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Reynolds Number Dependence of a Turbulent Duct Flow

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ABSTRACT

Different statistics of the longitudinal velocity fluctuation have been measured in the outer region of a fully developed turbulent duct flow over a large (two-decade) Reynolds number range. Whereas high order moments and spectra continue to evolve slowly at the highest Reynolds numbers, the mean VITA period and VITA ensemble averages, when scaled on outer variables, are approximately constant provided the Reynolds number is sufficiently large. This proviso has to be taken into account when studying the scaling of the bursting frequency in the near-wall region.

INTRODUCTION

In an earlier study (Shah et al, 1984) aimed at finding the relevant scaling parameters for the average "bursting" frequency in a fully developed turbulent duct flow, it became evident that the range of the Reynolds number $R (= U_0 d/\nu$, where U_0 is the centreline velocity, d is the duct half-width and ν is the kinematic viscosity) had to be sufficiently high. In particular, the inclusion of data at values of R too small for similarity (e.g. Townsend, 1976, p.53) to be achieved in the outer layer made it difficult to interpret bursting frequency measurements in the near-wall region [y^+ ($\equiv yU_\tau/\nu$) < 50 where y is the distance from the wall and U_τ is the friction velocity]. Since conclusions regarding the scaling of the bursting frequency have usually been based on relatively small Reynolds number ranges (e.g. Blackwelder and Haritonidis, 1983; Chambers et al, 1983) it seemed important to first establish the Reynolds number behaviour of the outer layer.

The use of two different ducts, one at the University of Newcastle and the other at the University of Grenoble, made it possible to achieve a two-decade (3×10^3 to 3×10^5) variation in the Reynolds number. Measurements of the longitudinal velocity fluctuation u were made at $y/d = 0.5$. We present results for moments and spectra of u, conditional VITA (variable interval time averaging) averages (e.g. Blackwelder and Kaplan, 1976) of u, the mean VITA period of u and the mean zero-crossing period of u. VITA was considered because it is a technique that has been often used (e.g. Blackwelder and Haritonidis, 1983; Chambers et al, 1983; Andreopoulos et al, 1984) to elucidate the scaling for the mean bursting period. The zero-crossing period is considered here because of : [i] the physical importance attached to this quantity by Badri Narayanan et al (1977), [ii] the conflicting experimental trends in the literature (e.g. Badri Narayanan et al, 1977; Sreenivasan and Antonia, 1977; Sreenivasan et al, 1984), and [iii] the possible implication with regard to the behaviour of the zero-crossing period of the wall shear stress fluctuation.

EXPERIMENTAL SET UP

The open return blower tunnel used for the duct flow investigation has been described in detail (Shah et al, 1984) and will not be repeated here. The inlet boundary layers were tripped at x = 40 mm (x is in the flow direction) using 1.6 mm dia. rods attached to the walls and spanning the full height of the duct. Previous hot wire measurements (Shah et al, 1984) indicated that the flow is fully developed at the measurement station (x = 215d) over the range $3 \times 10^3 \le R \le 3 \times 10^4$. Fully developed flow conditions were established on the basis of two main criteria. First, a linear decrease, with respect to x, of the static pressure was found for $x/d \gtrsim 85$ and all values of R. Secondly, normalised moments of u ($\overline{u^n}/u'^n$, where prime denotes the rms value) of order as high as eight, were essentially constant near the duct centreline for $x/d \gtrsim 150$ and all values of R.

Measurements were also carried out in an open return wind tunnel, originally used by Comte-Bellot (1963), at the Institut de Mécanique, University of Grenoble. The modified tunnel contains a distortion section that leads into an 8 m long, 2.4 m high and 0.18 m wide duct. Wall roughness elements, consisting of 9 lengths of 6 mm × 6 mm timber separated from each other by 50 mm, were attached to one wall in the inlet section of the channel and the measurements were made at 6 m from the last element. The roughness elements gave an equivalent x/d of about 100 at the measurement station and the flow was fully developed for the range $2 \times 10^4 \lesssim R \lesssim 3 \times 10^5$ (Gagne, 1980).

The measurements of u were made at $y/d \simeq 0.5$, using a Wollaston (Pt) hot wire (1.3 μm dia. and 0.25 mm length) operated with a constant temperature anemometer at an overheat ratio of 0.8. The output from the anemometer was conditioned, using a buck and gain circuit and low-pass filtered at the Kolmogorov frequency f_η [$= \overline{U}/2\pi\eta$, where $\overline{U}$ is the local mean velocity and $\eta \equiv (\nu^3/\overline{\varepsilon})^{\frac{1}{4}}$ is the Kolmogorov length scale; isotropy was used to estimate $\overline{\varepsilon}$]. The filtered signal was digitised at a sampling frequency of $2f_\eta$, of approximately 60 s duration, using an 11 bit plus sign A/D converter for the Newcastle data and a 14 bit plus sign A/D converter for the Grenoble data. The time series for the fluctuating voltage was converted into a time series for u on a PDP 11/34 computer using constants obtained from a velocity calibration on the duct centreline. Processing of the data was carried out using the PDP 11/34 and a VAX 11/780 computer.

DEPENDENCE OF CONVENTIONAL STATISTICS OF u ON R

Mean velocity profiles measured (Shah et al, 1984) over the range $3 \times 10^3 < R < 3 \times 10^4$ indicated a substantial logarithmic region of the form $\overline{U}^+ = A \log y^+ + B$ where $\overline{U}^+ = \overline{U}/U_\tau$ and A and B are log-law constants. The variation in A (5.6-5.8) and B (4.9-5.1) are of the same order as reported in other studies (e.g. Dean, 1978; Johansson and Alfredsson, 1982) and do not indicate a systematic dependence on R. The c_f values, obtained using the Preston tube, over the range $3 \times 10^3 < R < 3 \times 10^5$ agree to within ±4% with the values obtained using the empirical relation of Dean (1978), modified to suit the present definition of R, viz. $c_f = 0.0397\ R^{-0.23}$.

The dependence of R on bulk flow parameters such as the shape factor H ($\equiv \delta^*/\theta$, δ^* is the displacement thickness and θ is the momentum thickness) and the maximum velocity defect ratio $\Delta\overline{U}^+_{max}$ (estimated from the mean velocity profiles) was examined for the range $3 \times 10^3 < R < 3 \times 10^4$. The shape factor H decreased from 1.57 to 1.37 over this range and is in good agreement with the variation of H reported in Dean's (1978) survey. The present results and those of Dean indicate that H is a slowly varying function of R and will asymptote only at very high R.

A systematic decrease was observed in $\Delta\overline{U}^+_{max}$ with R for R < 7000. In particular, a value of 0.5 for $\Delta\overline{U}^+_{max}$ obtained for $R \simeq 3300$ was significantly lower than 0.85 ±0.05 obtained for $R \gtrsim 7000$. This is consistent with Dean's (1978) conclusion that $\Delta\overline{U}^+_{max}$ becomes constant

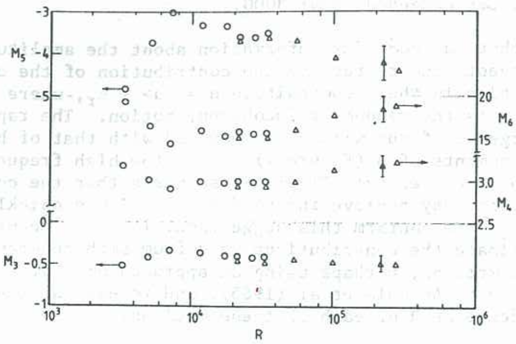

Figure 1 Normalised higher order moments of u. o, Newcastle duct; Δ, Grenoble duct. The maximum experimental uncertainty is indicated by I.

($\simeq$ 0.85) for R $\gtrsim$ 6000.

The distribution for u'/U_0 (not shown here) exhibits a very weak dependence on R (0.065-0.045) for the range $3 \times 10^3 < R < 3 \times 10^5$. The normalised high order moments of u (Figure 1) indicate, for R $\lesssim$ 7000, an increase in odd moments (M_3 and M_5) and a corresponding decrease in even moments (M_4 and M_6). For R > 7000, M_3 and M_5 decrease slowly with R whereas M_4 and M_6 increase slowly. The observed trends for M_3 and M_4 are consistent with the results of Comte-Bellot (1963) for the range $6.3 \times 10^4 < R < 2.6 \times 10^5$. Apart from the good overlap between the Newcastle and Grenoble data, Figure 1 indicates that high order moments continue to evolve slowly for the range of R considered.

Three different non-dimensional plots were used to examine the influence of the Reynolds number on $\phi(f)$, the spectral density of u defined such that

$$\int_0^\infty \phi(f)\, df = 1 \quad .$$

Results obtained for each of these are briefly mentioned below although a plot is shown for only one normalisation.

(i) Semi-log plots of $(u'^2/U_0^2)\ \omega\ \phi$ vs ω (= $2\pi fd/U_0$) highlight the relative contributions of different frequencies to (u'^2/U_0^2). We found a weak but systematic dependence on R for the energy containing part of the spectrum up to the highest R.

(ii) The log-log presentation of Figure 2 highlights the increase in the extent of the inertial range with increasing R and the continuing change of the high frequency part of the spectrum as R increases. Also evident in Figure 2 is the good overlap of the Newcastle and Grenoble data.

(iii) When ϕ is plotted vs f/f_η, the high frequency part of ϕ exhibits practically no dependence on R. A similar observation was made by Sreenivasan (1985) using data for boundary layer, pipe and grid flows. Strictly, one would expect the high frequency part of the Kolmogorov normalised spectrum to evolve slowly in a manner which reflects the small changes in high order moments of velocity derivatives (e.g. Champagne, 1978). It is possible however, that the experimental uncertainty is such as to mask any weak dependence of the high frequency part of the Kolmogorov normalised spectrum.

VITA FREQUENCY AND CONDITIONAL AVERAGE

VITA, as used here, is essentially as described by Blackwelder and Kaplan (1976). Briefly, VITA events are identified at times τ_n when the conditions $\dot{u} > 0$ and $\widetilde{u^2} - \tilde{u}^2 > ku'^2$ are satisfied. The dot denotes differentiation with respect to time and the tilde denotes averaging over a time interval T, such that

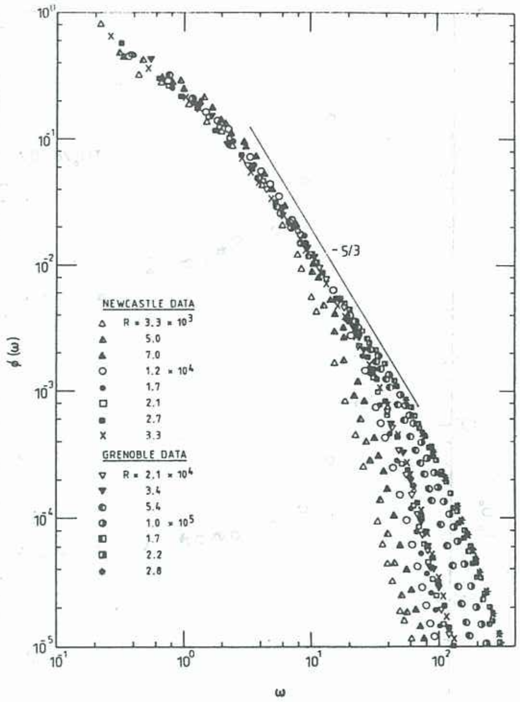

Figure 2 Reynolds number variation of the normalised u spectrum.

$$\tilde{u}(t,T) = T^{-1} \int_{t-T/2}^{t+T/2} u(s)\, ds \quad .$$

The exponential variation of VITA frequency f_b ($\equiv T_b^{-1}$) with k as in Blackwelder and Kaplan (1976) was verified. Since the main interest is in the behaviour of f_b with R, the choice of k is not too critical (Blackwelder and Haritonidis, 1983). We have used k = 1, the value which appears to have been widely used in the literature (e.g. Johansson and Alfredsson, 1982; Blackwelder and Haritonidis, 1983; Shah et al, 1984). We also considered a wide range of values of T, as suggested by Johansson and Alfredsson (1982).

The variation of $T_b U_0/d$ with R is presented in Figure 3 for several values of T. It is evident from this figure that the selection of T plays an important role and it is difficult to define a precise value of R at which $T_b U_0/d$ cease to depend on R. The dependence on R is more pronounced for smaller values of T, as for the water duct data of Alfredsson and Johansson (1984) for the range $6.9 \times 10^3 < R < 6.1 \times 10^4$. The increased dependence on R of $T_b U_0/d$ for smaller values of T is in qualitative agreement with the dependence of the high frequency part of the spectrum on R (Figure 2) since T is directly related to the duration of events (Alfredsson and Johansson, 1984).

Conditional averages of u, denoted by angular brackets, were obtained, subsequent to the identification of τ_n, using

$$\langle u(t) \rangle = \frac{1}{N} \sum_{n=1}^{N} u(\tau_n + t)$$

where N is the total number of detections.

Distributions of $\langle u \rangle$, obtained for a particular value of T, are shown in Figure 4. The value of N used to obtain the conditional averages varied from 100 to 1000, depending on R. These averages quickly become independent of R and, to avoid confusion, only a few distributions are shown in the figure. The shape and magnitude of $\langle u \rangle$ at R $\simeq$ 3300 are significantly different to those at higher

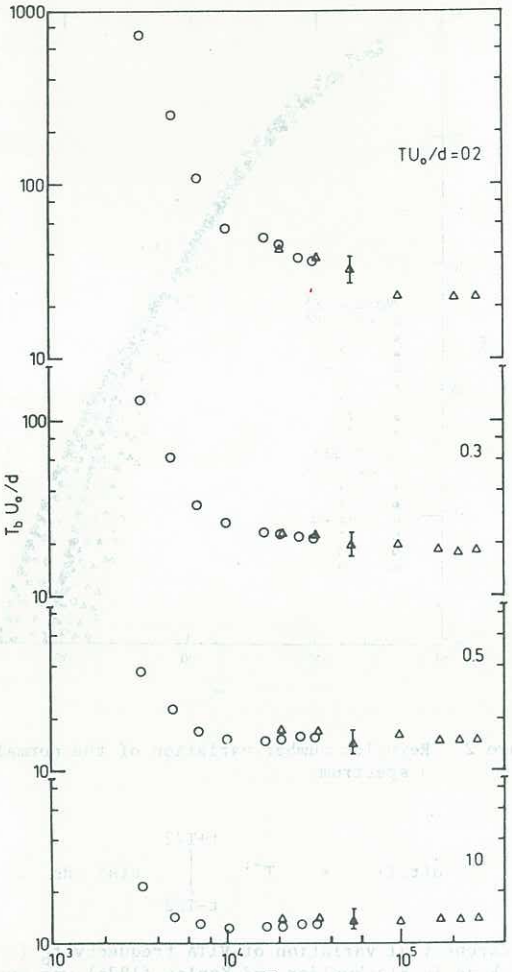

Figure 3 Reynolds number variation of the VITA period for different values of TU_0/d. o, Newcastle duct; Δ, Grenoble duct. The maximum experimental uncertainty is indicated by I.

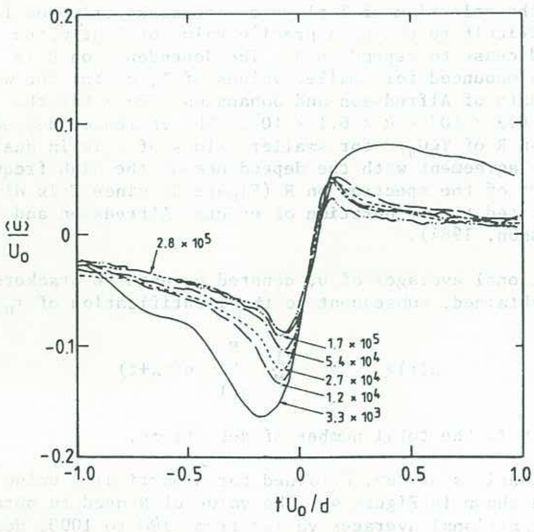

Figure 4 VITA averages of u for $TU_0/d = 0.3$.

Reynolds numbers. In the outer region of a boundary layer Antonia et al (1982) found that <u> became independent of the momentum thickness Reynolds number when the latter exceeded about 3000.

Note that <u> contains information about the amplitude of VITA events and represents the contribution of the coherent motion in the decomposition $u = \langle u \rangle + u_r$, where u_r represents the random or incoherent motion. The rapid convergence of <u> with R as compared with that of high order moments of u (Figure 1) and of the high frequency part of the spectrum (Figure 2) suggests that the coherent motion may achieve independence on R more quickly than u_r. To confirm this suggestion, it would be useful to estimate the contributions to u from both coherent and random motions, perhaps using an approach similar to that outlined by Antonia et al (1985), and to examine the dependence on R of each of these motions.

ZERO-CROSSING PERIOD

Badri Narayanan et al (1977) found that the zero-crossing length scale Λ (= $\overline{U}T_0/\pi$, where T_0 is the zero-crossing period) was 3 to 5 times the Taylor microscale λ (= $\overline{U}\, u'/\dot{u}'$) in several turbulent shear flows. In contrast, Sreenivasan et al (1984) found that $\Lambda \simeq \lambda$. They also pointed out that various sources of error, such as the signal dynamic range, discriminator characteristics, filter frequency and noise contamination may have affected the result of Badri Narayanan et al. These error sources were taken into account in the present experiments (details are given in Shah and Antonia, 1986) and the results for the zero-crossing period of u for both the Newcastle and Grenoble ducts are shown in Figure 5. Results for the zero-crossing period of the wall shear stress fluctuation in the present experiment (Newcastle duct only) and in the duct experiment of Sreenivasan and Antonia (1977) are included in Figure 5.

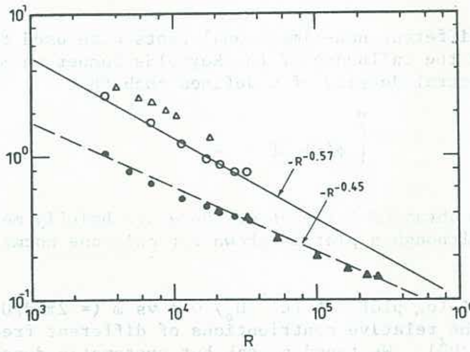

Figure 5 Reynolds number variation of the zero-crossing period T_0 for u and the wall shear stress fluctuation.
u (at y/d = 0.5): •, Newcastle duct; ▲, Grenoble duct.
wall shear stress: o, Newcastle duct; Δ, Sreenivasan and Antonia (1977).

The present data indicate that $\Lambda \simeq \lambda$ to within ±10% at all values of R. The good overlap between the Newcastle and Grenoble data is evident in Figure 5. The results at y/d = 0.5 suggest that $T_0U_0/d \sim R^{-0.45}$. This dependence is consistent with $\lambda \sim R^{-0.5}$ obtained on the assumption that the production and dissipation of turbulent energy are approximately in balance. The equality $\Lambda \simeq \lambda$ was also satisfied by the wall shear stress fluctuation and the $R^{-0.57}$ variation is reflected in the earlier data of Sreenivasan and Antonia (1977). The previous results corroborate Sreenivasan et al's (1984) conclusion that Λ contains no more information than λ at least in wall-bounded shear flows.

CONCLUSIONS

Normalised high order moments of u and the high frequency end of the u spectrum change slowly but systematically over a two-decade variation in Reynolds number. The mean zero-crossing period varies more markedly with Reynolds number but this variation reflects the equality between the zero-crossing length scale and the Taylor microscale.

The Reynolds number variation of the VITA period T_b (Figure 3) exhibits a similar trend to that obtained in the near-wall study of Shah et al (1984). This similarity, irrespective of position in the flow, is consistent with the existence of spatially coherent structures, possibly in the form of vortex loops or hairpin vortices, which are stretched across the shear layer.

The present evidence suggests that low Reynolds number effects observed in the outer layer cannot be ignored when enquiring into the scaling of the bursting frequency in the near-wall region.

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