Calculation of Strongly Swirling Jet Flows

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ABSTRACT

Computations of the mean-velocity and turbulent-stress distributions in the recirculating flow induced by a strongly swirling free jet are reported. The effects of turbulence are represented by both eddy-viscosity (k-ε) and algebraic Reynolds-stress models. The governing equations are solved by a finite-difference numerical method using quadratic upstream-weighted discretization of the convection term to minimise numerical diffusion. The calculations are compared with measurements obtained by laser anemometry. The general features of the flow are reasonably well predicted but discrepancies in the detailed mean velocity and turbulent-stress distributions indicate the need for further refinements in modelling the turbulence energy dissipation and the pressure-strain correlation in the Reynolds-stress closure. The results show relatively little difference between the mean-velocity fields calculated by the standard k-ε model and the stress model.

NOMENCLATURE

A_{ij}	'added' generation rate of stress $\overline{u_i u_j}$
c	turbulence-model constant
D	nozzle diameter
D_{ij}	deformation rate tensor
k	turbulent kinetic energy
ℓ	mixing length
p	mean static pressure
P	production rate of turbulent kinetic energy
P_{ij}	production rate of stress $\overline{u_i u_j}$
r	radial coordinate
T_{ij}	transport of stress $\overline{u_i u_j}$
u_i	fluctuating velocity component in x_i direction
U_i	mean velocity component in x_i direction
$\overline{u_i u_j}$	kinematic turbulent stress
U,V,W	mean velocity components in axial, radial and tangential directions
u,v,w	fluctuating velocity components in axial, radial and tangential directions
x	axial coordinate
x_i	Cartesian coordinate
ε	dissipation rate of turbulent kinetic energy
ε_{ij}	dissipation rate of stress $\overline{u_i u_j}$
ϕ_{ij}	pressure-strain correlation of stress $\overline{u_i u_j}$
ν	kinematic molecular viscosity
ν_t	kinematic turbulent viscosity
ρ	fluid density
σ	turbulent Schmidt number

INTRODUCTION

Swirling jet flows are found in many engineering applications. Of particular interest are flames in furnaces where recirculation induced by strong swirl is used to enhance mixing and promote stable, high-intensity combustion (Mahmud et. al., 1986).

Turbulent swirling jet flows have proved difficult to predict even under isothermal conditions. Serious problems arise with the modelling of turbulence and these are frequently compounded by an incomplete knowledge of the initial flow conditions at the jet

exit. Moreover, significant numerical errors may arise from false diffusion associated with upwind differencing of convection terms, especially for strongly swirling jets with recirculation.

For weakly swirling jets (no recirculation), eddy-viscosity turbulence models, such as the well-known k-ε model, fail to predict the pronounced effect of swirl on the rate of spread of the jet unless the model 'constants' are made a function of a swirling flow Richardson number (Rodi, 1979). The failure of the eddy-viscosity model is frequently attributed to the assumption of an isotropic viscosity. However, the more complex Reynolds-stress turbulence models, which account for stress anisotropy, afford no more than a marginal improvement (Launder and Morse, 1979).

In a recent study of strongly swirling free jets with recirculation, Leschziner and Rodi (1984) suggested that the standard k-ε turbulence model, without any swirl-related modifications, can yield satisfactory predictions provided realistic jet-exit conditions are prescribed. Unfortunately, for the flows examined, the jet-exit conditions were not known precisely; in particular, the radial component of velocity and the turbulence energy dissipation rate, both of which were found to influence the predictions significantly, were not available from the measurements. Consequently, no firm conclusions could be drawn regarding the validity of the turbulence model.

The present paper reports an evaluation of the standard k-ε model and an algebraic Reynolds stress model of turbulence using comprehensive measurements from a very recent study of a strongly swirling free jet (Sislian and Cusworth, 1986). The measurements provide a complete set of profiles for all flow properties close to the jet exit, from which initial conditions for the computations can be prescribed with some certainty. The computations employ the quadratic upstream-weighted differencing scheme to minimize numerical diffusion.

THEORETICAL MODEL

Governing Equations

The time-mean equation for conservation of mass and momentum in steady-state, isodensity turbulent flow can be written in Cartesian tensor notation as:

$$\frac{\partial U_i}{\partial x_i} = 0 \tag{1}$$

$$\frac{\partial (U_i U_j)}{\partial x_j} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j}\left[\nu\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right) - \overline{u_i u_j}\right] \tag{2}$$

where U_i and u_i are the mean and fluctuating velocity components in the direction x_i, p is the mean static pressure, ρ the density and ν the kinematic molecular viscosity. Closure of the equations is achieved through a turbulence model to represent the Reynolds stress correlations $\overline{u_i u_j}$. Here, two turbulence models are employed: the widely used k-ε eddy-viscosity model and an algebraic Reynolds-stress model.

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Turbulence Models

Eddy-viscosity turbulence models are based on a genera-lised Boussinesq hypothesis relating the stress to the rate of strain,

$$\overline{u_i u_j} = \frac{2}{3} k \delta_{ij} - \nu_t \left[\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right] \qquad (3)$$

where ν_t is the kinematic turbulent viscosity. In the k-ε model (Launder and Spalding, 1974,) the turbulent viscosity is expressed in terms of the turbulence energy k and its dissipation rate ε by

$$\nu_t = C_\mu k^2 / \varepsilon \qquad (4)$$

where C_μ is an empirical constant. The transport equations for k and ε are given by

$$\frac{\partial (U_j k)}{\partial x_j} = \frac{\partial}{\partial x_j} \left[(\nu + \nu_t) \frac{\partial k}{\partial x_j} \right] + (P - \varepsilon) \qquad (5)$$

$$\frac{\partial (U_j \varepsilon)}{\partial x_j} = \frac{\partial}{\partial x_j} \left[(\nu + \frac{\nu_t}{\sigma_\varepsilon}) \frac{\partial \varepsilon}{\partial x_j} \right] + (C_{\varepsilon 1} P - C_{\varepsilon 2}) \frac{\varepsilon}{k} \qquad (6)$$

where P is the production rate of turbulence energy given by

$$P = - \overline{u_i u_j} \frac{\partial U_i}{\partial x_j} \qquad (7)$$

The empirical constants C_μ, $C_{\varepsilon 1}$, $C_{\varepsilon 2}$, and σ_ε are assigned to values 0.09, 1.44, 1.92 and 1.22 respec-tively.

Reynolds-stress turbulence models are based on the exact transport equations for the Reynolds stresses which have the general form

$$T_{ij} = P_{ij} + \phi_{ij} - \varepsilon_{ij} \qquad (8)$$

where the various terms represent transport (convec-tion and diffusion), production, pressure-strain redistribution and dissipation of the stress $\overline{u_i u_j}$. Here, the model of Launder et. al. (1975) is adopted for the pressure-strain correlation and the dissipation. The algebraic form of the Reynolds-stress turbulence model is obtained by approximating the differential transport term T_{ij} as suggested by Rodi (1976):

$$T_{ij} = \frac{\overline{u_i u_j}}{k} (P - \varepsilon) \qquad (9)$$

The resulting system of algebraic equations for the stresses $\overline{u_i u_j}$ can be written as

$$\overline{u_i u_j} = \frac{2}{3} k \delta_{ij} + \frac{k/\varepsilon}{C_1 - 1 + P/\varepsilon} \left[(1 - C_2)(P_{ij} - \frac{2}{3} P \delta_{ij}) \right.$$
$$\left. - C_3(D_{ij} - \frac{2}{3} P \delta_{ij}) - C_4 k \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) + C_5 A_{ij} \right] \qquad (10)$$

where

$$P_{ij} = -(\overline{u_i u_k} \frac{\partial U_j}{\partial x_k} + \overline{u_j u_k} \frac{\partial U_i}{\partial x_k}) \qquad (11)$$

$$D_{ij} = -(\overline{u_i u_k} \frac{\partial U_k}{\partial x_j} + \overline{u_j u_k} \frac{\partial U_k}{\partial x_i}) \qquad (12)$$

and P is given by Eq(7) and k and ε are obtained from Eqs(5) and (6). A_{ij} represents the so-called 'added' generation which arises when the convection term is expressed in a cylindrical coordinate system. These terms are frequently large and are included as produc-tion terms by setting C_5 equal to $1 - C_2$. The empirical constants C_1, C_2, C_3 and C_4 are assigned the values 1.8, 0.76, 0.11 and 0.18 respectively.

Solution Method

The governing equations were written in cylindrical co-ordinates and reduced to a conservative finite-differ-ence form using the control-volume method over a rectangular grid covering one half plane of the axisymmetric flow domain. Quadratic upwind-weighted convective differencing was used to minimise numerical diffusion (Leonard, 1979). The finite-difference equations were solved by well-established computational methods (Gosman and Pun, 1974).

The coupled system of algebraic equations for the six Reynolds stresses was solved by standard matrix methods. The Reynolds stresses were introduced into the momen-tum equations by extracting as a diffusion term the eddy-viscosity contribution given by Eq(3) and treating the residue as a source term.

The computations were carried out on a non-uniform 30x 30 grid with a concentration of nodes in regions of high velocity gradient. The results are believed to be essentially grid independent.

Boundary Conditions

The measurements of mean velocity and turbulent stress-es in the strongly swirling free jet were obtained by laser anemometry (Sislian and Cusworth, 1986). Swirling motion was imparted to the axial flow by fixed, flat guide-vane swirlers placed at the nozzle exit. The nozzle-exit diameter was 25.4mm and the swirl number was calculated as 0.79.

The inlet conditions for the computations were taken from the measurements close to the jet exit, at an axial distance of 0.125D. The profiles of U, V, W and k were directly available and the turbulence energy dissipation rate, ε, was determined from the shear stress through the relationship

$$\varepsilon = C_\mu \frac{k^2}{\overline{uv}} \frac{\partial U}{\partial r} \qquad (13)$$

The analogous relationship involving k, W and the $\overline{vw}$ shear stress produced broadly similar values of ε (slightly higher but within a factor of two of those from $\overline{uv}$). Shear stress measurements were not available near the outer edge of the jet and in this region ε was prescribed from the mixing-length as

$$\varepsilon = k^{3/2} / \ell \qquad (14)$$

with ℓ taken as the length scale at the outer-most point where $\overline{uv}$ was measured. The turbulence length scale ($k^{3/2}/\varepsilon$) is small, typically about 2mm in the shear layer, and more closely related to the dimension of the guide-vane swirler located at the jet exit than to the jet dimension.

At the entrainment boundary (r/D=4), the static press-ure was assumed to be constant, U, W and k were taken as zero and V was calculated from the radial momentum equation with the continuity condition rV=const. At the outflow boundary, which was located well beyond the recirculation zone in a region of forward flow (x/D=7), the axial derivative of static pressure was assumed to be zero and U was calculated from the axial momentum equation. In practice, the results proved to be in-sensitive to the conditions imposed at the entrainment and outflow boundaries.

RESULTS AND DISCUSSION

The calculated and measured U- and W-velocity profiles at axial distances D, 2D and 3D downstream of the nozzle exit are compared in Figs.1 and 2. As can be seen, the qualitative features of the flow are reason-ably well predicted. However, some discrepancies occur near the nozzle exit where the initial rates of decay of the peaks in the U- and W-velocity profiles are underpredicted by both turbulence models. At down-stream stations, a worthwhile improvement in the predi-ction of the W-velocity profile is achieved with the Reynolds stress turbulence model. The length of the recirculation zone is well predicted by both turbulence models. However, the prediction of the algebraic Reynolds stress model is sensitive to 'added' genera-tion and the length of the recirculation zone is de-creased significantly (to about 2D) when the added generation is neglected, and increased, with large negative velocities on the axis within the zone, when C_5 is increased beyond $1 - C_2$.

The profiles of turbulence energy at the same axial locations considered above are compared in Fig.3. The predictions are generally rather poor. At the outer edge of the jet the turbulence energy is underpredicted while in the central region of the flow it is overpredicted by a factor of about two. The prediction of the Reynolds stress turbulence model is only a marginal improvement on that of the k-ε model. The improvements, namely an increase in turbulence energy at the outer edge of the jet and a decrease in the central region, reflect the sensitivity of the stress model to curvature-related anisotropy and the destabilizing or stabilizing influence of swirl on the turbulence.

The discrepancies in the predictions are believed to be due primarily to deficiencies in the modelling of the turbulence dissipation. Because the swirl is imparted to the flow by guide vanes located at the nozzle exit, the turbulence structure at the inlet to the flow domain is highly complex. As noted above, the inlet condition for ε, Eq(13), yields large values of ε (small length scales) in the shear layer and consequently low turbulent viscosities of the order of only 40ν. That the initial decay of the peaks in the U- and W-velocity profiles is underpredicted suggests that the turbulent viscosity increases too slowly as the flow develops downstream of the inlet, presumably because the dissipation rate decreases too slowly in response to the rapidly changing turbulence structure. Indeed, Hanjalic et. al.(1979) have previously suggested that a single ε equation is inadequate to model the evolution of the range of length scales present in complex turbulent flow and proposed a multiple-length-scale (multiple ε) model for turbulence dissipation. In the context of the present flow, the length scales of the turbulent eddies near the nozzle exit range from the small scales associated with the swirl generator (about 2mm) through to the large scales associated with the fluid entrained at the jet boundary (about 10mm). A computation with lower values of ε at the inlet confirmed the expected increase in the rate of decay of the peaks in the velocity profiles and also revealed a decrease in the turbulent energy in the central region of the flow as a consequence of the lower velocity gradients and lower turbulence generation in the shear layer. However, in order to reproduce the measured peak in the U-velocity profile at D, the value of ε at the inlet would have to be decreased by a factor of about five, and an error of this magnitude in the value of ε obtained from Eq(13) seems unlikely. Furthermore, with ε decreased by a factor of five the spreading rate of the jet is grossly overpredicted. Finally, it is interesting to speculate that the measurements would have made fewer demands on the turbulence model had the swirl generator been located well upstream of the nozzle exit.

A number of corrections to the dissipation equation have been proposed, generally involving ad-hoc modifications to the empirical constants and/or the generation term. Launder et. al (1981) proposed the use of $\nu_t(\partial U_i/\partial x_j)^2$ in place of P in the ε equation for both eddy-viscosity and Reynolds-stress models. Recently, Rhode and Stowers (1985) employed the modification in an assessment of turbulence models for confined swirling jets and found an improvement with their Reynolds stress model but an inferior performance with the k-ε model. In the present flow, the modification produced inferior predictions with both k-ε and Reynolds stress models. Although the modification has been reported to work well on relatively simple flows, such as weakly swirling jets, it appears to lack generality and performs particularly poorly on complex flows with rapid acceleration and/or deceleration. Hah (1983) proposed the use of the turbulence anisotropy as the generation term in the ε equation for the Reynolds stress model. Preliminary calculations with the modification have not produced any definite conclusions: first, the model is particularly sensitive to the empirical constants and those reported by Hah seem to be inappropriate for use with the present algebraic stress model (Hah employed an unusual representation of stress transport); and secondly, the stability of the iterative solution scheme is seriously degraded.

Profiles of the turbulent stresses normalized by the kinetic energy are compared in Figs.4-6. The qualitative features of the stress profiles are reasonably well predicted although a number of discrepancies are evident. The most obvious discrepancies arise in the shear stresses at the outer edge of the jet where $\overline{uv}$ and $\overline{vw}$ are overpredicted and $\overline{uw}$ has the wrong sign. The discrepancies in $\overline{uv}$ and $\overline{uw}$ are not unrelated because the two stresses are quite strongly coupled through their respective production terms and the pressure-strain correlation. In the pressure-strain correlation of Launder et. al. (1975) the so-called 'rapid' term (the one involving the coefficient C_2 in Eq(10)) is dominant and the shear stresses are simply related to their respective production terms.

The production of $\overline{uv}$ contains a term $\overline{uw}(W/r)$, the only term in which the swirl velocity appears explicitly, which Launder and Morse (1979) argue is responsible for the increased entrainment and rate of spreading in weakly swirling jets. Here it should be noted that $\overline{uw}$ is found from measurements (Launder and Morse, 1979; Ribeiro and Whitelaw, 1980) in weakly swirling jets to be large and positive and therefore represents a source of generation in the equation for $\overline{uv}$. The measurements of Sislian and Cusworth (1986) suggest that the action of $\overline{uw}$ in strongly swirling jets is opposite to that in weakly swirling jets, at least over the first few nozzle diameters downstream of the jet exit. Consequently, the increase in entrainment due to strong swirl must arise indirectly through the primary generation term in the $\overline{uv}$ equation, namely $-\overline{v^2}\partial U/\partial r$, as a result of an increase in $\overline{v^2}$ through the generation term $2\overline{vw}(V/r)$ in the $\overline{v^2}$ equation.

The production of $\overline{uw}$ contains the primary generation term $-\overline{u^2}\partial W/\partial z$ and secondary generation terms $-\overline{uv}\partial W/\partial r$ and $-\overline{vw}\partial U/\partial r$. Near the outer edge of the jet the primary generation term is negative and both secondary generation terms are large and positive, leading to the erroneous positive values for the $\overline{uw}$ correlation. Evidently, changes are needed in modelling the pressure strain correlation. The present work suggests that the contribution from the isotropic term should be increased (by increasing the coefficient C_4) in order to obtain negative values of the $\overline{uw}$ correlation. Launder and Morse (1979) also concluded from their study of weakly swirling jets that there are shortcomings in the modelling of the pressure-strain correlation. In passing, it is interesting to note that the eddy-viscosity turbulence model which contains only the isotropic term predicts a correctly-signed $\overline{uw}$ correlation and reasonable values for the other shear stresses. This fact may explain the observation of Leschziner and Rodi (1984) that the standard k-ε turbulence model appears to perform best for strongly swirling jets while in weakly swirling jets swirl-related modifications are necessary.

CONCLUSIONS

The following main conclusions can be drawn from the present study of a strongly swirling free jet flow:
The initial development of the jet is markedly sensitive to the turbulence dissipation rate, ε, at the nozzle exit. The standard ε equation does not adequately represent the rapid evolution of the turbulence structure near the nozzle exit.
The algebraic stress model affords no more than a marginal improvement over the k-ε model in predicting the mean-velocity field.
The model of the pressure-strain correlation in the Reynolds stress closure fails to reproduce the measured shear stress distribution at the outer edge of the jet. An increase in the isotropic contribution to the correlation would seem to improve the predictions.

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REFERENCES

Gosman, A.D. and Pun, W.W. (1974): Calculation of re-circulating flows, Imperial College, Mechanical Engineering Department Report HTS/74/2.

Hah, C. (1983): Calculation of various diffuser flows with inlet swirl and inlet distortion effects, AIAA Journal, Vol. 21, pp.1127-1133.

Hanjalic, K., Launder, B.E. and Schistel, R. (1979): Multiple-time-scale concepts in turbulent trans-port modelling, Turbulent Shear Flows 2, Edited by Bradbury et. al., Springer Verlag, Berlin, pp.36-49.

Launder, B.E.,and Morse, A. (1979): Numerical predic-tion of axisymmetric free shear flows with a Reynolds-stress closure, Turbulent Shear Flows 1, Edited by Launder and Whitelaw, Springer Verlag, Berlin, pp.279-294.

Launder, B.E.,and Spalding, D.B. (1974): The numerical computation of turbulent flows, Computer Methods in Applied Mechanics and Engineering, Vol.3, pp.269-289.

Launder, B.E., Leschziner, M.A. and Sindir, M. (1981): Comparison of computation with experiment, AFOSR-HTTM-Stanford Conference on Complex Turbulent Flows 1981, pp.1390-1404.

Launder, B.E., Reece, G.J. and Rodi, W. (1975): Progress in the development of a Reynolds-stress turbulence closure, Journal of Fluid Mechanics, Vol.68, pp.537-566.

Leonard, B.P. (1979): A stable and accurate convective modelling procedure based on quadratic upstream interpolation, Computer Methods in Applied Mechanics and Engineering, Vol.19, pp.59-98.

Leschziner, M.A. and Rodi, W. (1984): Computation of strongly swirling axisymmetric free jets, AIAA Journal, Vol. 22, pp. 1742-1747.

Mahmud, T., Truelove, J.S. and Wall, T.F. (1986): Flow characteristics of swirling coaxial jets from divergent nozzles, submitted to ASME Journal of Fluids Engineering.

Rhode, D.L. and Stowers, S.T. (1985): Turbulence model assessment for the confined mixing of co-swirling concentric jets, Proc. AIAA 21st Joint Propulsion Conference (Monterey, California), AIAA Paper No. 85-1271.

Ribeiro, M.M. and Whitelaw, J.H. (1980): Coaxial jets with and without swirl, Journal of Fluid Mechanics, Vol. 96, pp.769-795.

Rodi, W. (1976): A new algebraic relation for calculat-ing the Reynolds stress, ZAMM, Vol. 56, pp.T219-T221.

Rodi, W. (1979): Influence of buoyancy and rotation on equations for the turbulent length scale, Proc. of the 2nd Symposium on Turbulent Shear Flows (Imperial College, London), pp.10.37-10.42.

Sislian, J.P. and Cusworth, R.A. (1986): Measurement of mean velocity and turbulent intensities in a free isothermal swirling jet, AIAA Journal, Vol.24, pp. 303-309.
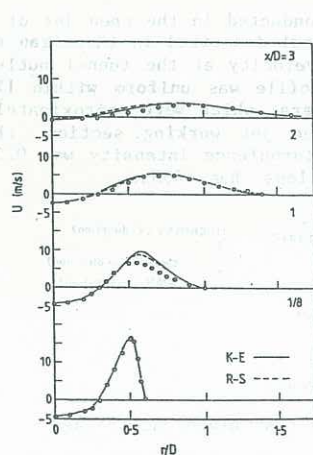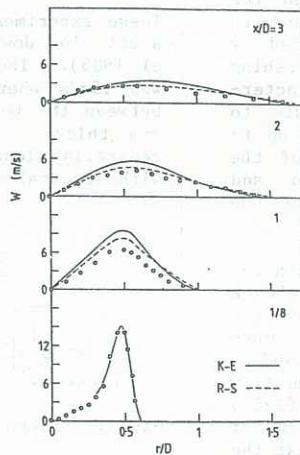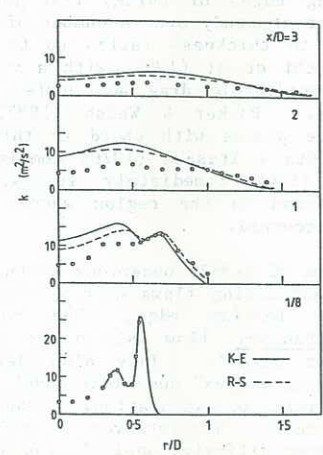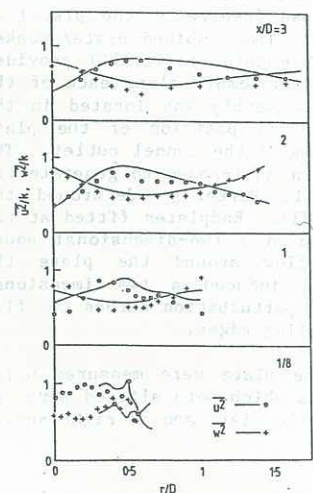
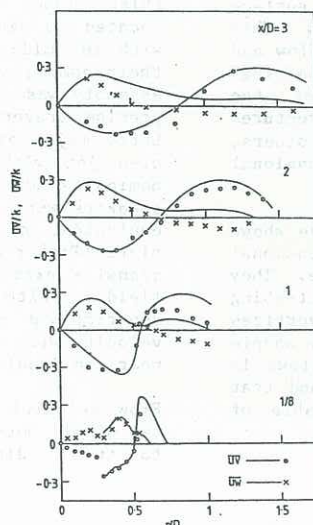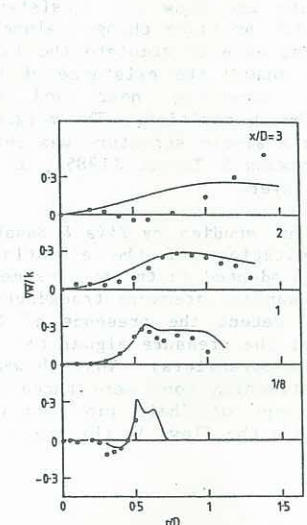Fig.1 Mean axial velocity

Fig.2 Mean tangential velocity

Fig.3 Turbulence energy

Fig.4 $\overline{u^2}$ & $\overline{w^2}$ normal stresses

Fig.5 $\overline{uv}$ & $\overline{uw}$ shear stresses

Fig.6 $\overline{vw}$ shear stress