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Air Entrainment at Spillway Aerators

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SUMMARY

The mechanism of air entrainment into a spillway aerator is discussed and a method for computing the air flow in the duct below the aerator is presented.

INTRODUCTION

When water flows over a long, ungated spillway there are several flow regions (Wood, 1985)(fig. 1). Initially the boundary layer from the spillway grows toward the free surface. In this region the flow above the boundary layer can be treated as irrotational. The outer edge of the boundary layer is irregular and when this outer region and its turbulence reaches the free surface air entrainment commences. Downstream of this point the air-water mixture slowly spreads until it reaches the spillway floor. For high head dams this point is a long way from the dam crest and the velocities next to the spillway surface may be sufficiently large to cause cavitation. This cavitation is prevented if small quantities of air are in the layers next to the spillway surface and for this reason cavitation does not occur downstream of this point. (fig. 1) Upstream of this point air may be introduced artificially by aerators (fig. 2). These aerators normally consist of a ramp, an offset in the spillway surface, a main duct and the spillway side duct. The drag from the underside of the nappe induces an airflow through the main duct and aerates the layers next to the spillway surface.

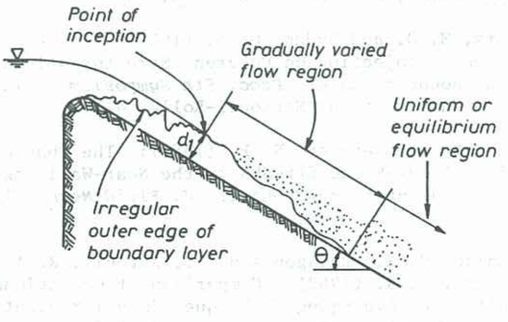

Figure 1 - The region of the flow on the spillway.

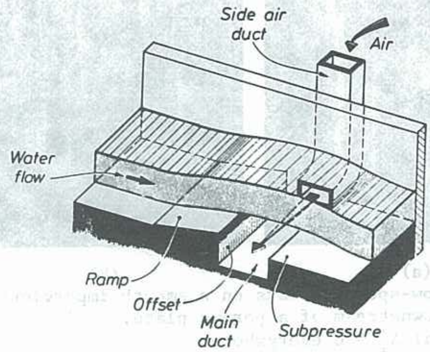

Figure 2 - An aerator.

Aerator Design

At the present time aerator design proceeds with a model of a section of the spillway and the relevant parameters for these studies are:

(1) The geometry of the inlet duct (fig. 2)
This determines the head loss in the flow from the inlet duct to the main duct or the area under the nappe.

(2) The geometry of the aerator and the spillway (fig. 2)
that is: θ, the spillway slope; ϕ, the inclination of the ramp with reference to the spillway floor; ℓ_r[m], the ramp length; t_s[m], the step or offset height; k_s[m], the equivalent sand roughness of the ramp; A_d[m^2], the duct area below the spillway surface; and B[m], the spillway width.

(3) The flow properties
that is: Q[m^3/s], the total discharge; or q m^2/s, the discharge per unit width; or u m/s, the average velocity upstream of the ramp, some measure of the turbulence in the ambient flow; and h[m], the water depth in the approach flow.

(4) The fluid properties
μ[Ns/m^2] the dynamic viscosity of water; σ N/m, the surface tension of water; ρ_w[Kg/m^3], the density of water; ρ_a[Kg/m^3], the density of air; and g[m/s^2], the acceleration of gravity.

The required design parameters are: q_a[m^3/sm], the air demand from the aerator; c, the air concentration on the floor downstream of the aerator; Δp[N/m^2], the difference between the atmosphere and air pressure beneath the nappe; and ℓ_m, the water jet length.

On these models the underpressures are controlled by a valve on the air inlet system. This moves the variable Δp in with the group of independent variables (1-4).

Conventional dimensional analysis then yields for air demand the air concentration at a chosen point downstream of the aerator and the length of the water jet.

$$\frac{q_a}{q_w}, c, L = \phi[\text{Dimensionless geometric variables, Fr, Re, Tu, We, } \rho_a/\rho_w] \quad (1)$$

where $Fr\ (\frac{u}{\sqrt{gh}})$ is the Froude number,

$Re\ (\frac{\rho_w uh}{\mu})$ is the Reynolds number,

$We\left(u(\frac{\rho_w h}{\sigma})^{\frac{1}{2}}\right)$ is the Weber number,

$Eu\left(u(\frac{\rho_a}{\Delta p})^{\frac{1}{2}}\right)$ is the Euler number,

Tu is the measure of the turbulence in the flow, and

$\frac{\rho_a}{\rho_w}$ is the density ratio.

The density ratio is a constant and for large Reynolds numbers [> 10^5] and large Weber numbers [> 400] (Kobus, 1984, Pinto, 1984) the effect of changes in these numbers is small. Thus for a geometrically similar model to satisfy the above criteria we get

$$\beta,\ c,\ L = \phi[Fr,\ \Delta p/\rho gh,\ Tu] \qquad (2)$$

where $\beta = q_a/q$.

Apart from determining the jet length the Froude Number is important in determining the instability of the jet and $\Delta p/\gamma h$ [a form of pressure gradient] is important in determining an air bubble's rise velocity. Indeed it can be shown that the rise velocity of air bubbles in fluid with pressure gradient (which is not necessarily hydrostatic) may be written as

$$u\ |u| = -\frac{4\ g}{3C_d\ d}\left[\frac{\rho_a}{\rho} + \frac{1}{\rho g}\frac{\partial p}{\partial z}\right] \qquad (3)$$

where z is positive, measured vertically upwards,
d is the bubble diameter,
and C_d is the bubble drag coefficient.

This equation shows the strong effect of $\partial p/\partial z$ on the bubbles rise velocity. Indeed, if $\partial p/\partial z = -\rho_a g$, [i.e. the pressure variation is as in the atmosphere] the bubble rise velocity is zero and the spread of the air water layer must be due to turbulent diffusion alone. For a value $\partial p/\partial z$ greater than this value the turbulent diffusion must be retarded. This accounts for the strong dependence of $q_a/q[\beta]$ on $\Delta p/\rho gh$ obtained from both model and prototype measurements. Indeed it is the balance between the upward turbulent diffusion and the downward pressure gradient which affects the air entrainment.

Typical model results are shown in figures 3 and 4 for one of the shapes used on the model of the Clyde Dam (spillway slope 53°) (Tan, 1984, Low, 1985) and for the Foz Do Areia (spillway slope 15°) (Pinto, 1982) aerator. These results, and those for most aerators tested to date, fit the empirical equation

$$\beta = K_1[F - F_c] \qquad (4)$$

where F is the Froude number based on the velocity and depth just upstream of the aerator, K_1 is a constant, and F_c is a critical Froude number which is a function of the dimensionless under pressure and may be written as

$$F_c = K_2 - K_3\left[\frac{\Delta p}{\rho gh}\right]^{\gamma} \qquad (5)$$

where K_1, K_2 K_3, and γ are constants determined from the sectional model.

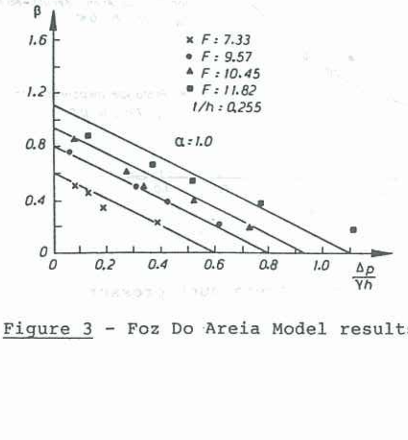

Figure 3 - Foz Do Areia Model results.

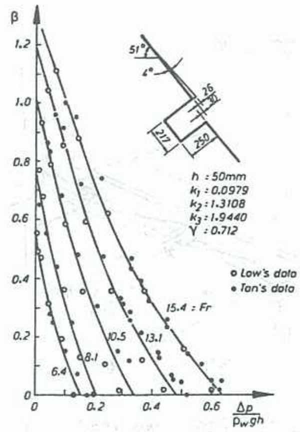

Figure 4 - Clyde Dam Model results.

For the Foz Do Areia aerator Pinto et al (1982) not only obtained the set of model results but also made some remarkable prototype measurements. The model and prototype measurements gave approximately the same values of K_1, K_2 and γ but while the model results showed a variation of K_3 with the dimensionless ramp height [$\ell_T \sin\phi/h$ or t/h] the prototype measurements showed K_3 as a constant.

It is believed that this difference is caused by the varying levels of turbulence on the model and prototype. On the prototype the turbulence level is that appropriate to the developing boundary layer & with the aerator position fixed it will be almost constant.

For the model there was a rectangular duct with a slide gate immediately upstream of the test section. This gate-duct combination determined the flow depth and hence the flow contraction. The variation of K_3 is consistent with the phenomena of the decrease in relative turbulence with an increasing contraction (Prandtl, 1952).

The analysis of the prototype measurement is shown in Fig. 5.

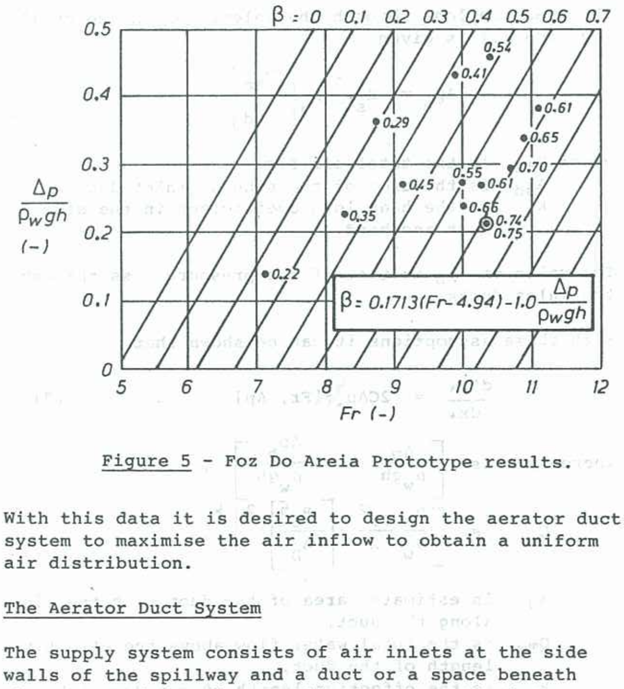

Figure 5 - Foz Do Areia Prototype results.

With this data it is desired to design the aerator duct system to maximise the air inflow to obtain a uniform air distribution.

The Aerator Duct System

The supply system consists of air inlets at the side walls of the spillway and a duct or a space beneath the aerator (Fig. 6). In this duct beneath the aerator the air flow streamlines close to the duct floor and near the inlet gradually rises as the air flow decreases until affected by the air demand on the underside of the nappe they move in the downstream direction.

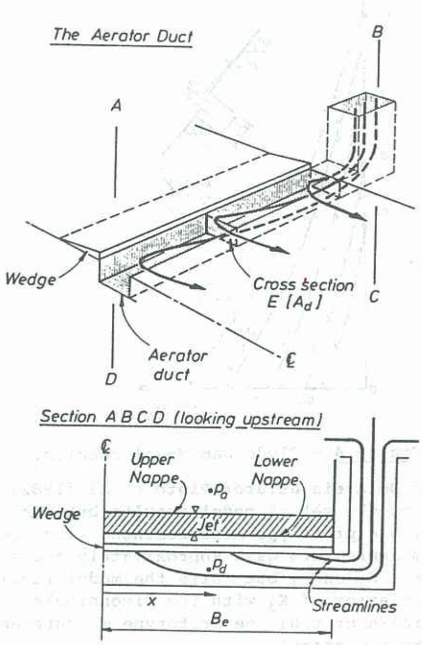

Figure 6 - The aerator duct system.

Using an equation of the form of Equation (2) then the air flow, the air flow distribution and the nappe underpressure can be estimated by assuming:

(1) The effective area in which the air is flowing along the duct is assumed to be a constant and can be estimated. It seems reasonable that the entrance into the area beneath the nappe be taken as this cross-section.
(2) The air flow in the duct is one-dimensional and thus the velocity does not vary with position in the duct cross-section.
(3) The duct under the aerator is sufficiently short that the head loss in this portion of the flow is small enough to be neglected. This implies that as the discharge decreases the pressure head increases.

The pressure loss through the inlets Δp_b in the spillway sidewall is given

$$\Delta p_b = K_s \frac{1}{2} \rho_a \left[\frac{Q_{Ta}}{A_{sd}}\right]^2 \qquad (6)$$

where Q_{Ta} is the total air flow
A_{sd} is the area of the side or inlet duct
K_s is the head loss coefficient in the side duct and bend.

The value of Δp_b is that of the pressure loss through the inlet ducts.

With these assumptions it can be shown that

$$\frac{d\Delta p_*}{dx_*} = 2C\Delta p_*^{1/2}\phi[Fr, \Delta p] \qquad (7)$$

where Δp_* is $\left[\frac{\Delta p}{\rho_w gh} - \frac{\Delta p_b}{\rho_w gh}\right]$,

C is $\left[\frac{\rho_a}{\rho_w}\frac{Fr^2}{2}\left[\frac{B_e h}{A_D}\right]^2\right]^{1/2}$,

A_D is estimated area of the duct with the flow along the duct,
Q_{Tw} is the total water flow above the effective length of the duct,
B_e is the effective length of the duct (this will be one half the distance between the spillway wall for a duct with an aerator inlet on both walls), and
x_* is x/B_e, where x is the distance measured from where the air flow along the duct is zero. (For a spillway with an aerator inlet at either side of the spillway the origin of x is at the spillway centreline.)

If the centreline pressure difference (i.e. the pressure where there is no flow along the duct) is assumed then integrating Equation (7) numerically gives the pressure distribution under the nappe. The local air demand can then be computed and the integration of this demand gives the total air flow. The total air flow then enables an estimate of the centreline pressure difference (this is the pressure difference due to the head loss in the inlet ducts in Equation (6). The process can then be repeated until the computation of the centreline pressure difference is to within the desired accuracy. After completion of these calculations the value of β at positions along the duct can be computed. For the particular case where γ = 1 the analytic solution below is obtained for the under pressure distribution and for the total air demand as

$$\Delta p_*(x) = \Delta p_b + \frac{\beta}{K_3}\left[\frac{\exp[2C[K_3\beta]^{1/2}x] - 1}{1 + \exp[2C[K_3\beta]^{1/2}x]}\right]^2 \qquad (8)$$

and

$$\frac{Q_{Ta}}{q_w B_e} = \frac{[\beta]^{1/2}}{C\sqrt{K_3}}\left[\frac{\exp[2C[K_3\beta]^{1/2}] - 1}{\exp[2C[\beta K_3]^{1/2}] + 1}\right] \qquad (9)$$

It is worth commenting on this last equation. For a particular position down the spillway the Froude number is approximately constant and it is desirable to obtain the greatest air discharge and as uniform an air distribution as possible and the equation suggests that this means:

(1) maximising the function β. This implies designing an aerator to maximise the value of $[K_1(Fr - F_c)]$. This implies minimising the value of $K_2\ \Delta p/\gamma h$. The value of Δp_b is associated with the head loss in the aerator inlets. To minimise this loss the inlets should be as streamlined as physically possible.
(2) Secondly, the value of C should be as small as possible and this implies maximising the duct area.

The form of these results can be verified using the bend loss coefficient obtained from the Foz Do Areia prototype results [0.536], the experimental values of K_1, K_2 K_3 and γ, and assuming that the main duct size is the same as the inlet duct size. With these assumptions the underpressure is computed and compared with the prototype measurements in Fig. 7.

Finally this computation procedure is applied to an air duct system with an air demand of the Foz Do Areia (Fig. 8)

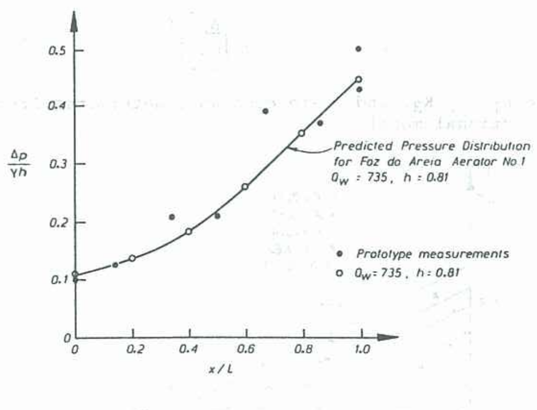

Figure 7 - Foz Do Areia duct pressures.

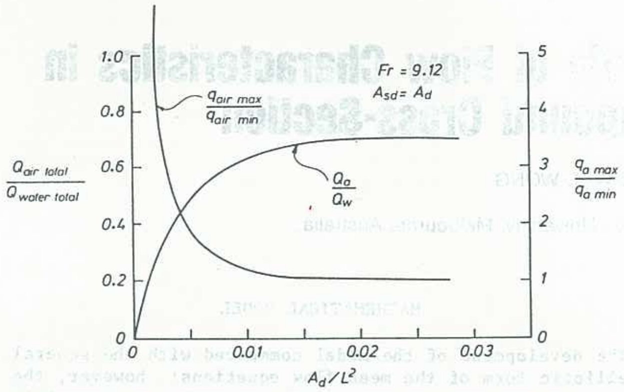

Figure 8 - The effects of variations of duct areas.

Figure 8 shows that increasing the dimensionless duct area $[A_d/L^2]$ from 0.005 to 0.01 yields a 35% increase in the dimensionless air demand and greatly improves the uniformity of the airflow over the cavity length $[q_{air}max/q_{air}min$ changes from 1:91 to 1:15]. This implies an increase in the minimum air flow per unit width of approximately 80%.

CONCLUSION

The underpressure beneath the nappe of an aerator gives a pressure gradient which opposes the upward turbulent diffusion of the air bubbles and restrains the air entrainment. This makes aerator modelling difficult.

The effects of the underpressure can be minimised by aerator design, by streamlining the inlets to the aerator duct and providing a large main duct area.

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