption that the final scour state for a discharge q_i has exactly the dimensions of an intermediate scour state for a larger flow rate q . In other words, similarity of scour hole shape and position is assumed for the time development of a scour hole, even with a variable discharge. This means that if the final scour shape for q_i yields D_i, h_i and I (where I = tanβ), then, with $q > q_i$, one can have the same D and h , but also a sediment transport rate out of the hole predicted from eqn.14 with J = I. The major problem with this model is that h in eqns 9 and 12 is unknown. [Note that the scour field is initially filled to the plane of the top of the steps]. From geometric considerations,

$$h = L\sin\gamma - (L\cos\gamma - \ell_2)\sin\beta \quad (18)$$

A substitution for ℓ_2 in eqn.18 can be derived from eqn.13. The resultant expression for h can then be substituted into eqn.9, which is then solved iteratively for So. (The convergence using the previously evaluated So value is rapid). With h then known, S can be solved for using eqn.12. Now, the scour dimension D is approximately given by

$$D \cong [S + (h-\ell_1 JL)]\cos\gamma \quad (19)$$

in which

$$\ell_1 \cong 0.452\ \ell_2 \quad (20)$$

The model was initially employed by setting d_{90} = 0.0049 m and using the same slopes as in the series B tests. For each of these slopes, after solving for D , h , and I for a series of incremented flow rates, corresponding values of q_b were evaluated for each of the series B q values. Consequently, for each combination of slope and flow rate investigated in series B, a relationship between q_b and D was able to be established. Lines representing this relationship are plotted, together with the series B results, in Figure 5. As the model was not calibrated with the test series B results, the correspondence shown in Figure 5 is really an evaluation of the accuracy of the model. The reasonable correspondence to the test results indicates that the model describes the transport phenomenon fairly well.

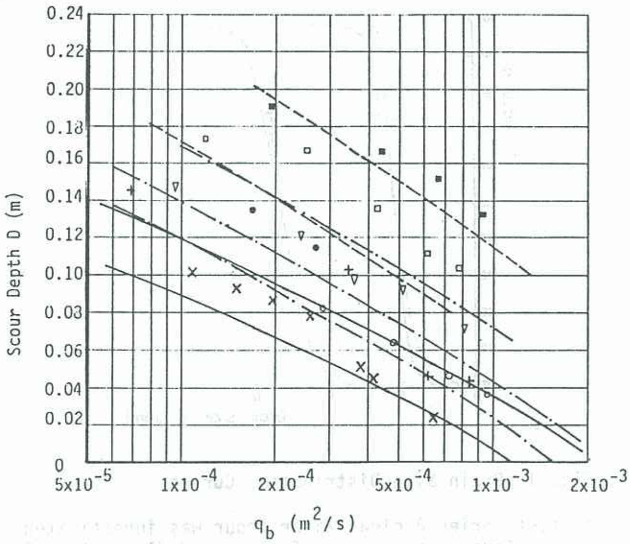

Fig. 5 Scour Variable D vs Sediment Transport Rates (Compared With Results of Scour Model)

The model was then run with a number of hypothetical discharges, slopes, and sediment sizes. These results were pooled with those used to derive the lines in Figure 5, and used to derive a general predictive equation for the loss coefficient α. (q_{bmax} and D_{max} values predicted by the model were also required). Multiple regression of the model results yielded

$$\alpha = e^{-B} \quad (21)$$

where

$$B = \frac{2.4206\ J^{0.244}\ L^{0.109}\ q^{0.221}\ (D/Dmax)^{1.16}}{g^{0.1105}\ d_{90}^{0.4405}} \quad (22)$$

[Note, J is the channel slope].

Equations 21 and 22 were then employed, with the values of D observed in the series B tests and with values of D_{max} predicted by the model (these tended to be ≅ 1.04 x the values observed), to predict values of the loss coefficient α corresponding to each series B test. These are plotted versus the observed values of α (these latter already having been used in Figure 4) in Figure 6. As can be seen, the correspondence between predicted and observed is reasonably good. A better fit is given by:

$$\alpha_{obs} = 1.0065\ \alpha_{pred} + 0.0368 \tag{23}$$

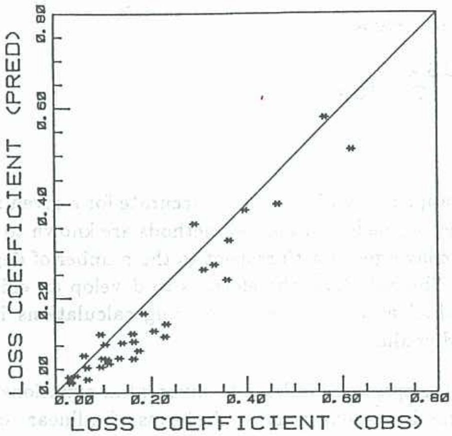

Fig. 6 Loss Coefficient: Predicted (eqns.21,22) vs Observed

Thus, eqns.21,22,23 and 14 can be employed to predict sediment transport rates in a step-pool stream.

5 CONCLUSIONS

The following conclusions may be stated from the material presented above.

- An empirical adjustment is usually made to sediment transport formulae to allow for flow energy lost to the transport process because of form losses.
- The Smart/Jaeggi formulae has been adapted through the inclusion of a loss coefficient for predicting sediment transport in step-pool mountain streams. The loss coefficient allows for flow energy dissipated in the pools by turbulent mixing, which is thus unavailable for transport.
- The most suitable step-pool sediment transport loss coefficient was derived from the results of a sediment transport model based on clear water scour in step-pool systems. The accuracy of the predictive equation developed was satisfactorily checked with independent sediment transport measurements, although it was possible to obtain a slightly improved accuracy with the measurements.

ACKNOWLEDGEMENTS

The work reported in this paper was carried out in the Department of Agricultural Engineering, Lincoln College, NZ, and in the Laboratory of Hydraulics, Hydrology and Glaciology (VAW), ETH Zürich, Switzerland, whose directors are Professors T. Ward and D. Vischer respectively. The Lincoln work was financed by the Ministry of Works and Development, NZ. This paper is published with permission of the Commissioner of Works.

REFERENCES

Ashida, K; Takahashi, T and Sawada, T (1976): Sediment yield and transport on a mountainous small watershed. Bull. Disaster Prev. Res. Inst., Kyoto Univ., Vol. 26, Pt.3, No. 240, pp 119-144.

Ashida, K; Takahashi, T and Sawada, T (1981): Processes of transport in mountain stream channels. In: Erosion and sediment transport in Pacific rim steeplands (Proc. I.A.H.S. Christchurch Symposium, Jan. 1981. Ed. by T.R.H. Davies and A.P. Pearce), pp 166-178.

Meyer-Peter, E and Müller, R (1948): Formulas for bed load transport. Proc. 2nd IAHR Congress, Stockholm.

Rouse, H (1965): Critical analysis of open channel resistance. Journal of the Hydraulics Div, ASCE, Vol 91, HY 4, pp 1-25.

Smart, G M (1984): Sediment transport formula for steep channels, Journal of Hydraulic Engineering, ASCE, Vol. 110, No. 3, pp 267-276.

Smart, G M and Jaeggi, M N R (1983): Sediment transport on steep slopes. Mitteilung Nr. 64 der VAW, ETH, Zuirch, Switzerland.

Whittaker, J G (1982): Flow and sediment movement in stepped channels, Ph.D. thesis, Lincoln College, University of Canterbury, NZ.

Whittaker, J G (1985): Sediment transport in step-pool streams. 2nd International Gravel Rivers Workshop, Colorado, August 1985.

Whittaker, J G (1987): Design and behaviour of stepped channels. Mitteilung der VAW, ETH, Zürich, Switzerland (in preparation).

NOMENCLATURE

c	friction factor (= u_m/u_*)
d	grain size
d_n	grain size such that n% of grains are smaller than
g	acceleration due to gravity
h	head driving the scour process
k_r	Strickler roughness coefficient for grain resistance
k_s	Strickler roughness coefficient for total resistance to flow
ℓ_1	distance from vertical through crest of upstream step to point of maximum scour depth
ℓ_2	distance from vertical through crest of upstream step to end of scour
n	Manning roughness coefficient
q	specific discharge
q_b	specific sediment transport rate (m²/s)
s	ratio ρ_s/ρ
u_*	shear velocity
B	function defined in eqn. 21
D	maximum scour depth measured from plane through top of steps
Fr	Froude number
I	critical slope at downstream end of scour hole for q, (I = tanβ)
J	slope (J = tanγ))
L	distance, in plane of channel, between steps
R	hydraulic radius
S	maximum scour depth measured below a vertically horizontal plane through the end point of the scour hole
S_o	maximum scour depth measured vertically from the water surface

GREEK SYMBOLS

α	energy loss coefficient used to adapt sediment transport equation for step-pool streams
β	angle of sediment sloping from downstream end of scour hole to next step
γ	angle of channel
θ	flow parameter (defined in eqn.2)
ξ	form loss coefficient
ρ	density of water
ρs	density of sediment
φ	sediment transport parameter