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Nonlinear Water Waves Generated by a Moving Pressure Band

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ABSTRACT

Water waves generated by a moving pressure band is solved by a Boundary Element Technique. The free surface condition involves the use of complete unsteady Bernoulli Equation. As the computation grids are Lagrangian waves of large amplitudes and slopes can be computed. Numerical flow visualizations are presented to show the sequential development of streamline patterns for transient waves generated by the pressure band and the interaction of these waves with stepped channel floor. The effects of nonlinearity are illustrated by mode of unsteady flow and the smooth transition from low to high Froude number.

1 INTRODUCTION

The linearized problem for waves generated by a travelling disturbance have been solved long ago and essential works were reported by Lamb (1945). A consistent second order analysis was given by Tuck (1965) in which he solved for the steady state water wave generated by a submerged cylinder. The analysis was carried out using series expansion of the free surface equation so that the application of free surface boundary condition was transferred from the surface of the wave to the calm free surface. This imposed a restriction on the solutions and was shown by the appearance of singularities in the resulting stream function.

Mathematical solutions of large amplitude wave are restricted to cases where the wave profiles and wall boundaries are relatively simple. One example is the solution for solitary wave. For more complex problems one has to seek solution from numerical technique. This is not to imply that the numerical solution of fully nonlinear wave is without difficulties of its own: The large movements of the free surface boundary make the traditional Finite Element and Finite Difference techniques very difficult to applied. There are numerical instabilities which are caused by the absence of viscosity. The stability criterion associated with the absence of diffusion term has been illustrated by Hirt (1966). On the other hand the free surface itself is also the source of numerical instability. As the result, some forms of smoothing technique have been employed to maintain stability.

The solution of nonlinear waves generated by a submerged cylinder was carried out numerically by Haussling and Coleman (1979). The technique used was a finite difference scheme incorporating the boundary-fitted technique to handle large deformation of free surface. Their results showed that the wave amplitude of nonlinear waves were significantly higher than that of the linear wave and consequently higher lift and drag than the linear theory were predicted.

The differences between linear and nonlinear waves should also be examined with respect to the Froude number of the source. There are three characteristic lengths associated with this type of flow: the depth of the channel, the depth of immersion and the length of the body. A more simple system in which the depth of immersion is absence is that of a moving pressure band.

Consider the the pressure band travels at Froude number equals to 1. In the frame work of the linearized theory the wave generated by the pressure can be decomposed by way of a Fourier Series. Thus one sees the dispersion for waves of short wave length. The long wave will have a velocity approximately equal to the square root of gravity times the depth of the channel. In other words these long waves travel with the pressure band. The result is the building up of the wave amplitude ahead of the pressure band until the wave amplitudes grow beyond the validity of linearized wave theory.

The analysis of the above waves system in the frame work of nonlinear wave theory will bring in the effect of amplitude. That is the velocity of the wave is now dependent on the amplitude and the shape of the wave. It can no longer be analysed by Fourier Series. It seems that nonlinear wave of finite amplitude has to be studied in reference of the source in which the waves are generated. It is envisage that nonlinear wave theory will give further information on the build-up of wave amplitudes ahead of the pressure band when the pressure is traveled at Froude number around unity.

The aim of this paper is to carry out numerical experimantations on the waves generated by a moving pressure band for different Froude numbers. The change of Froude number is brought about by a different velocity of the pressure band and by the variation of depth in an irregular channel.

2 GOVERNING EQUATIONS AND NUMERICAL TECHNIQUE

Consider the generation of plane water waves in a channel of finite depth by a pressure distribution in the atmosphere. The variables discussed here are nondimensionalized so that the gravity, the depth of the flow channel and the density of water are made unities.

2.1 Equation of Motions

This potential flow problem can be expressed by the values of the enclosing boundaries. In this problem the free surface S and the channel floor B enclosed the domain of the fluid flow. In this formation the free surface is represented by a vortex sheet and doublet distribution are placed on the channel floor. The expression for the velocity induced by the presence of vortex sheet and doublet distribution is given by a boundary integral equation where the integrations are carried out along the free surface and the channel floor.

$$u - iv = \frac{1}{2\pi i}\int_S \frac{\gamma_1 ds_1}{z - z_1} + \frac{1}{2\pi i}\int_B \frac{\mu_1 \exp(i\theta_1)\, ds_1}{(z - z_1)^2} \tag{1}$$

where

S represents the line integration along the free surface and

B represents the line integration along the wall boundary.

u and v are the horizontal and vertical components of the velocity vector;

z is the complex coordinates;

the subscript 1 is associate with the dummy variables in the integration;

$\gamma 1$ is the vorticity on the sheet;

$\mu 1$ is the doublet density on the rigid boundary;

s1 is the length measured along the line of integration;

z1 is the coordinates of a point on the sheet;

S represents the line integration along the free surface and

B represents the line integration along the wall boundary.

Let uj + ivj be the velocity on the wetted side of these boundaries. According to Plemej's formulae, this velocity is defined by:

$$u_j - iv_j = \frac{1}{2\pi i}\int_S \frac{\gamma_1 ds_1}{z - z_1} - \tfrac{1}{2}\gamma_j \exp(i\theta) + \frac{1}{2\pi i}\int_B \frac{\mu_1 \exp(i\theta_1)\, ds_1}{(z - z_1)^2}$$

on the free surface, and

$$u_j - iv_j = \frac{1}{2\pi i}\int_S \frac{\gamma_1 ds_1}{z - z_1} + \frac{1}{2\pi i}\int_B \frac{\mu_1 \exp(i\theta)\, ds_1}{(z - z_1)^2} + \tfrac{1}{2}\frac{\partial \mu_j}{\partial z} \tag{2}$$

on the wall boundary

There there two type of boundary conditons: On the free surface the velocity component tangential to the free surface inclined at angle θ is expressed as the derivative of the velocity potential ϕ with respect to the arc length of the free surface, that is:

$$\frac{\partial \phi_j}{\partial s} = u_j \cos\theta_j + v_j \sin\theta_j \tag{3}$$

The no flow free slip boundary condition is applied to the channel floor. This implies that the normal velocity component is zero, thus:

$$0 = u_j \sin\theta_j - v_j \cos\theta_j \tag{4}$$

The dynamics of the free surface flow is governed by the unsteac Bernoulli equation. Apply this to a fluid particle on the free surface the Lagragian Bernoulli Equation is given by:

$$\left[\frac{d\phi_j}{dt}\right] = \tfrac{1}{2}(u_j^2 + v_j^2) - \eta - p(x,t) \tag{5}$$

where p(x,t) is the given pressure distribution.

2.2 Outline of Solution

The principal variables are the velocity potential ϕ, the vorticity γ, the doublet strength μ and the coordinates of the free surface z (= x + iη). Equations (2), (3), (4) and (5) are a set of simultaneous equations which will be solved numerically as an initial value problem. The solution begins by the time integration of equation (5) to give the values of ϕ in the next time step. The new positions of fluid markers on the free surface are also computed according to their velocities. This advance of time will be followed by the evaluation of γ and μ. These variables are related with the velocity vector as expressed in equation (2) and is therefore implicit in equations (3) and (4). The discretization of the integral equation in (2) and its substitution into equations (3) and (4) will result in a set of linear equations which can be readily solved. Detail of the technique is given by Soh (1985).

3 SOME OBSERVATIONS OF COMPUTED RESULTS.

3.1 Format for Figures.

All figures are plotted with the abscissa on the channel floor. The calm free surface will a line one unit above the abscissa. The locations for the pressure distributions are represented by shaded areas above the free surface. For convenient of presentation, the horizontal scales are exaggerated.

3.2 Stationary Pressure Band

For a stationary pressure band which has a smooth pressure distribution, the hydrostatics solution for the depression of the free surface is given by,

$$y = 1 - p(x). \tag{6}$$

However for a stepped pressure distribution, the steep pressure gradient at the ends of the pressure band will cause instability on the free surface in these regions. Consider a pressure band of 10 unit width and has a constant pressure distribution of unity is suddenly imposted on the otherwise calm free surface. The movements of the free surface is most severe at the edge of the pressure band where the pressure gradient is the highest. Thus the depression started at the edge of the pressure band. Concurrently, wave crests have been generated just outside of the depression and beginning to travel away from the pressure band. At 4 unit of time, as shown in figure 1 the free surface in these regions has become vertical. Subsequently in 5 unit of time the instability of the flow has caused the wall of water to fall into

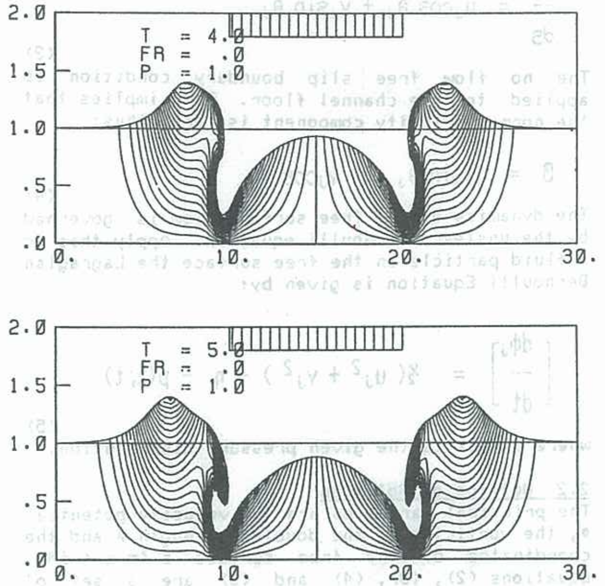

Figure 1

the cavity. This process will generate eddies and form entrapped bubbles in water. The flow is highly turbulent even in the present ideal environment where the effect of the air jet which usually associate with this type of pressure distribution has been neglected. The continue overturning of the free surface near the edges of the pressure band has prevented the depressed free surface from obtaining a hydrostatics profile as indicated in equation (6). It is to note that the streamline patterns do not indicate the directions of the flow in the same way as the case of a steady state. For example the point of inflexion found at (9.0, 0.9) and (21.0, 0.9) are not stagnation points.

3.3 Moving Pressure Band.

The effect of Froude number can be demonstrated by allowing the pressure band to move along the free surface at different velocity. Consider a pressure band of 5 unit width and has a moderate pressure coefficient of 0.2 moving toward the right. The velocity corresponds to Froude number equal to one, as shown in figure 2. A transient bow wave which has an amplitude of 0.59 unit and travel ahead of the pressure at Froude number equals to 1.21, that is 0.21 relative to the pressure band. For a correspond solitary wave with this amplitude it velocity has to be 1.29. One would expect that this bow wave will eventually achieve the form and velocity of a solitary wave. A transient stern wave of much smaller amplitude of 0.05 is found moving in the opposite direction.

This above phenomenon occures for the velocity of the pressure band below unity. For higher velocity, it takes longer for the transient bow wave to develop. For example it occurs at about 40.0 unit time for a velocity of one unit, this has increases to 60.0 and 80.0 for the velocities of 1.1 and 1.2 respectively. So far no transient bow wave has been computed at the velocity of 1.5. At this velocity the maximum crest height is maintained at a low value of 0.14. It is possible that an asymptote has been reached and that the transient bow wave will no longer exist.

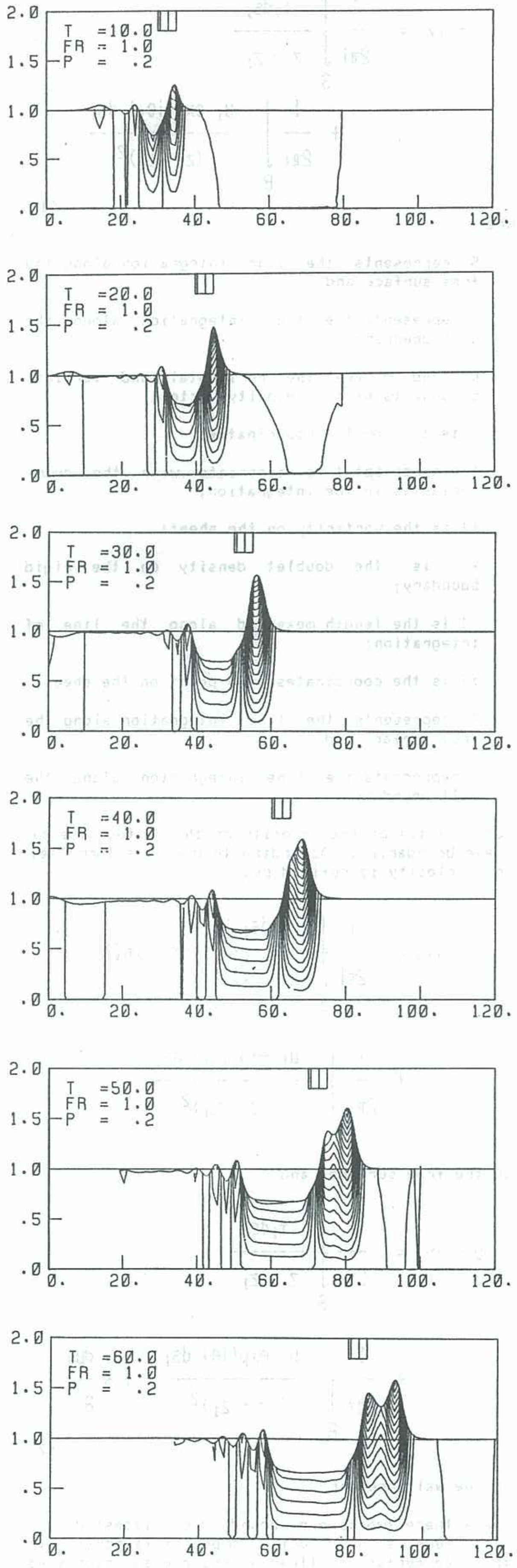

Figure 2

The height of the maximum wave crest for various velocities are tabulated in table 1. The digits appear for velocities 1.3 and 1.4 are vetical ruled to indicate that they are the values obtained so far and have not reached steady state values.

Table 1 Maximum wave height.

Velocity	Amplitude
0.5	0.22
1.0	0.59
1.1	0.70
1.2	0.78
1.3	0.37
1.4	0.21
1.5	0.14

3.4 Stepped Channel Floor.

The change of Froude number could be brought about by the change in channel depth. Thus a pressure band travelling with Froude number of 0.5 will reach 1.0 when the depth of the channel floor is raised so as to reduce the depth by half. The transition from Froude number of 0.5 to 1.0 happens very quickly as shown in figure 3. The mode of separation of the leading crest when the pressure bend is over the raised channel floor is identical to that of figure 2. It seems that the flow is heavily dependent on the local depth as the case of one dimensional channel flow. The influence of the trailing waves, to the free surface under the pressure band is at most secondary. This is reflected in the Boundary Integral Equation that, for any distance over the length of the depth of the channel, the influence of waves surrounding a point in question is inversely proportional to the square of the distance.

As the pressure band moves away from the raised channel floor, the increase in the depth of water causes the wave to accelerate. This results in the generation of a transient bow wave moving faster than the pressure band.

4 SOME REMARKS ON NUMERICAL EXPERIMENTATION.

The unsteady flow pattern for the case of stationary pressure band has demonstrated that steady state condition in nonlinear wave may not exist. This is also to be expected for moving pressure band which has a pressure coefficient greater than one. The transient characteristic of the waves generated by a moving pressure band of moderate pressure coefficient of 0.2 has a dominant transient bow wave that move ahead of the pressure. The time scale for this transient to develop is proportional to the velocity of the pressure band. The asymptot in which this transient bow wave cease to exist is found to be about the velocity of 1.5. More computing effort is needed to establish a more accurate limit.

Although the technique is capable to compute for the overturning of waves, as in the case of stationary pressure band, no overturning of wave are observed from the moving pressure bands. The transition of Froude number from low to high values is relatively smooth. This is a feature that cannot be visualized from linear theory. For the case of stepped channel, the wave characteristics is mainly influenced by the local depth of the channel. The net effect by the raised platform is the generation of a transient bow wave moving ahead of the pressure band and this results in accelerating the propagation of wave energy.

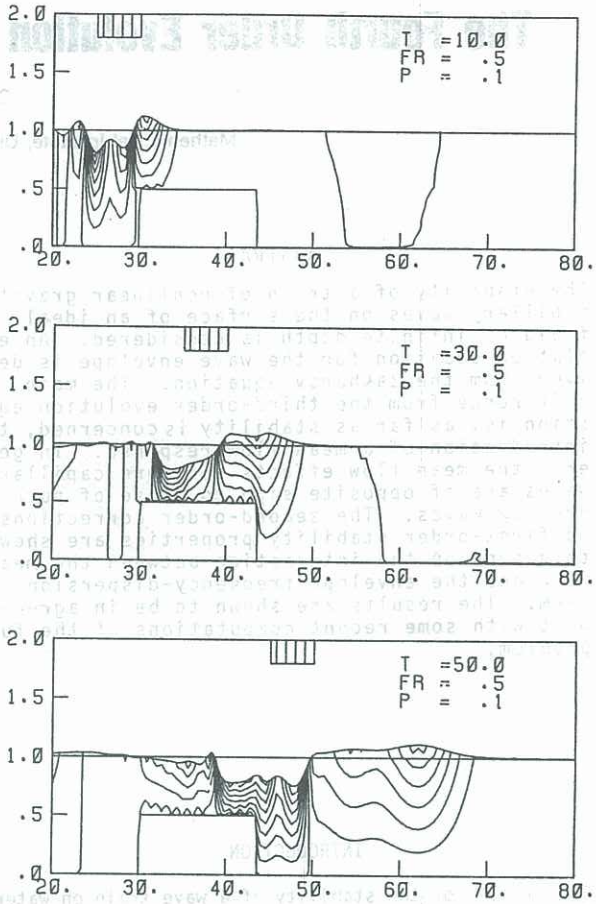

Figure 3

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