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The Studies on the Correction Coefficients in the Entrance Region for the Laminar and Turbulent Radial Diffuse Flow Between Two Parallel Disks

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ABSTRACT

In this paper, the general calculating methods are given for inlet length, loss of pressure and correction coefficients of entrance region for laminar and turbulent flow between two parallel disks, with the help of momentum integral equation. The flow quantity, by taking account of the effect of entrance region, is also derived. The authors take the following as examples: the laminar velocity distribution is $f(\eta)=2\eta-\eta^2$, the turbulente one is $f(\eta)=\eta^{1/7}$, in boundry layer. The authors have done some experiments. The laminar theoretical results agree very well with the author's experiments. The turbulent theoretical result is checked with "Mohn's (1930)" experiment. It is proved that the formulae given in this paper is simple and reliable.

INTRODUCTION

The radial diffusive flow between parallel disks is very common in hydraulic technology, for instance, the flow of valve "Oki(1959), Usava(1958), the flow between slipper plate and slipper bearing, between block cylinder and valve plate. Besides these it can be seen that flow in air micrometer "Nakayama(1954), Hagiwara(1952)". And it is possible to use for the studies on the vaneless diffuser "Brown (1947)", axial bearings "Show (1949), Fuller (1947)", spherical bearings "Allen(1953)".

Radial diffuser in one form or another have been investigated experimentally by "Brow(1947) Allen(1953), Comolet(1952), Mohn(1930), Paivanas(1955), Welanetz(1956), Savage(1964), Woolard(1954), Oki(1958), Nakayama(1954), Hagiwara(1952), Wang, Z Q(1982), Liu, Z B(1984)". Some analyses are for laminar viscous flow in which the acceleration terms in the equations of motion are assumed to be small in comparision to the viscous terms, and are therefore negleted. The hydraulic method is also used to treat the flow as one-dimensional and utilize a modified Bernoulli equation which includes an additional term accounting for the pressure drop due to friction by means of friction factor or coefficient. Pressure distributions calculated by "Woolard(1954)"using the hydraulic method for laminar and turbulente flow in radial diffuser with parallel disks, are compared with an experiments obtained by "Mohn(1930) "for the flow of water in a double-disk valve element. Deep studies have been done for the air micrometer by "Hagiwara(1952) and Nakayama(1954)". But they just studied on laminar flow in both theory and experiment. In this paper, the authors pay more attention to the studies of laminar and turbulente flow in entrance region. They are derived that the general equations which inlet length satisfies, while the entrance region effect is accounted, and the general expression of coefficients of entrance region effect. The authors have done a lot of experiments. The laminar theoretical results agree quite well with the authors experiments. The turbulent experiments happen the flow contr action But the author's turbulent results agree better than "Woolard's (1952)" turbulent result comparision with "Mohn's(1930)" experiment.

NOMENCLATURE

- Ce —— correction coefficient of flow quantity of the entrance region effect
- h —— gap between disks
- p_o —— total pressure at inner radius r_1
- p, p_1, p_2, p_e —— static pressure at r, r_1, r_2, r_e
- Q —— flow quantity
- r —— radial direction coordinate
- r_1 —— inner radius of disk
- r_2 —— outer radius of disk
- r_e —— inlet length
- R_e —— correction Reynolds number at r_1 $(u_{m_1} h/\nu).(h/r_1)=Qh/(2\pi r_1^2 \nu)$
- u —— radial velocity component

*It is one of the problems that surported by the fund of Chinese Academy of Science.

u_o —— velocity of potential flow outer boundry layer

u_m —— mean velocity at r $(Q/2\pi rh)$

u_{m1} —— mean velocity at r_1 $(Q/2\pi r_1 h)$

U —— nondimensional velocity of potential flow outer boundry layer (u_o/u_m)

U_1 —— nondimensional velocity of potential flow at r_1 (u_o/u_{m_1})

z —— axial direction coordinate

γ —— correction coefficient of pressure drop of the entrance region effect

δ —— boundry layer thickness

δ_1 —— displacement thickness

δ_2 —— momentum thickness

Δ_1 —— nondimensional displacement thickness $(2\delta_1/h)$

Δ_2 —— nondimensional momentum thickness $(2\delta_2/h)$

λ —— coefficient of total pressure head

λ_ξ —— coefficient of partial pressure head

λ_p —— coefficient of pressure drop

ν —— motion viscocity

ξ —— nondimensional local radius (r/r_1)

ξ_e —— nondimensional inlet length (r_e/r_1)

ρ —— density

τ_w —— shearing stress at the wall

p_{th} —— theoretical pressure

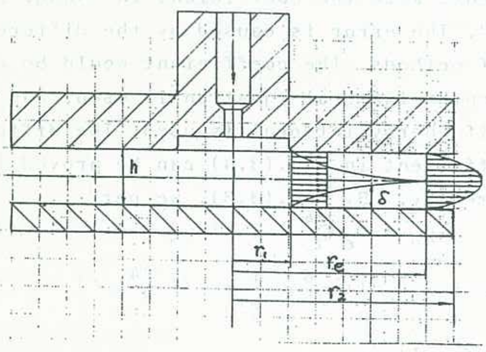

Fig 1: Illustration of radial diffusive flow between parallel disks.

1 FOUNDATION OF THEORY

We assume that:

(1) The fluid is incompressible (ρ=const.): The flow is steady and constant physical characteristic.

(2) The inlet is smooth. The radius of air chamber is significant larger than the gap $(h/r_1 << 1)$.

(3) It is formed in the entrance region that boundry layer is axisymmetrical about the plane of z=h/2. The flow is turbulence (average-time turbulence) inner boundry layer, and is potential flow outer boundry layer.

(4) At the end of entrance region the full filled time-average turbulence is formed. And its velocity distribution is power law (Actually it is logarithmic law).

By reference "Wang, ZQ(1982)", we know that the momentum integral equation at the wall in entrance region is as below:

$$\frac{d\delta_2}{dr}+\frac{1}{u_o}\frac{du_o}{dr}(2\delta_2+\delta_1)+\frac{\delta_2}{r}=\frac{\tau_w}{\rho u_o^2} \tag{1.1}$$

wheree $\delta_1=\int_0^\delta\left(1-\frac{u}{u_o}\right)dz$, $\delta_2=\int_0^\delta\frac{u}{u_o}\left(1-\frac{u}{u_o}\right)dz$

From the flow continuity, we get

$$U=\frac{u_o}{u_{m_1}}=\frac{1}{(1-A\Delta)\xi} \tag{1.2}$$

$$\text{Where}\quad A=\int_0^1(1-f)d\eta\,,\quad \eta=\frac{z}{\delta} \tag{1.3}$$

Thus, from the flow continuity, again, we get

$$u_{m_1}/u_m=r/r_1=\xi$$

$$\text{So}\quad U=U_1u_{m_1}/u_m=\xi U_1 \tag{1.4}$$

By (1.2), (1.4), we get

$$U=1/(1-A\Delta) \tag{1.5}$$

If it is assumed that the general expression of velocity distribution in boundry layer is

$$u/u_o=f(\eta)=\eta^n \tag{1.6}$$

Besides satisfing the general boundry conditions, $f(\eta)$ should satisfy the additional condition at the end of entrance region, i.e. when $\Delta\to1$, the flow is full filled homogeneous diffusive one.

The shearing stress at the wall which is respectively with power law, see "Hermann Schlichiting (1979)", are surposed

$$\tau_w=B(n)\left(\frac{\nu}{u_o\delta}\right)^m\rho u_o^2 \tag{1.7}$$

Where $m=\frac{2n}{1+n}$ —— resistance exponent

$B(n)$ —— characteristic number concerned with R_e, when $n=\frac{1}{6}\sim\frac{1}{7}$, $B(n)$=0.0395---0.0118.

When the flow is laminar, the shearing stress at the wall is defined by Newton shearing stress law:

$$\tau_w=\left(\frac{\partial f}{\partial\eta}\right)_0\left(\frac{\nu}{u_o\delta}\right)\rho u_o^2 \tag{1.8}$$

If we assume $m=1, B(n)=\left(\frac{\partial f}{\partial\eta}\right)_0$, in egn.(1.7), it will be changed in the form of eqn.(1.8), so eqn (1.7) is suitable for both laminar and turbulent flow. The power law of velocity distribution is for turbulence. But the laminar velocity distribution can be 2nd-power law, 3rd-power law and sine law.

2 TURBULENT INLET LENGTH AND CORRECTION COEFFICIENTS OF ENTRANCE REGION EFFECT

(1) Turbulent Inlet Length

The eqn.(1.1) can also be written in dimensionless form as

$$\frac{d\Delta_2}{d\xi}+\frac{1}{U}\frac{dU}{d\xi}(2\Delta_2+\Delta_1)-\frac{\Delta_1+\Delta_2}{\xi}=\frac{2\tau_w}{\rho u_o^2}\left(\frac{r_1}{h}\right)\qquad(2.1)$$

From (1.7), we obtain

$$\frac{2\tau_w}{\rho u_o^2}\left(\frac{r_1}{h}\right)=\frac{2^{m+1}B(n)}{R_e^{\,m}}\Delta^{-m}U^{-m}\xi^{m}\left(\frac{h}{r_1}\right)^{m-1}\qquad(2.2)$$

Where $\Delta_1=A\Delta$ $\qquad\Delta_2=(B-A)\Delta$ (2.3)

Here $B=\int_0^1(1-f^2)d\eta$ (2.4)

Substitute (2.2),(2.3), (2.4) and (1.5) into eqn.(2.1), we gain

$$\frac{d\xi}{d\Delta}=\frac{R_e^{\,m}}{2^{m+1}B(n)}\cdot\frac{(B-A+AB\Delta)\Delta^m\xi}{(1-A\Delta)\left((1-A\Delta)^m\xi^{m+1}+\frac{B}{2^{m+1}B(n)}R_e^{\,m}\Delta^{m+1}\right)}\qquad(2.5)$$

ξ can be calculated from the above eqn.(2.5).

(2) <u>The Coefficient of Loss of Pressure.</u>

The partial coefficient of loss of pressure is defined as

$$\lambda_\xi=-\frac{\partial p}{\partial\xi}/(\tfrac{1}{2}\rho u_{m_1}^2)\qquad(2.6)$$

when $\xi\geqslant\xi_e$, $\Delta\equiv1$,

By eqn.(1.1), (1.2), (1.5), (2,2). we gain,

$$\lambda_\xi=\frac{2^{m+1}B(n)\left(\frac{h}{r_1}\right)^{m-1}}{R_e^{\,m}(1-A)^{2-m}}\xi^{m-2}-\frac{2}{(1+2n)(1-A)^2}\cdot\frac{1}{\xi^3}\qquad(2.7)$$

The coefficient of total pressure head along the pipe is defined as

$$\lambda=\frac{1}{\xi}\cdot\frac{P_o-P}{\frac{1}{2}\rho u_{m_1}^2}\qquad(2.8)$$

When $\xi<\xi_e$, by Bernoulli equation, we get

$$P_o-P=\tfrac{1}{2}\rho u_o^2\qquad(2.9)$$

Substitue eqn. (2.9) to (2.8)

$$\lambda=1/(\xi^3(1-A\Delta)^2)\qquad(2.10)$$

The coefficient of loss of pressure, when $\xi<\xi_e$

$$\lambda_p=\lambda\xi=1/(\xi^2(1-A\Delta)^2)\qquad(2.11)$$

When $\xi=\xi_e$, $\Delta\equiv1$,

$$\lambda_{pe}=1/(\xi_e^2(1-A)^2)\qquad(2.12)$$

When $\xi\geqslant\xi_e$, loss of pressure can be written as

$$\lambda_p=\frac{2^{m+2}B(n)\left(\frac{h}{r_1}\right)^{m-1}}{R_e^{\,m}(1-A)^{2-m}(m-1)}(\xi^{m-1}-\xi_e^{m-1})+\frac{1}{(1+2n)(1-A)^2}\left\{\frac{1}{\xi^2}-\frac{1}{\xi_e^2}\right\}+\lambda_{pe}\qquad(2.13)$$

(3) <u>The Correction Coefficient of Flow Quantity of Entrance Region Effect.</u>

When $\xi<\xi_e$, by eqn. (2.11), we obtain

$$P_o-P=C_e\frac{2^{m+2}B(n)(h/r_1)^{m-1}}{R_e^m(1-A)^{2-m}(1-m)}\xi^{m-1}\cdot\tfrac{1}{2}\rho u_{m_1}^2\qquad(2.14)$$

Where
$$C_e=\frac{R_e^{\,m}(1-A)^{2-m}(1-m)}{2^{m+2}B(n)(h/r_1)^{m-1}(1-A\Delta)^2}\xi^{-(1+m)}\qquad(2.15)$$

When $\xi\geqslant\xi_e$

$$C_e=\left(\frac{\xi}{\xi_e}\right)^{1-m}-1+\frac{R_e^{\,m}(1-A)^{2-m}(1-m)\xi^{1-m}}{2^{m+2}B(n)\left(\frac{h}{r_1}\right)^{m-1}(1+2n)}\left\{\frac{2n}{\xi_e^2}+\frac{1}{\xi^2}\right\}\qquad(2.16)$$

The mean velocity at inner radius can be got from eqn.(2.14).Thus the flow quantity, which taking the account of entrance region effect, can be got

$$Q=\left\{\frac{\{\pi(1-A)\}^{2-m}(1-m)\,h^{4-m}\,\Delta p}{C_e\,2^{m-1}\,B(n)\,\xi^{m-1}\mu^m\rho^{1-m}}\right\}^{1/(2-m)}\qquad(2.17)$$

3 COMPARISION OF CALCULATIONG EXAMPLES WITH EXPERIMENTS

(1) <u>Laminar Entrance Region</u>

It is assumed that the velocity distribution in boundry layer is similar in the entrance region. Thus

$$\frac{u}{u_o}=f(\eta)=2\eta-\eta^2\qquad(3.1)$$

By eqn.(1.3) and (2.4), we get

$A=1/3$, $B=7/15$, $B(n)=f'(0)=2$

And if we assume m=1, substitute A, B,B(n) in eqn.(2.5), we gain

$$\frac{d\xi}{d\Delta}=\frac{R_e}{40}\cdot\frac{\Delta(6+7\Delta)\,\xi}{(3-\Delta)\{(3-\Delta)\xi^2+7/40\cdot Re\ \Delta^2\}}\qquad(3.2)$$

It is the equation which we derived in reference "Wang, Z Q(1985)". From eqn(1.8) and the definition of λ_ξ ,λ_p the general expression of λ_p can be written as

$$\lambda_p=\frac{24}{R_e}\ln\xi+\frac{6}{5}\frac{1}{\xi^2}+\gamma\qquad(3.3)$$

where
$$\gamma=\left\{\frac{9}{(3-\Delta)^2}-\frac{6}{5}\right\}\frac{1}{\xi^2}-\frac{24}{R_e}\ln\xi\qquad(3.4)$$

When $\xi\geqslant\xi_e$, $\gamma=\gamma_e$.

The coefficient of second term in eqn.(3.3)is differente with the coefficient in "Wang, Z Q (1982)". The error is caused by the different uses of methods. The coefficient would be 6/5 if momentum integral equation is used, and 54/35 if energy integral is used. The difference of coefficient in eqn.(3.4) can be proved in the same way. By eqn.(3.3), we get

$$p_o-p=C_e\frac{24}{R_e}\ln\xi\cdot\tfrac{1}{2}\rho u_m^2\qquad(3.5)$$

When $\xi<\xi_e$, $C_e=1+\left(\frac{6}{5}\frac{1}{\xi^2}+\gamma\right)/\left(\frac{24}{R_e}\ln\xi\right)$ (3.6)

When $\xi\geqslant\xi_e$,$\gamma=\gamma_e$

While ξ is constant, C_e is only the function of R_e. The larger R_e, is the bigger C_e is.When $\xi\geqslant\xi_e$, $C_e\doteq1$; when $\xi<\xi_e$, the effect of C_e for flow quantity should not be ignored.

Using the experimental element, see Fig.1, (r_1=21 mm, r_2=80 mm), the present authors have done a lot of experiment in air at various gap. The results are shown in Fig.2. It is can be seen that the theoretical values agree very well with experiments.

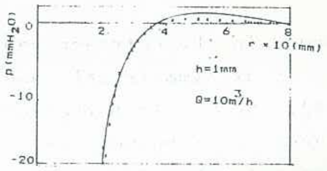

Fig.2.Comparision of p_{th} With Author's Experiment.

(2) Entrance Region of Turbulence

For the entrance region of turbulence, we assume that velocity distribution in boundry layer is as below power law. As an example, we take $n=1/7$, $B(n)=0.0232$. From (1.3), (2.4), we have $A=1/8$, $B=2/9$. By eqn.(2.5), we get

$$\frac{d\xi}{d\Delta}=0.2517R_e^{\frac{1}{4}}\frac{\Delta^{\frac{1}{4}}(7+2\Delta)\xi}{(1-\Delta/8)\{(1-\Delta/8)^{\frac{1}{4}}\xi^{5/4}+4R_e^{\frac{1}{4}}\Delta^{5/4}\}} \tag{3.8}$$

We use Rung-Kutta method to calculate eqn.(3.8) and obtain inlet length ξ_e which varies with R_e, see Fig.4. In Fig.5, we shown ξ varies with Δ at different R_e.

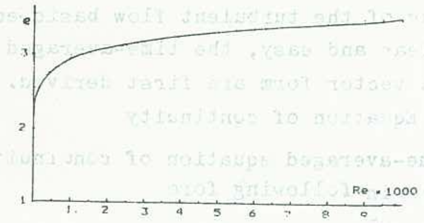

Fig 4. Inlet Length varies with R_e

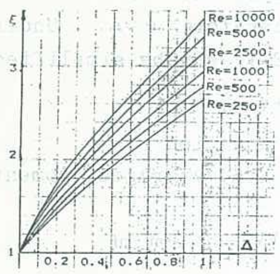

Fig 5. ξ varies with Δ

When $\xi<\xi$, by eqn.(2.11),

$$\lambda_p=\frac{1}{\xi^2(1-\Delta/8)^2} \tag{3.9}$$

When $\xi=\xi_e$, $\lambda_p=\frac{64}{49}\cdot\frac{1}{\xi^2}$ (3.10)

When $\xi>\xi_e$

$$\lambda_p=\frac{0.1859(r_1/h)^{3/4}}{R_e^{\frac{1}{4}}}\{\frac{1}{\xi_e^{3/4}}-\frac{1}{\xi_e^{3/4}}\}+\frac{64}{63}\frac{1}{\xi^2}+\frac{128}{441}\cdot\frac{1}{\xi_e^2} \tag{3.11}$$

When $\xi<\xi_e$, by eqn.(2.15),

$$C_e=\frac{5.3792\,R_e^{\frac{1}{4}}}{(r_1/h)^{3/4}\xi^{5/4}(1-\Delta/8)^2} \tag{3.2}$$

When $\xi\geqslant\xi_e$, by eqn.(2.16),

$$C_e=(\frac{\xi}{\xi_e})^{3/4}+\frac{4.1839R_e^{\frac{1}{4}}\xi^{3/4}}{(r_1/h)^{3/4}}\{\frac{1}{\xi^2}+\frac{2}{7}\cdot\frac{1}{\xi_e^2}\}-1 \tag{3.13}$$

By eqn.(2.17),

$$Q=7.7737\{\frac{h^{15}\Delta p^4}{C_e^4\mu}\frac{\xi^3}{\rho^3}\}^{1/7} \tag{3.14}$$

From the expression of C_e, we can see that C_e of turbulence is only the function of R_e while ξ=const. But it differ from laminar flow because when $\xi>>\xi_e$, C_e does not limit to 1. Thus the conclusion will be got that C_e effects a lot for flow quantity of turbulence whenever ξ is, i.e. the effect of entrance region should not be ignored. That is the reason why we study the entrance region.

In order to compare with "Woolard's (1930)" theoretical value, we derive loss of pressure from eqn.(3.11). The author's theoretical turbulent result is shown in Fig.6. It can be seen that the latter agree with "Mohn's (1930)" experiment better than the former.

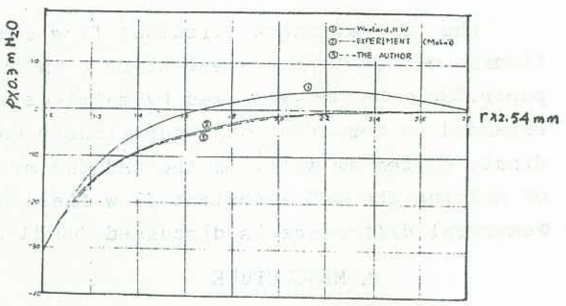

Fig. 6: Comparision of Pressure Distribution of the Author With Mohn's experiment and Woolard's Pressure Distribution.

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