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The Response of a Laminar Shear Layer on a Flat Plate to Transverse Surface Vibrations

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Introduction

The work presented in this paper pertains to the response of a steady laminar boundary layer to transverse surface vibrations of a flat plate. It is part of a broader investigation into unsteady viscous flows and unsteady forced convective heat transfer originating from a time dependent disturbance which is external to the fluid - a flexible flat plate thus excites an already established laminar shear layer and no bluff body separation exists. Unsteady viscous flows have received increasing attention since the sixties and most of the problems considered to date have been concerned with a rigid body oscillating in its own plane in the direction of the fluid flow, or with the flow over a stationary body where a sound field has been applied in a direction parallel to the body surface. Some earlier work [1,2] has been specifically concerned with the influence of localised, normal oscillations on the steady laminar flow over a flat plate, but the theoretical results did not explain all the observed experimental phenomena - for instance the positions of maximum mean velocity variations were not identified in the theory. The present work is not only concerned with localised vibrations but also with the global response of the entire vibrating surface. The equations governing the fluid flow are presented in a non-inertial, orthogonal, curvilinear coordinate system which is fixed to the surface of the oscillating plate, and the significance of the resultant governing non-dimensional groups discussed. The steady and fluctuating components of the longitudinal fluid velocity at various positions on the plate have been analysed for both sinusoidal excitation, and excitation with band-limited white noise. For the sinusoidal excitation, the plate was specifically excited at frequencies (a) below the fundamental natural frequency, (b) corresponding to the first and second natural modes of vibration, to simulate both resonant and non-resonant forced response situations.

Theoretical Considerations

The problem is modelled as a flat plate in a free stream (U_∞). A fixed cartesian coordinate system (X,Y) is defined such that X is measured along the surface of a fixed plate and Y is measured normal to the plate from its surface. The fluid velocity components (U,V) are then defined in the X and Y directions respectively. One should note at this stage that capitals are used for fixed coordinates and absolute velocities, whilst lower case letters are used for the accelerating coordinate system and the relative velocities. Now consider a plate in a free stream where the leading edge ($0 \le X \le X_1$) is fixed, whilst the rest of the plate (ie. $X > X_1$) is allowed to vibrate normal to the on-coming flow. Let us assume that the vibrations do not affect the potential flow around the plate and hence the free stream velocity can be considered to be equivalent to that associated with a fixed flat plate. The plate vibrations can be modelled by a smooth, continuously differentiable function Y=F(X,t). At this stage F(X,t) is assumed to be an arbitrary function with only a limitation on its amplitude, ie. $|F(X,t)| \le \alpha$, where α is a typical (but small) amplitude of vibration. The governing equations for an incompressible fluid are then expressed as (in component form - note: $(*)_{,X} = \partial(*)/\partial x$)

$$U_{,X} + V_{,Y} = 0$$

$$U_{,t} + U\,U_{,X} + V\,U_{,Y} = -\rho^{-1}\,p_{,X} + \nu(\,U_{,XX} + U_{,YY}\,)$$

$$V_{,t} + U\,V_{,X} + V\,V_{,Y} = -\rho^{-1}\,p_{,Y} + \nu(\,V_{,XX} + V_{,YY}\,)$$

with boundary conditions,

$$Y = F(X,t): \qquad U = 0,\ V = F_{,t}$$

$$Y \to \infty: \qquad U = U_\infty$$

In the above equations, ρ is the fluid density, p is the fluid pressure, and ν is the kinematic viscosity. To simplify the boundary conditions, the problem is transformed to an orthogonal curvilinear coordinate system which is fixed on the surface of the plate. The method used is similar to the technique described in [1]. The new coordinate system (x,y) with corresponding velocity components (u,v) is defined such that x is the distance measured along the surface of the plate to the origin of y, where y is then measured normal to the plate at that point on the surface. The coordinate system is now accelerating and therefore non-inertial. The velocity components at any instant are tangential and normal to the plate surface, and measured relative to the plate. The coordinate system might seem ambiguous when the plate appears as a concave body but this, however, poses no difficulty because we are only concerned with the very thin viscous layer adjacent to the plate, and the plate vibrations are assumed to be small.

In the derivation of the equations of motion with respect to the (x,y) coordinate system, we shall assume that terms which involve products of F(X,t) with itself or its derivatives are negligible. Then, the independent variables are related by,

$$X = x - yF_{,x}$$

$$Y = y + F$$

and the dependent variables are related by,

$$U = u - yF_{,tx} - vF_{,x}$$

$$V = v + F_{,t} + uF_{,x}$$

The Navier-Stokes equations in the orthogonal curvilinear coordinate system in the accelerating frame of reference of the plate are then given by,

$$u_{,x} + (1 - yF_{,xx})v_{,y} - F_{,xx}v = 0 \quad \text{(Continuity Equation)}$$

$$(1 - yF_{,xx})u_{,t} + uu_{,x} + (1 - yF_{,xx})vu_{,y} - F_{,xx}uv - (2\,F_{,tx}v + yF_{,ttx})$$
$$= -\rho^{-1}\,p_{,x} + \nu\big[(1 + yF_{,xx})u_{,xx} + (1 - yF_{,xx})u_{,yy} + F_{,xxx}(yu_{,x} - v)$$
$$- F_{,xx}(u_{,y} + 2v_{,x}) - 2yF_{,txxx}\big]$$

(x-Momentum Equation)

$$(1 - yF_{,xx})v_{,t} + uv_{,x} + (1 - yF_{,xx})vv_{,y} + F_{,xx}u^2 + 2\,F_{,tx}u + F_{,tt}$$
$$= -(1 - yF_{,xx})\rho^{-1}\,p_{,y} + \nu\big[(1 + yF_{,xx})v_{,xx} + (1 - yF_{,xx})v_{,yy}$$
$$+ F_{,xxx}(yv_{,x} + u) + 3F_{,xx}u_{,x}\big]$$

(y-Momentum Equation)

Boundary Conditions,

$$y = 0: \qquad u = v = 0$$

$$y = \delta: \qquad u = U_\infty - yF_{,tx}; \quad v = -U_\infty F_{,x} - F_{,t}$$

It should be noted that δ is the position at which the solution to the above equations merges into the potential flow. In order to completely define the problem, the x-conditions and initial conditions should also be specified.

We now proceed to non-dimensionalise the equations by using typical length and time scales l_x, l_y and τ for the independent variables, and typical velocity and pressure scales u_o, v_o and p_o for the dependent variables. The function F will be scaled by using α. One can show that the term $(1 \pm yF_{,xx})$ can be approximated by unity. From an analysis of the normalised continuity equation one can conclude that $(l_x v_o)/(l_y u_o)=1$. Thus, the normalised equations in non-dimensional variables simplify to,

$$u_{,x} + v_{,y} = 0 \quad \text{(Continuity Equation)}$$

$$(St)u_{,t} + uu_{,x} + vu_{,y} - (\varepsilon/Stl_{xy})F_{,xx}uv - (\varepsilon/l_{xy})\{2F_{,tx}v + (St)yF_{,ttx}\}$$
$$= -(p^*)p_{,x} + (l_{xy}^2/R_x)\big[(l_{xy}^{-2})u_{,xx} + u_{,yy} + (\varepsilon/Stl_{xy}^{\,3})F_{,xxx}(yu_{,x} - v)$$
$$- (\varepsilon/Stl_{xy}^{\,3})F_{,xx}\{(l_{xy}^2)u_{,y} + 2v_{,x}\} - (\varepsilon/l_{xy}^{\,3})2yF_{,txxx}\big]$$

(x-Momentum Equation)

$$(St)v_{,t} + uv_{,x} + vv_{,y} + (\varepsilon l_{xy}/St)F_{,xx}u^2 + (\varepsilon l_{xy})2F_{,tx}u + (\varepsilon St l_{xy})F_{,tt}$$
$$= -(l_{xy}^2 p^*)p_{,y} + (l_{xy}^2/R_x)[(l_{xy}^{-2})v_{,xx} + v_{,yy}$$
$$+ (\varepsilon/St l_{xy}^3)F_{,xxx}\{yv_{,x} + (l_{xy}^2)u\} + (\varepsilon/St l_{xy})3F_{,xx}u_{,x}]$$

(y-Momentum Equation)

The dimensionless groups in the equations are defined as,

$$St = l_x/(u_o\tau),\ \varepsilon = \alpha/(u_o\tau),\ l_{xy} = l_x/l_y,\ p^* = p_o/(\rho u_o^2)$$

and $R_x = u_o l_x/\nu$.

One should note that there are 2 additional groups in this type of unsteady two-dimensional flow, ie. St and ε. St is the ratio of the local fluid acceleration to the convective acceleration, whilst ε is the ratio of the plate acceleration to the local fluid acceleration. In most circumstances pertaining to unsteady viscous boundary layer flows due to transverse plate oscillations whilst we expect the St parameter to generally be large, ε will always be small. However, under certain conditions ε can have a significant effect on the dynamics in much the same manner as viscosity sometimes affects the flow through the parameter R_x.

Note: One should note that care must be taken when comparing solutions to the above sets of equations with experimentally measured velocity components. The calculated velocity components (u,v) are relative to a moving frame of reference whilst any measured velocity components are most probably relative to a fixed laboratory frame of reference. Thus, the transformation equations for the dependent variables should be used.

Experimental Arrangements

The boundary layer wind tunnel used for the tests is of a blow-down, open circuit, low speed type as described by Mehta and Bradshaw[3]. The working section is rectangular (380 mm x 250 mm) with less than 0.75% free stream variation across it. Flow velocities of up to 45 m/s can be sustained in the working section within a 1% variation over a two hour period. The free stream turbulence levels are typically 0.09% with most of the spectral content below 200 Hz. The primary spectral peak was identified with the fan rotational speed. The experiments reported on in this paper were conducted at a flow velocity of 4 m/s, and the sound pressure level in the working section at this speed was typically 78 dB.

An aluminium flat plate was mounted horizontally, 80 mm above the test section floor. The plate was 1111 mm long, 375 mm wide and 18 mm thick with a polished upper surface. The plate was supported at the upstream end on a heavy I-beam steel structure, and the downstream end was connected to an electromagnetic exciter via a line contact to minimise the excitation of cross modes. A wooden leading edge with a 7:1 semi-ellipse profile was isolated from the vibrating flat plate via a silicone rubber gap of ~ 1 mm - the leading edge was supported via side supports bolted to the test section floor. Great care was taken to avoid any mechanical contact between the working section of the wind tunnel and the vibrating flat plate structure. The experimental rig is illustrated schematically in Fig 1.

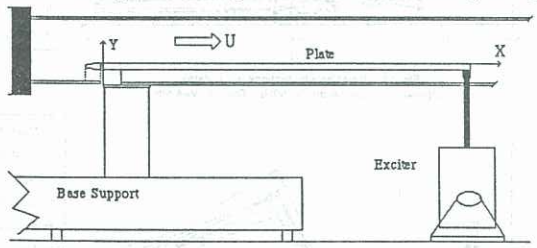
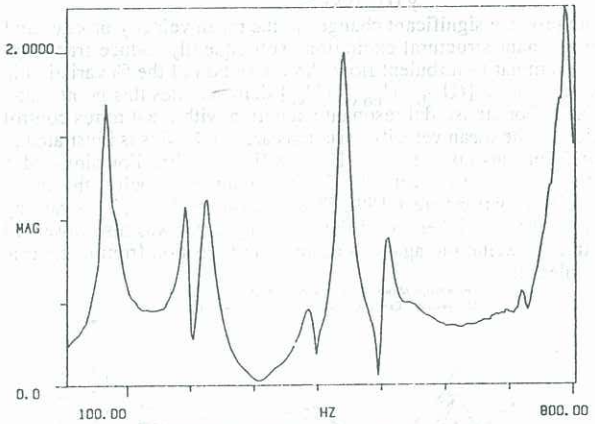


Fig 1. Schematic illustration of experimental set up

Mean velocities and velocity fluctuations in the boundary layer were measured with a TSI miniature hot film (1261A-10). The probe was traversed across the test section (transversely and longitudinally) and positioned with a digital height gauge. The distance that the probe could be moved in the boundary layer could be determined to an accuracy of 0.02 mm. Miniature B&K accelerometers (type 4374) were mounted at suitable locations on the bottom side of the plate to measure the transverse plate velocity. The plate was excited with a B&K Exciter System (types 4801,4812, 2707, 1047) and a random noise generator. A Data 6000 Waveform Analyser was used to process the data together with an HP 86B micro-computer. During sinusoidal excitation, the fluctuating signals from the hot-film anemometer and the accelerometers were phase averaged by using the excitation signal as a trigger in the acquisition of the time records - this allowed for the measurement of components which had a constant relative phase to the excitation signal (the rms quantities evaluated in a 5 Hz bandpass spectra around the excitation frequency were found to be nearly equal to the rms quantities evaluated from the phase averaged time records, indicating that the phase averaged time records were dominated by the components in the bandpass frequency).

A free stream velocity of 4 m/s was chosen for all the tests reported on here. At this velocity the boundary layer along the plate was laminar. Static pressure variations along the plate were measured to ensure that there were no strong pressure gradients. A small beneficial pressure gradient was, however, observed to be present. The experimental measurements (fluid velocity and plate vibrational velocity) were obtained along the centre line of the plate at distances of X = 298, 387, 479, 633 and 931 mm from the leading edge. During the sinusoidal excitation tests, the peak value of the plate velocity at the excitation point was kept constant with feed-back control by the B&K exciter controller at a chosen control velocity. Excitation frequencies of 60 Hz and 180 Hz were chosen for a control velocity of 4 mm/s (peak), and 20 Hz and 60 Hz were chosen for a control velocity of 5 mm/s (peak). The 60 Hz and the 180 Hz frequencies were in proximity to the first two longitudinal natural frequencies of the plate whilst the 20 Hz excitation was non-resonant. The plate natural frequencies can be identified from the transfer function (measurement point acceleration / drive point acceleration) at X = 633 mm which is illustrated in Fig 2. The two resonant frequency excitations (60 and 180 Hz) were outside the unstable region of Schlicting's[4] neutral stability curve, whilst the 20 Hz excitation was inside the unstable region for X > 388 mm. White noise (2 - 800 Hz) was used to excite the plate for the broadband experiments, and the average spectra for the fluid velocity fluctuations and the plate velocity were obtained.

Fig 2 Plate structural response transfer function at X = 633 mm



Discussion of Experimental Results

Mean Velocity Profiles Boundary layer profiles are presented in the form U/U_∞ vs $y\,(U_\infty/\nu\, x)^{1/2}$. Without vibrations, the flow (at 4 m/s) is laminar with a small beneficial pressure gradient, as illustrated in Fig 3. There is very little change in the mean velocity profiles when the plate is excited with a control velocity of 4 mm/s at 60 or 180 Hz, as illustrated in Fig 4. The maximum velocity variation is ~ 1% in relation to the free stream velocity. Sinusoidal resonant excitation does not appear to produce any significant change to the boundary layer profiles at this control velocity. With the larger control velocity (5 mm/s), there is little effect with non-resonant (~20 or 120 Hz) sinusoidal forced excitations, but the effects are somewhat more dramatic when the forced vibration coincides with a natural frequency. This is illustrated in Fig 5 for 60 Hz resonant excitation. Small variations in the vibrational level of the

Fig 3 B.L. Profiles for Different x, Without Vibration

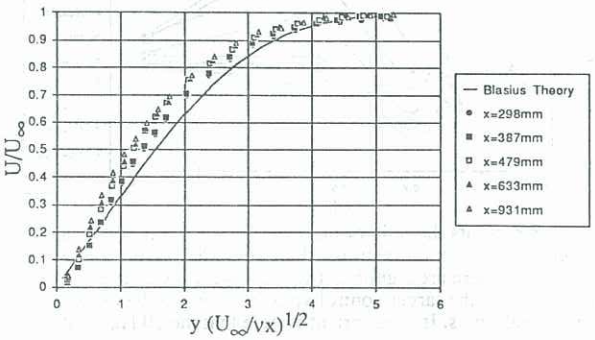


Fig 4 B.L. Profiles for Different x,
- U(fs)=4m/s, Excitation f=60 Hz, Control V=4mm/s

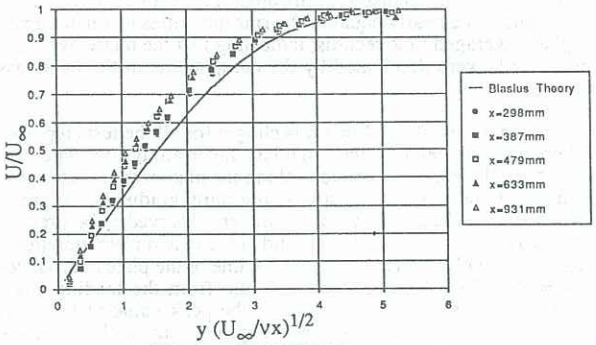


Fig 5 B.L. Profiles for Different x,
- U(fs)=4m/s, Excitation f=60Hz , Control V=5mm/s

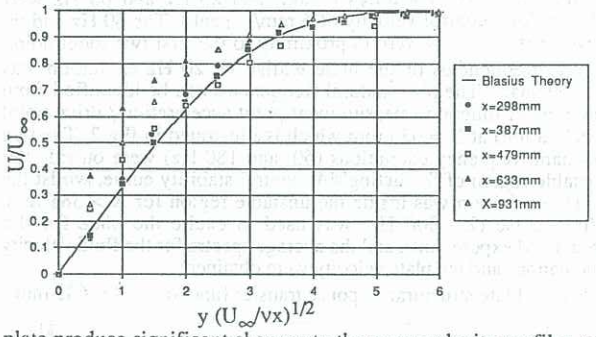


plate produce significant changes to the mean velocity profiles and the resonant structural excitations subsequently induce transition from laminar to turbulent flow. An inspection of the % variation in mean velocity $\{(U_{vib} - U_{no\ vib})/U_\infty\}$ demonstrates this point quite clearly. For sinusoidal resonant excitation with the 4 mm/s control velocity, the mean velocity variations are ± 1%. This is illustrated in Fig 6 for various values of $Nu = y\ (U_\infty/\nu\ x)^{1/2}$. For sinusoidal resonant excitation with the 5 mm/s control velocity, the mean velocity variations are ± 15%. This is illustrated in Fig 7 for varying Nu values. A thickening of the boundary layer was also observed with this excitation, again substantiating transition from laminar to turbulent flow.

Fig 6 Percentage Mean Velocity Variations [(U(v)-U(nv))/U(fs)],
- U(fs)=4m/s, Excitation f=60Hz, Control V=4mm/s

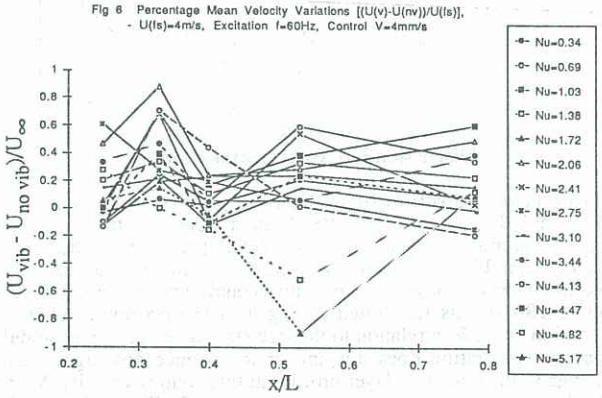


Fig 7 Percentage Mean Velocity Variation [(U(v)-U(nv))/U(fs)],
- U(fs)=4m/s, Excitation f=60Hz, Control V=5mm/s

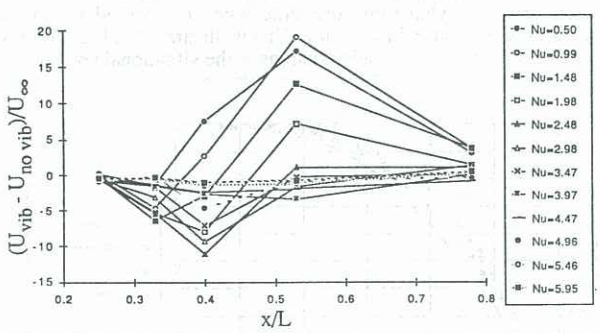
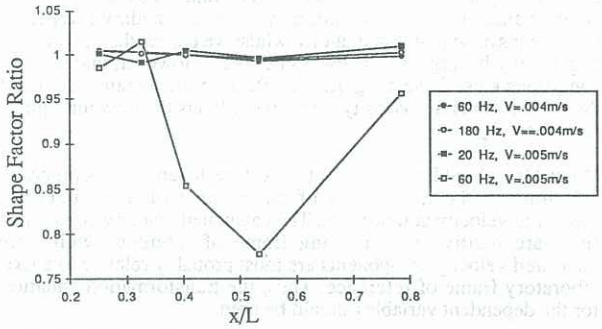


Figure 8 presents the ratio of the shape factor (ratio of displacement thickness (δ_1) to momentum thickness (δ_2)) with and without vibrations. There are significant variations for resonant excitation of the plate with the larger control velocity as would be expected with transitional flows. It is important to note that the 60 Hz excitation is outside the unstable region of the neutral stability curve [4], and that for local plate velocities of the order of 4 - 6 mm/s (control velocity of 4 mm/s), no instabilities or turbulent bursts could be observed. However, an increase in the local plate velocity level to ~ 10 mm/s (control velocity of 5 mm/s) produced turbulent bursts and subsequent transition to turbulent boundary layer flow. This observation suggests that linear stability theory cannot explain the finite disturbances produced by the surface oscillations. A simple analysis of a steady boundary layer not separating from a transversely oscillating surface revealed that a probe fixed in the laboratory reference frame would observe variations which are an order of magnitude smaller than are observed in the current experiments. This suggests that the observed mean velocity variations are dependent upon the vibrational magnitude, and are due to non-linear effects associated with fluctuations in the velocity field.

Fig 8 Ratio of Shape Factor with vibration to Shape Factor without vibration



Velocity Fluctuations The boundary layer velocity fluctuations are presented in the form u'_{rms}/U_{plate} vs x/L (u' is the rms velocity fluctuation, U_{plate} is the local rms plate velocity at position x, and L is the plate length) in Figures 9 -12 for a range of values of Nu. It is interesting to note that the fluctuations are all amplified with respect to the local plate velocity. The amplification is greatest for resonant vibrations with the larger control velocity of 5 mm/s (Fig 12), and the x-dependence of the fluctuations are also well correlated with the x-dependence of the plate velocities for this particular case. It is important to note that maxima can be observed in the boundary layer velocity fluctuations when the data (u'_{rms}/U_{plate}) is presented as a function of Nu at a specific spatial location on the plate. This point is illustrated in Fig 13 for 60 Hz (the fundamental natural frequency) excitation for both control velocities.

Fig 9 Velocity Fluctuations u'/U(plate),
- U(fs)=4m/s, Excitation f=60Hz, Control V=4mm/s

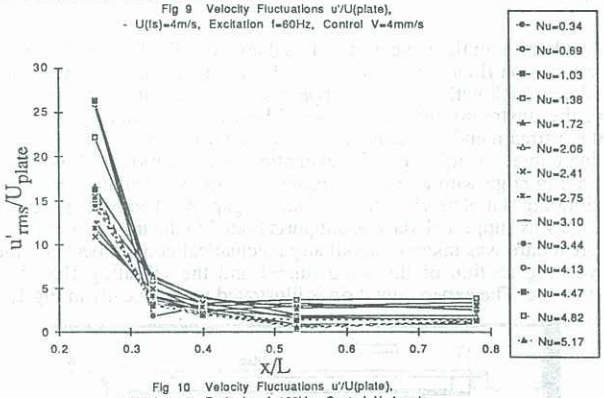
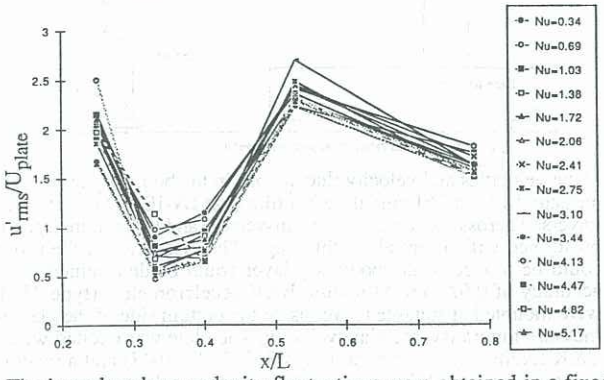


Fig 10 Velocity Fluctuations u'/U(plate),
- U(fs)=4m/s, Excitation f=180Hz, Control V=4mm/s



The boundary layer velocity fluctuations were obtained in a fixed, inertial reference frame. The streamwise velocity fluctuations observed by a probe in this reference frame are therefore made up of (i) a mean component which is time independent relative to the accelerating plate but time dependent to a fixed reference frame (this

Fig 11 Velocity Fluctuations u'/U(plate), - U(fs)=4m/s, Excitation f=20Hz, Control V=5mm/s

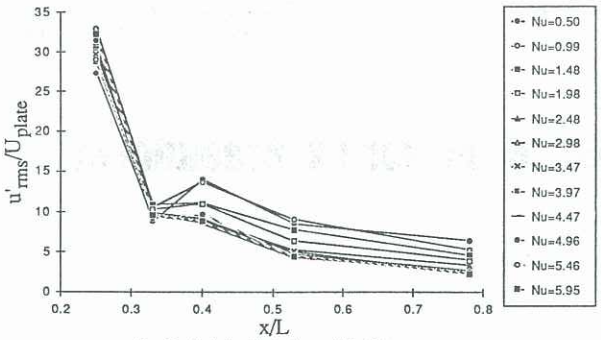


Fig 12 Velocity Fluctuations u'/U(plate), - U(fs)=4m/s, Excitation f=60Hz, Control V=5mm/s

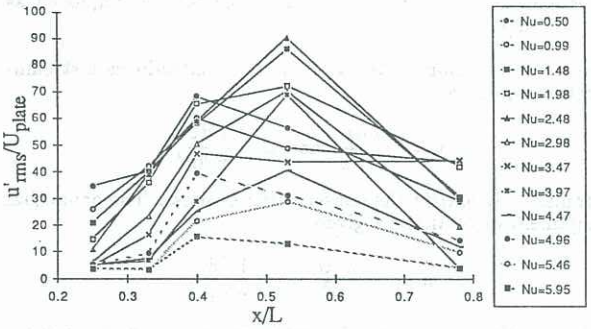
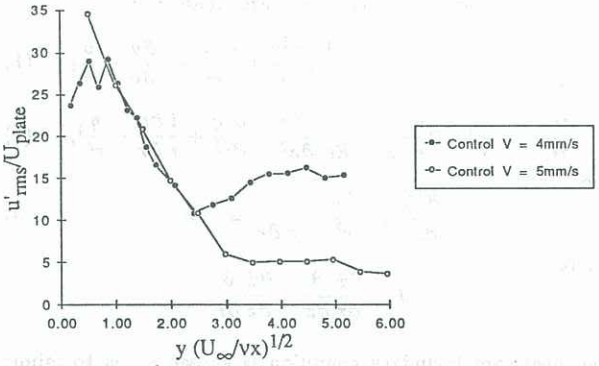


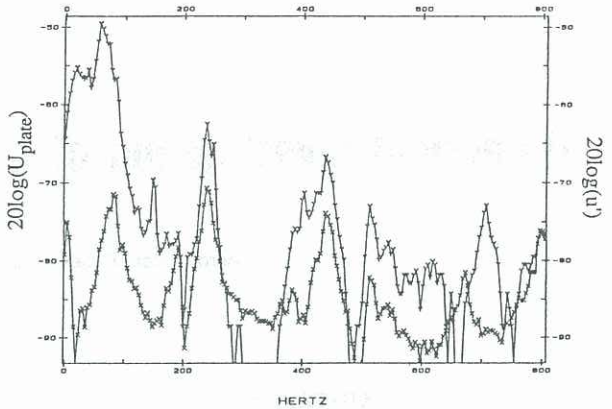
Fig 13 Velocity Fluctuations u'/U(plate) at X=298mm, - U(fs)=4m/s, Excitation f=60Hz



component can be approximated as a steady boundary layer which oscillates in contact with the surface without separation, when observed from a fixed frame of reference), (ii) additional velocity fluctuations (as observed from the fixed reference frame) which are induced in the flow and are a function of the plate vibrational amplitude. As for the mean velocity profiles, an analysis of the oscillating boundary layer illustrated that the velocity fluctuations due to (i) are generally an order of magnitude smaller than those observed, and that the vibrating plate significantly enhances the localised velocity fluctuations in the flow. It should be noted that the fluid velocity measurements were obtained with a single hot film probe and that the two orthogonal velocity components (U,V) were therefore not resolved (naturally the probe would respond to an effective velocity which is made up of both the streamwise and the normal components). Hence, this and the possible interaction between the streamwise convected velocity fluctuations and the locally induced velocity fluctuations account for correlation between the x-dependence of the fluctuations and the x-dependence of the plate velocity only being observed for the resonant vibrations with the larger control velocity of 5 mm/s. The results therefore suggest that for large resonant surface vibrations the induced velocity fluctuations are controlled by the local plate velocity, whereas at lower resonant and non-resonant excitations the velocity fluctuations are controlled both by the local plate velocity and the velocity fluctuations convected downstream by the mean flow.

When the plate is excited with band limited white noise (2-800 Hz), the natural frequencies within the band are excited. The experimental results demonstrate that the fluid is also excited at these frequencies, as is illustrated in Fig 14 - the velocity fluctuations have their largest components at the resonant frequencies, and the fluctuations are much larger than the plate velocity. This observation was true for all

Fig 14 Spectra of plate (-x-) and fluid velocity (-Y-) for X = 298 mm, Y = 0.11 mm (Nu = 0.1)



the measurement positions. Another important observation is that the amplification of the fluid velocity fluctuations is largest for the fundamental mode - this observation was also true for the case of sinusoidal excitation.

Conclusions

The main conclusions relating to the work reported on in this paper are:

1. This type of unsteady two-dimensional flow produces two additional non-dimensional groups. They are (i) St - the ratio of the local fluid acceleration to the convective acceleration, and (ii) ε - the ratio of the plate acceleration to the local fluid acceleration.
2. Variations in the mean boundary layer velocity profiles are only of the order of ±1% for resonant excitation with the lower control velocity of 4 mm/s (local plate velocities ~ 4-6 mm/s). For non-resonant excitation the variations are even smaller.
3. For the larger control velocity of 5 mm/s (local plate velocities ~ 10 mm/s), variations of the order of ±15% were observed in the mean boundary layer velocity profiles during resonant excitation - the boundary layer was changing to a transitional profile. The neutral stability curve, however, shows that the excitation is in a region where the disturbances are not amplified. It is therefore very important to note that linear stability theory cannot predict the observed phenomena.
4. The velocity fluctuations are generally much larger than the plate velocities. The amplifications are, as expected, dependent upon the magnitude of the plate velocity with the largest amplification being observed at the fundamental frequency.
5. At large local plate velocities there is good correlation between the x-dependence of the plate velocity and the resulting velocity fluctuations. This suggests that these large local plate velocities control the fluid velocity fluctuations. At the lower plate velocities, however, the correlation is not so apparent, suggesting that the fluid velocity fluctuations are due to a combination of local structural excitation and fluctuations convected with the mean flow.

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