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Separation of Viscous Jets using Boundary Element Methods

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ABSTRACT

Viscous jets are not usually observed to separate from a sharp edge in the manner expected theoretically. In the present paper the separation of a creeping jet emerging from a tube with a rounded exit is considered. As a separation criterion, in the absence of surface tension, we propose that the traction normal to the nozzle surface drops to zero at the separation point. Boundary-element calculations then show a behaviour that agrees with experimental data and with previous finite-element computations. They also permit the Michael condition to be observed at separation, so that the discrepancy between finite-element calculations and theory is removed.

THE SEPARATION PROBLEM

In many processes a liquid stream leaves a conduit and generates a free surface; a change from the no-slip boundary condition to the free-surface flow occurs at some point. Two types of separation are shown in Fig. 1. In case (a) a calculation by Michael (1958) shows that the separation angle (Fig. 1a) should be exactly 180° ($\alpha = 0$), and that the viscous stresses at the separation point vary like $d^{-1/2}$, where d is the distance from the separation point A. Experimentally and computationally separation does not always occur as at Fig. 1(a) (Tanner 1985). Another possible model for separation is shown at Fig. 1(b) and this is explored in the present paper; here separation takes place at the unknown point A.

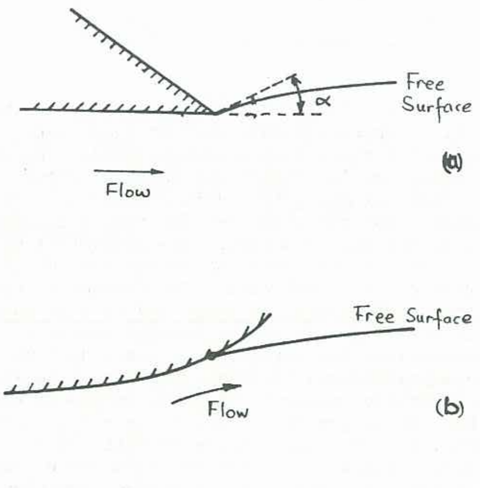

Fig. 1: Singular Points in Flow. (a) Flow at a sharp, well-defined separation point, (b) General separation point.

Early studies on separation were made by Nickell et al (1974) and Bush and Tanner (1983), who gave finite-element and boundary-element solutions respectively to the basic problem of a creeping jet of fluid exiting from a long tube. All of these numerical solutions and experimental data show that at a "sharp-edged" tube exit, the initial angle (α) of the free jet surface is at some small (approximately 12 degrees) finite angle to the tube centreline. In Fig. 2 we show finite-element and boundary-element creeping flow calculations and the experimental data of Batchelor and Horsfall (1971) in which the Reynolds number (Re) was about 10^{-8}, plus our own data with Re < 10^{-3}. At moderate (1-10) Re the results are essentially the same (Gear et al, 1983), so we believe that the Reynolds number is not an essential parameter. Similarly, small surface tension effects do not change the separation angle much and, hence, inertia and surface tension are both ignored in the present study.

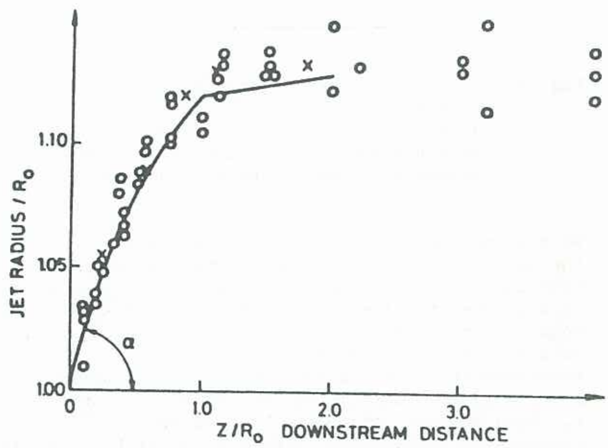

Fig.2: Comparison of finite-element computed jet shape (——) with our own experiments (X) and those of Batchelor and Horsfall (O). In all cases Re < 10^{-3}, and surface tension parameter $\rho\sigma R_0/\eta^2 < 10^{-5}$. The initial separation angle α is between 9 and 14° for the experiments, and is about 12° for the computations.

The theoretical result of Michael (1958) for creeping flow separation at a sharp edge gives the result $\alpha = 0$, which is at variance with experiment and computation. Here we try to reconcile theory, experiment and computation for creeping flow. In view of the stress singularity at the separation point, it is tempting to relax either the Newtonian constitutive equation or the no-slip boundary condition. Rejection of Newtonian fluid physics is a last resort, and we shall retain the Newtonian fluid assumption and the no-slip boundary condition; Silliman and Scriven (1980) have shown that relaxing the no-slip condition still yields a finite separation angle. Our present hypothesis assumes that every exit is not perfectly sharp, but has a finite radius. Jean and Pritchard (1980) show photographs of separation from the rounded exit to a plane channel, and it is clear that the rounding affects the downstream surface shape. In this paper we numerically investigate the effect of rounding the tube exit on the jet shape.

Fig. 3 shows the basic geometry. Let the tube radius be R_o and the exit rounding radius be r. The final swelled radius is R_f, and the swelling ratio χ is R_f/R_o. We suppose that separation takes place at an unknown point A, which can be specified by the separation angle θ (Fig. 3). From dimensional arguments, in creeping flow with no surface tension or gravity, we find

$$\chi = R_f/R_o = \chi(r/R_o,\theta). \tag{1}$$

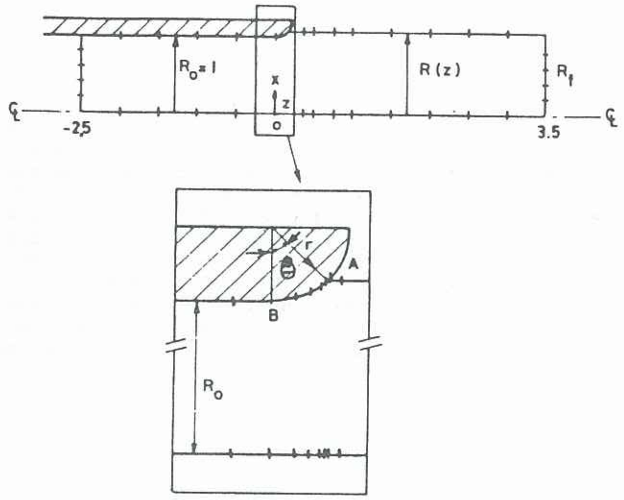

Fig. 3: Initial jet profile and discretization of the boundary for $r/R_o = 0.2$ and $\theta = 45°$. (+ indicates the end points of elements.) Generally the arc A-B contained four elements graded as shown in the enlarged sketch.

We now need to consider the stresses in the fluid near the separation point A. Just downstream of A (Fig. 3), both the normal (t_n) and the tangential (t_s) components of the traction vector are zero, and the velocity normal to the free surface (v_n) is zero. Further, once separation has occurred, we shall suppose that no reattachment to the surface occurs. Just upstream of A we could also demand that v_n is zero and that t_n or t_s could be zero, thereby giving several possible criteria for separation. There is no difficulty with v_n; it, and also the tangential speed v_s, must be zero upstream of A if we adopt the no-slip assumption. The situation with t_n and t_s is not so clear, and will be explored. One expects intuitively that a compressive normal stress is needed to keep the jet stuck to the wall, and when the normal stress is tensile, there is an expectation that separation will occur. Therefore we propose as a separation criterion that t_n is zero just upstream of A; the shear stress at point A is singular according to the Michael theory, and hence it is difficult to believe that t_s is zero at this point which would be an alternative criterion. We now turn to the numerical investigation.

METHOD OF ATTACK AND RESULTS

Our main interest is to predict the final jet profile and find the separation conditions. The boundary conditions of the flow change from no-slip to zero traction at the exit, and this leads to rapid changes of velocity and stress in the fluid nearby. Since the final solution depends largely on the representation of the flow near the exit, a fine grid is required throughout the region for domain-type solutions. The use of fine grids will make the problem a large one in terms of computing time and space. With boundary-element methods, we can refine the surface element spacing without much enlarging the computational problem. Therefore the boundary-element method is ideal for this type of problem and will be used (Bush and Tanner, 1983).

Since the point of attachment of the free surface to the nozzle exit boundary is unknown prior to solution, we used a method of trial and error. A number of flows with different assumed angles of separation θ were analysed for r/R_o = 0.01, 0.1, 0.2, and 0.5 using a boundary-element program. Details of the formulation and the performance of the program can be found in an earlier report (Tanner, 1985). In each flow the fluid boundary was divided into 56 elements. Figure 3 shows a typical initial geometry used.

Calculations were made at zero Reynolds number for axisymmetric jets. Figure 4 shows the tangential and normal stresses on the last fixed element of the fluid at the exit boundary at different prescribed separation angles θ for $r/R_o = 0.2$. The corresponding graphs

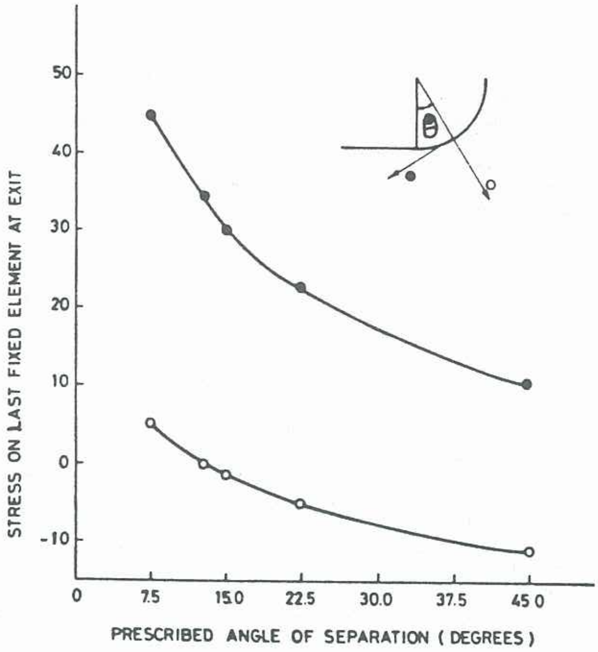

Fig. 4: Dimensionless stresses on the last fixed elements at exit at different angles of separation: Tangential ●; Normal ○. Note the positive directions are as in sketch. The stresses are normalized by the factor $\eta\bar{w}/R_o$, where $\bar{w}$ is the mean velocity in the upstream tube. Here $r/R_o = 0.2$ in this case.

for r/R_o = 0.01, 0.1, 0.5 are very similar to that for $r/R_o = 0.2$; therefore they are not shown here. Of the separation conditions mentioned above, the shear stress criterion is unimportant in this analysis; the shear stress all along the surface is never zero (Fig. 4). Recall that the plots are for the last fixed element on the exit boundary. The tangential velocity of the fluid near the point of separation but just downstream of it is not zero. The tangential traction at the point of separation jumps due to a singularity at A, in a similar way to the Michael analysis. (In our computations the stresses can reach high values, with appropriate mesh refining, but cannot be singular at the separation point.) There is no way of choosing θ so that the tangential stress is zero just upstream of A (Fig. 4). For this reason we will discard the alternative separation condition given above, and further consider only the normal traction condition.

The angles (θ_c) at which the normal stresses on the last wetted element on the exit boundary fall to zero for various radius ratios (r/R_o) are given in Table I. The free surface was not found to penetrate into the nozzle exit boundary anywhere, so that there was no reattachment once separation had occurred. This is in agreement with our earlier postulate. Therefore, according to our normal traction separation criterion, we conclude that the values given in Table I are the

TABLE I. Computed separation angles and jet swelling for various radii of rounded exits.

Radius ratio r/R_o	Separation angle θ_c (degrees)	Final jet swelling χ
0	-	1.126
0.01	11.6	1.128
0.1	12.0	1.132
0.2	12.7	1.138
0.5	14.0	1.153

approximate separation angles (θ_c; degrees). Notice that these angles do not vary much although the geometry of the exit varies from almost sharp-edged ($r/R_o = 0.01$) to obviously rounded ($r/R_o = 0.5$). The radius ratio r/R_o has little effect on the separation angle.

Figure 5(a) shows the final jet profiles at various rounded exits. Figure 5(b) shows the enlarged jet shapes near the exit boundaries. The final jet expansions are also shown in Table I.

DISCUSSION

Experimental results of Batchelor and Horsfall (1971) showed that for a Newtonian free jet of $Re \simeq 10^{-8}$ and surface tension parameter $\rho\sigma R_o/\eta^2 < 10^{-5}$, the angle of separation of the flow from a "sharp-edged" exit was in the range 9-14 degrees to the centreline (Fig. 2). The radius ratio r/R_o of a typical "sharp-edged" exit for experimental work is in the order of 0.01 or less. Our computed angle of separation as $r/R_o \to 0$ is about 11.6 degrees. This agrees well with the experimental angles of Fig. 2, which show a 9-14 degree angle of separation. The Michael theory would demand that the initial departure from the solid wall be tangential. As far as we can see (Fig. 5(b)), this is so; the angle apparent in experiments hence appears to be due to the rounded exit actually present in any real jet. Thus, the general conclusion is that the observations, computations, and the Michael theory may be reconciled by assuming a very small rounding of the tube exit. It does not explain why the nominally sharp-edged computation does not conform to the Michael condition. In this case the flow far from the edge presumably dominates the jet shape to such an extent that in a computation with a nominally sharp edge, no notice is taken of the Michael condition, and the computation simply ignores it. This is, perhaps a fortunate outcome in terms of computer effort in the simulation of real jets.

For the prediction of swelling, Table I shows that the nominally sharp edge (Tanner 1985) and the exit with a radius ratio r/R_o of 0.01 give nearly the same results for swelling (1.128 - 1.126) and for jet shapes. Therefore it will not usually be necessary to model the edge as carefully as we have done here. Finally, the vanishing of the stress of traction normal to the surface appears to be a useful separation criterion, reminiscent of the peeling of a layer of tape from a surface, for very viscous fluids.

While much more remains to be done, we have shown that it is now feasible to model numerically a realistic separation in highly viscous fluids, thus making some progress towards reconciling theory, computation and experiment.

ACKNOWLEDGEMENT

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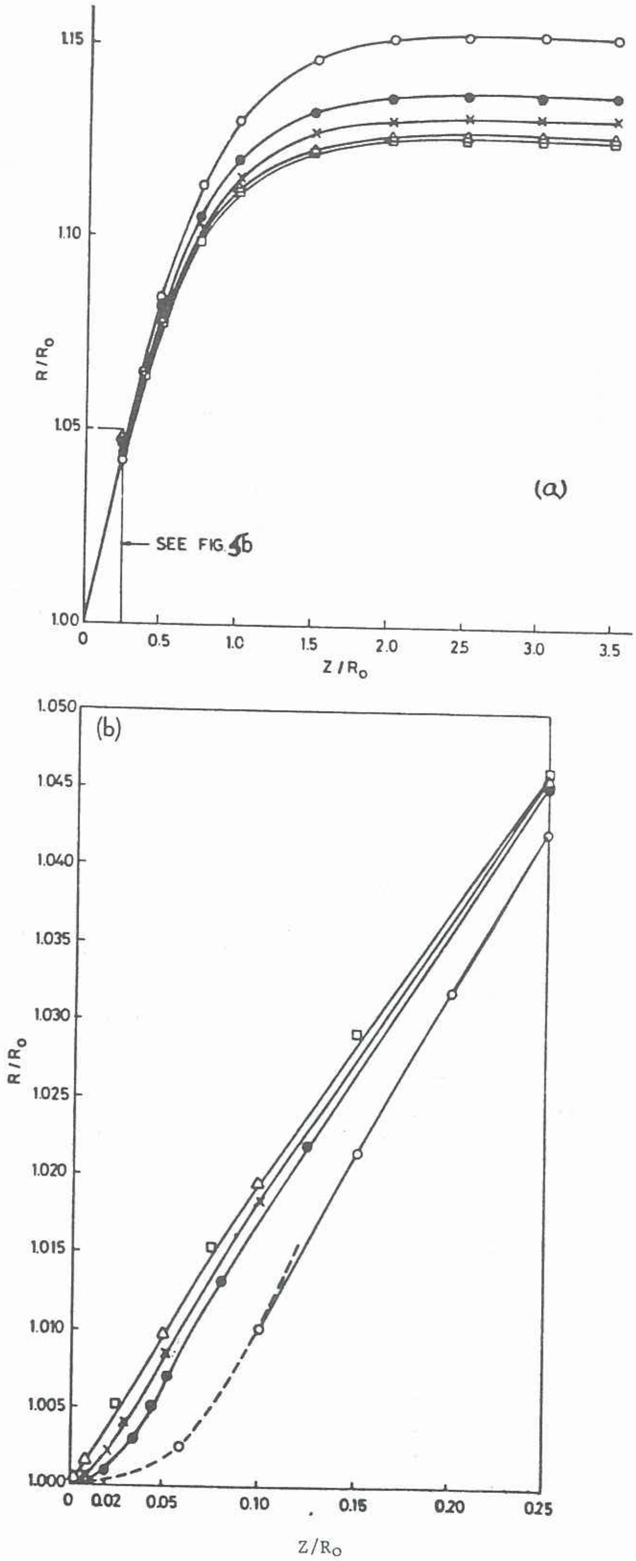

Fig. 5: (a) Computed jet shape for various exit radius ratios. ○ $r/R_o = 0.5$; ● $r/R_o = 0.2$; χ $r/R_o = 0.1$; △ $r/R_o = 0.01$; ⊡ $r/R_o = 0$. Origin of z is at B, Fig. 3. (b) Computed jet shape near exit boundary: ○ $r/R_o = 0.5$; ● $r/R_o = 0.2$; χ $r/R_o = 0.1$; △ $r/R_o = 0.01$; ⊡ $r/R_o = 0$. Origin of z is at B, Fig. 3. Dashed curve is shape of solid surface for $r/R_o = 0.5$. The separation is nearly tangential here.

NOMENCLATURE

d	distance from separation point
r	radius of tube lip
R_o	tube radius
R_f	final jet radius
Re	Reynolds number, $2\rho\bar{w}R_o/\eta$
t_n, t_s	normal and tangential traction vector components, respectively
v_n	velocity normal to free streamline
$\bar{w}$	mean velocity in tube
x	radial coordinate
z	axial coordinate

Greek Symbols

α	separated jet surface-angle to centreline (Fig.2)
η	viscosity
θ, θ_c	separation, critical separation angle
ρ	density
σ	surface tension coefficient
χ	swelling ratio R_f/R_o

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