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Wake Singularity Potential Flow Models of Two-Dimensional Separated Flows

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ABSTRACT

In an extension of an earlier wake source potential flow model for separated flow, new boundary conditions and conformal mappings are described which permit wider applications, including lifting airfoils, with only the base pressure coefficient as an empirical input. Examples are presented for the circular cylinder and for the Joukowsky airfoil fitted with either an upper surface spoiler or a lower surface split flap. Predicted pressure distributions are compared with wind tunnel data and good agreement is found.

INTRODUCTION

There are many two-dimensional free streamline potential flow models of bluff body wake or cavity flows. One proposed by the author (Parkinson and Jandali (1970) henceforth PJ) has the advantages of simplicity and wide applicability. In this conformal mapping model the contour mapped is the body wetted surface plus an additional contour in the wake providing a slit or cusp at the flow separation points. The part of the original contour exposed to the wake is ignored unless it already conforms to this requirement. The resulting contour is mapped to a circle by a sequence of transformations for which the overall derivative has simple zeros at the points on the circle corresponding to the flow separation points in the physical plane. Then if these are made stagnation points of the flow in the circle plane tangential flow separation is achieved in the physical plane because of the doubling of the angles. The flow model consists of uniform flow plus a doublet for the basic circle, two sources on the wake portion of the contour and their image sink, and a vortex for the circulation in lifting configurations, (Jandali and Parkinson (1970), henceforth JP), at the center. Tangent flow over the body wetted surface and uniform flow at infinity are achieved automatically, and the 5 unknowns (1 vortex strength, 2 source strengths and positions) are determined by the remaining boundary conditions. For symmetrical flows only 4 unknowns are involved and these are given by requiring separation at specified points and pressure. The separation pressure is assumed to be the (constant) base pressure, given empirically as in all potential-flow models. The method gives good simulations of pressure distribution and force for engineering purposes and because of its simplicity can be used in iterative combination with boundary-layer theory (Bluston and Paulson (1972)).

However, there are some unsatisfactory features of the original wake source model. For bluff sections with a continuously curved contour, so that flow separation is boundary-layer controlled, the position of the separation points is also specified empirically, while for lifting airfoils fitted with normal spoilers the number of unknowns to be solved for is kept at four by arbitrarily locating one of the two wake sources close to the transform stagnation point corresponding to the airfoil trailing edge, investigation having shown the result to be relatively insensitive to this source location. Although the model successfully predicts pressure distributions on a wide variety of sectional shapes, it would clearly be desirable to reduce the empiricism involved, and for the airfoil sections it would be useful to extend the applications from normal spoilers to the more relevant cases of inclined spoilers and split flaps. These possiblities are considered in the following sections.

BOUNDARY CONDITIONS FOR SEPARATED POTENTIAL FLOW

Finite Pressure Gradient and Streamline Curvature at Separation

The wake source model does not of course represent the solution of a complete boundary-value problem, since conditions along the free streamline boundaries are unspecified except at the separation points. However, this is not an important deficiency since the main interest in the problem is in the evaluation of the loading on the body, and the separating flow does start out in the right direction with the right fluid velocity, and approaches the right asymptotic value at downstream infinity. Furthermore, no precise definition can in fact be given to the free-streamline boundary conditions since the actual shear layers the streamlines simulate are in unsteady motion as they develop into an organized wake vortex system.

Given, then, that the model is necessarily incomplete it is still desirable that it should require as little empiricism as possible. It has been shown by Woods (1961) that the curvature of the boundary streamline at separation, and the corresponding streamwise pressure gradient, are in general positive infinite or negative infinite, with a single intermediate special case of finite curvature and pressure gradient. Negative infinite curvature is possible only for separation at a sharp edge, and if the curvature is positive infinite, so is the streamwise pressure gradient. Therefore, the occurrence of the special finite curvature at separation would appear to be the most natural possibility for separation from a curved surface, and this can be used in the wake source model to eliminate the empirical specification of the separation point.

Circular Cylinder

As mentioned in PJ this finite pressure gradient criterion was tried unsuccessfully for the case of laminar separation from the circular cylinder, and in fact would not have worked for the other circular cylinder examples in that paper. However, more data are now available for the case of turbulent separation from experiments on a circular cylinder by Nakamura and Tomonari (1982), and a search for other data for laminar separation has led to four sets from four different laboratories shown in Hoerner (1965). Figure 1 shows a comparison of these data with theoretical predictions from the wake source model using the criterion of finite pressure gradient at separation. (This criterion links the separation angle β_s to the base pressure coefficient C_{pb}, still given empirically, by the locus equation $C_{pb} = 1 - 9/4 \sin^2 \beta_s$). Theoretical and experimental pressure distributions are compared for the turbulent and one of the laminar cases, and the proximity of the data for the separation

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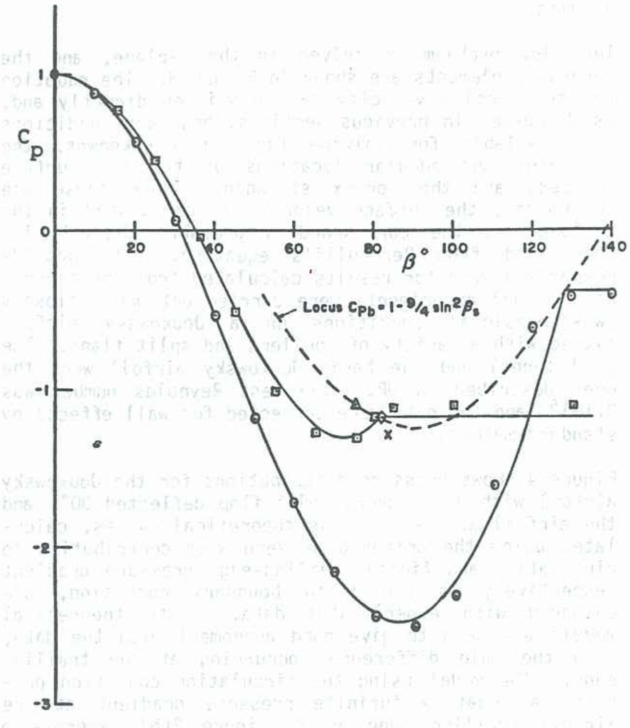

Figure 1. Criterion of finite curvature at separation for circular cylinder. ———, theory; □, Δ, ◇, x, Hoerner (1965) subcritical Reynolds number experiments; o, Nakamura & Tomonari (1982) experiments, Reynolds number $1.7(10)^6$.

points to the theoretical locus is shown for the other three laminar cases. The agreement is seen to be quite good, although the experimental pressure gradient for the turbulent case is more positive than the finite theoretical gradient prior to separation. Nevertheless, it appears that the criterion can in some cases lead to realistic results from the model. Further, the criterion suggests an additional boundary condition for those lifting airfoil cases with separation in which the separating streamline has naturally positive curvature; e.g. the case of airfoil stall or the flow at the trailing edge of an airfoil with split flap.

Trailing Edge Pressure Gradients for Airfoils with Spoilers or Split Flaps

It is of interest to compare observed pressure gradients at separation with the predicted behaviour using the criteria described in the previous section. In PJ the flow past a normal flat plate is treated. Here one expects negative infinite streamline curvature at separation because of the vortex-like flow around the edges, and the observed pressure gradient is in close agreement with the predicted negative infinite value there. Again in PJ for the circular cylinder examples treated, in which the finite pressure gradient criterion does not work, the observed pressure gradients near separation (boundary layer effects influence gradients at separation) are compatible with the predicted positive infinite value at separation.

For airfoils with upper-surface spoilers or lower-surface split flaps the significant separation condition is on the opposite airfoil surface at the trailing edge. An airfoil with deflected spoiler is usually at a positive angle of attack, so that the airfoil lower surface faces the oncoming flow producing trailing-edge separation conditions corresponding to the production of a negative infinite pressure gradient there. Observed pressure gradients are compatible with this, as can be seen in Figure 5. An airfoil with deflected split flap is also usually at a positive angle of attack, so that the airfoil upper surface faces away from the oncoming flow and the most natural trailing-

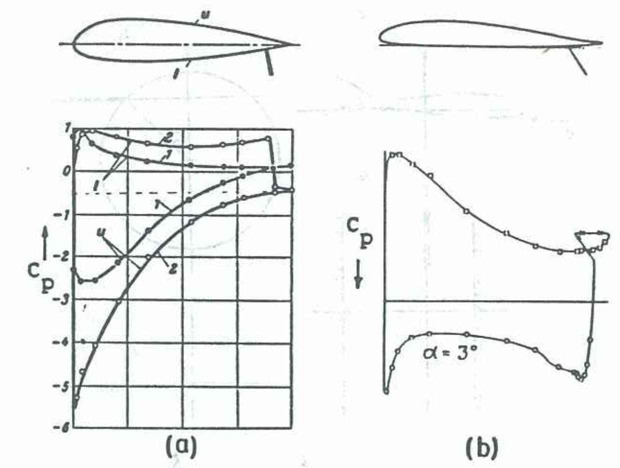

Figure 2. (a) Pressure distribution on airfoil with (2) and without (1) split flap, Schlichting & Truckenbrodt (1979). (b) Pressure distribution on airfoil with split flap, Wenzinger (1937).

edge separation condition would appear to be finite positive (concave upward) curvature of the separating streamline, corresponding to a finite positive pressure gradient. On examining several sources of data on airfoils with split flaps it was found that the finite positive trailing-edge pressure gradient does occur, for example as shown in Figure 2(a) taken from Schlichting and Truckenbrodt (1979), but apparently only at relatively high angles of attack α. (In the description for Figure 2(a) α was not given but is almost certainly greater than 10°. Note also that pressure coefficient C_p is plotted positive up in the figure, the reverse of the usual convention). At lower angles of attack as shown in Figure 2(b), taken from Wenzinger (1937), an interesting change consistently occurs. On the airfoil upper surface the pressure increases (suction decreases) towards the trailing edge, but overshoots the constant base pressure value so that a negative infinite gradient is observed at separation as the flow adjusts to the base pressure. Therefore, it appears that the criterion of finite pressure gradient at separation may provide a useful fifth airfoil boundary condition, but only for the case of an airfoil with split flap at high angle of attack.

Circulation Boundary Condition

A more general fifth boundary condition is therefore needed for airfoils with spoilers or split flaps, and it is reasonable to suppose that it should be related to the circulation, since the vortex strength Γ is the additional unknown in the wake source model for lifting bodies. In the real flow the wake region makes no contribution to the time-averaged airfoil circulation, which is therefore a consequence of the unseparated flow upstream of the airfoil trailing edge and the spoiler (or split flap) tip. If, then, in the flow model the wake region is also required to make no contribution to the airfoil circulation, the upstream flow should be a better simulation of the real flow. A contour integral expressing this requirement then provides a fifth boundary condition which has been found to lead to good predictions from the flow model for most practical configurations of Joukowsky airfoils with spoilers or split flaps, with only C_{pb} as empirical input. However, a few configurations required an adjustment to C_{pb} before a solution could be obtained, and Yeung (1985) has proposed a modified fifth boundary condition which avoids this additional empiricism. In Yeung's condition the wake contribution to the circulation, instead of zero, is set equal to the average of values calculated for two simplified flow models in which the requirement of specified separation velocity is relaxed first at the airfoil trailing edge, second at the spoiler (or split flap) tip.

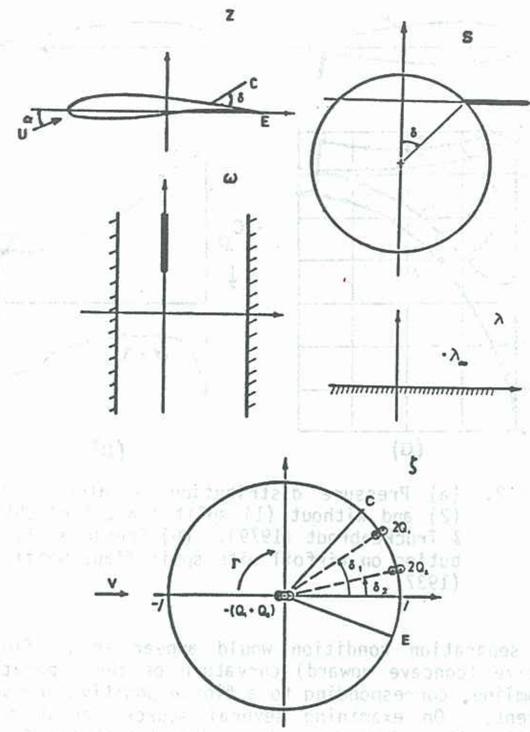

Figure 3. Physical and transform planes for Joukowsky airfoil with spoiler.

WAKE SOURCE MODEL FOR AIRFOILS WITH SPOILERS OR SPLIT FLAPS

Conformal Mapping Sequence

The original airfoil-spoiler model described in JP used conformal transformations which could deal only with normal spoilers. A new mapping sequence shown in Figure 3 permits the mapping of a Joukowsky profile of arbitrary thickness and camber fitted with a spoiler of arbitrary size, chordwise location, and inclination δ into a circle. The key configuration in the sequence is in the s-plane, a circle with a flat fence at angle δ. Moving back in the sequence, a translation, rotation, and Joukowsky transformation map the circle with fence into the Joukowsky profile with spoiler in the z-plane. Moving forward in the sequence from the s-plane, use is made of the fact that the circle and fence are on coordinate curves in a bipolar coordinate system. The field exterior to the circle and fence can therefore be mapped to the interior of an infinite strip of the ω-plane, with a semi-infinite slit along the imaginary axis, by a Karman-Trefftz transformation. The segments of the circle above and below the real axis map into the right and left boundaries of the infinite strip, and the fence maps into the slit. The point at infinity in the s-plane (and in the physical z-plane) becomes the origin in the ω-plane. The infinite strip with slit can be regarded as the interior of a degenerate polygon, a suitable subject for a Schwarz-Christoffel transformation to the upper half λ-plane, in which λ_∞ now represents the point at infinity in the z-plane. Finally, by a translation and scaling, followed by a bilinear transformation and a rotation, the upper half λ-plane is mapped into the exterior of the unit circle in the ζ-plane, with the point at infinity now preserved in the overall transformation from the z-plane to the ζ-plane.

With only minor geometric changes the sequence can be adapted to mapping a Joukowsky profile fitted with a lower-surface split flap into the unit circle. The method can be extended to an airfoil of arbitrary profile fitted with a spoiler or split flap by inserting a transformation originated by Theodorsen (1931) into the sequence between the airfoil and s-planes.

Results

The flow problem is solved in the ζ-plane, and the necessary elements are shown in Figure 3. The equation for the complex velocity can be written directly and, as discussed in previous sections, boundary conditions are available for solving for the 5 unknowns, the strengths and angular locations of the two surface sources, and the vortex strength. Once these are determined, the surface velocity is calculated in the z-plane and the corresponding pressure distribution determined from Bernoulli's equation. To provide comparison data for results calculated from the theory, wind tunnel experiments were carried out under closely two-dimensional conditions on a Joukowsky airfoil fitted with a variety of spoilers and split flaps. The wind tunnel and the basic Joukowsky airfoil were the ones described in JP. The test Reynolds number was $3(10)^5$, and the data were corrected for wall effects by standard methods.

Figure 4 shows pressure distributions for the Joukowsky airfoil with a 20% chord split flap deflected 30°, and the airfoil at $\alpha = 4°$. Two theoretical curves, calculated using the criteria of zero wake contribution to circulation and finite trailing-edge pressure gradient respectively as the fifth boundary condition, are compared with experimental data. Both theoretical models are seen to give good agreement with the data, with the main differences occurring at the trailing edge. The model using the circulation condition predicts a negative infinite pressure gradient at the airfoil trailing edge as in Figure 2(b), whereas a solution was obtained using the finite pressure gradient condition only by allowing a less negative C_{pb} than the experimental value of -0.55. In our experiment the actual pressure gradient at the airfoil upper surface trailing edge could not be determined because the airfoil was constructed for spoiler tests in which that portion was in the wake and was therefore artificially thickened to strengthen the trailing edge portion. The effect of the artificial thickening is seen in Figure 4 at the data point marked with a star, representing the last tap on the airfoil upper surface. It should be noted that in the theoretical curve using the pressure gradient condition the calculated trailing-edge gradient becomes a finite continuation of the upstream gradient, as in the curve in Figure 2(a).

In Figure 5 pressure distributions are shown for the airfoil with a 5% spoiler at 45° located at 70% chord. Theoretical curves are compared with experimental data for two angles of attack, and good agreement is seen in both cases except just upstream of the spoiler, where in the experiments the adverse boundary layer pressure gradient has produced a constant-pressure separation bubble instead of the potential-flow stagnation-point region. As would be expected, this bubble is larger at $\alpha = 12°$ than at $\alpha = 6°$. Yeung's modified circulation boundary condition was used in the theoretical calculations. Results similar to those of Figures 4 and 5 were obtained for other configurations tested. Details can be found in Yeung (1985).

DISCUSSION

The results presented show that the improved wake source model can give good predictions of pressure distribution on non-lifting bluff bodies with boundary-layer-controlled separation (Figure 1) and on lifting airfoils with geometrically-controlled separation (Figures 4,5) with only the base pressure coefficient as empirical input. Some interesting questions remain unanswered. In particular it would be worthwhile to investigate the range of system parameters in which the criterion of finite curvature and pressure gradient at separation works. In this connection it should be mentioned that the sequence of conformal transformations for Figure 3 can be modified to give a model of airfoil stall, and work has been started on this problem.

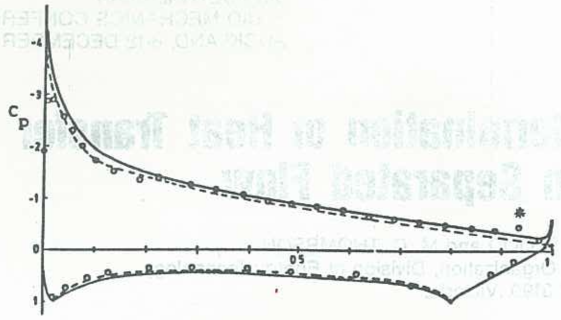

Figure 4. Pressure distributions on Joukowsky airfoil with 20% split flap. δ = 30°. α = 4°. ———, theory with circulation condition; -----, theory with finite curvature condition; o, experiment.

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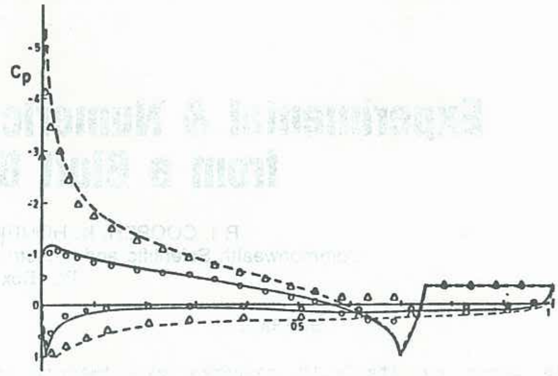

Figure 5. Pressure distributions on Joukowsky airfoil with 5% spoiler at 70% chord. δ = 45°. ———, theory; o, experiment: α = 6°. -----, theory; Δ, experiment: α = 12°.

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