9th AUSTRALASIAN
FLUID MECHANICS CONFERENCE
AUCKLAND, 8-12 DECEMBER 1986

Flow Past a Two-Dimensional Battened Sail

BARRY G. NEWMAN and BRUNO UGOLINI

Department of Mechanical Engineering, McGill University, Montreal, Quebec, Canada.

INTRODUCTION

Inviscid thin aerofoil theory was applied to two-dimensional membranes or 'sails' at small incidence by Thwaites (1961), Nielsen (1963) and others. When compared with experimental measurements of lift and tension as a function of incidence the agreement with theory is only passable for 'flat' sails with values of the excess-length ratio less than about 0.01 which are sails of small camber (Newman 1986). For more highly cambered sails, the increase in boundary layer thickness near the trailing edge counteracted to some extent by the effect of separation bubbles near the leading edge combine to make the predicted lift and tension significantly less than the predicted values. However as will be shown in this paper the direct relationship between lift and tension is in much better agreement with experiment.

In the present paper the theory is extended to sails with stiffness in bending, by inventing an idealized solution for a fully battened two-dimensional sail. The batten geometry which will give the ideal loading of constant pressure difference across the membrane (Stack, 1944) is worked out and as would be expected applies at zero incidence. For this design at non-zero incidence and more generally for any batten geometry the theory of Nielsen has been redeveloped by Wong (1985).

An ideal battened sail for which the stiffness and tension effects are of comparable magnitude has been mounted in the same experimental facility originally used by Newman and Low (1984) for unstiffened sails. The tests have been done at chordwise Reynolds numbers of 9.5×10^4 and 1.4×10^5.

It is envisaged that the present approach may be useful in developing a rational design method for tapered battens. This is currently being done in an empirical manner on wind surfers, hang-wing gliders and similar craft. It is also to be found on Chinese junks of ancient design where full chord chordwise battens are made of round bamboo poles of various lengths laid parallel to one another and stitched to the sails (see, for example, the model of a Fukien Chinese junk in the Science Museum, London). This type of batten gives a stiffness which is greatest at the mid-chord position and decreases towards the luff and leach. In this respect it resembles the ideal batten which is developed here.

THEORY

Preliminary analysis

Consider a planar two-dimensional sail of unit chord which is slightly stiff in bending (Fig. 1). Assuming that the flow is irrotational and that the slope of the membrane and the incidence are small, thin aerofoil theory may be used. The camber line is represented by an array of vortices which are effectively on the chord line (Glauert, 1926).

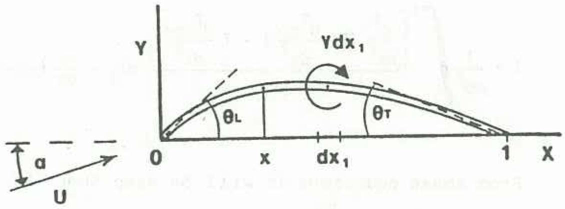

Fig. 1: Two-dimensional membrane in a flow at incidence α.

The equilibrium of element dx at x, y is due to a balance of pressure, tension T and shear forces N in the elastic membrane (Fig. 2).

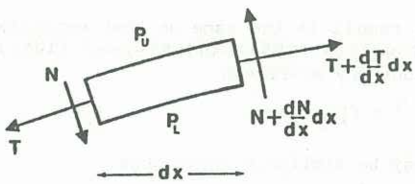

Fig. 2: Equilibrium of a two-dimensional membrane.

$$p_L - p_U + \frac{dN}{dx} = \frac{T}{R} = Ty'' \qquad (1)$$

where R is the local radius of curvature of the membrane.

For a membrane which is thin the theory for a simple beam in bending applies and

$$N = \frac{dM}{dx} = - \frac{d}{dx}(EIy'')$$

where E is the modulus of elasticity for bending and I is the second moment of the cross section per unit span about the neutral axis ($= a^3/12$). Note that the pressure term in this equation is second order and may also be neglected.

$$p_L - p_U - \frac{d^2}{dx^2}\left(EI\frac{d^2y}{dx^2}\right) + T\frac{d^2y}{dx^2} = 0 \qquad (2)$$

Equations similar to (2) will be found in Lighthill (1960), Datta and Gottenberg (1975) and Katz and Weihs (1978) where it is applied to similar unsteady problems. For small slope the tension is affected only by the skin friction and is therefore effectively constant (Newman and Goland, 1981).

The thin aerofoil boundary condition is (see, for example, Newman, 1986).

$$\alpha + \frac{1}{\rho U^2}\int_0^1 \frac{(p_L - p_U)dx_1}{2\pi(x_1 - x)} = y' \qquad (3)$$

The length of the membrane

$$\ell = \int_0^1 (1 + (\frac{dy}{dx})^2)^{1/2} dx \tag{4}$$

and the excess length ratio

$$\varepsilon = \frac{\ell - 1}{1} = \frac{1}{2}\int_0^1 (y^1)^2 dx \tag{5}$$

Hence

$$\varepsilon/\alpha^2 = \frac{1}{2}\int_0^1 \frac{d(y/\alpha)^2}{dx} dx \tag{6}$$

Substitution gives

$$1 + \frac{1}{\rho U^2}\int_0^1 \frac{\left[\frac{d^2}{dx^2}(EI\,\frac{d^2y/\alpha}{dx^2}) - T\,\frac{d^2y/\alpha}{dx^2}\right]_1}{2\pi\,(x_1 - x)} dx_1 = \frac{d}{dx}(y/\alpha). \tag{7}$$

From these equations it will be seen that the shape

$$y/\alpha = f(C_T, x, \frac{EI}{\rho U^2})$$

where I may also depend on x and

$$\alpha/\varepsilon^{1/2} = f(C_T, \frac{E}{\rho U^2}) \tag{8}$$

where a(x) is specified.

This result is the same as that established earlier by Thwaites (1961) and Nielsen (1963) for sails without any stiffness

$$\alpha/\varepsilon^{1/2} = f(C_T) \tag{9}$$

It may be similarly shown that

$$C_L = \int_0^1 \frac{(p_L - p_U)dx}{q} = \varepsilon^{1/2} f(C_T, \frac{E}{\rho U^2}) \tag{10}$$

and $$C_m = \int_0^1 \frac{p_L - p_U}{q} x\, dx = \varepsilon^{1/2} f(C_T, \frac{E}{\rho U^2}) \tag{11}$$

Case with constant loading

If the pressure difference $p_L - p_U = \Delta p$ is independent of x the solution is particularly simple. This case of uniform loading is likely to be beneficial from a practical point of view. Indeed it formed the basis of the very early NACA 16 series of aerofoils (Stack 1944).

$$\int_0^1 \frac{\Delta p}{x_1 - x} dx_1 = \Delta p \ln \frac{1 - x}{x}$$

and equation (3) becomes

$$\frac{dy}{dx} = \alpha + \frac{\Delta p}{2\pi\rho U^2}[\ln(1 - x) - \ln x]$$

$$y = \alpha x + \frac{\Delta p}{2\pi\rho U^2}[-(1 - x)\ln(1 - x) - x\ln x] \tag{12}$$

so that the trailing edge condition $y = 0$ at $x = 1$ proves that the case requires $\alpha = 0$ (Stack, 1944).

Putting this back into equation (2), after two integrations assuming zero bending moment at the leading and trailing edges.

$$\frac{x(1 - x)}{2} + \frac{EI}{2\pi\rho U^2}\left(\frac{1}{x(1 - x)}\right) - \frac{T}{2\pi\rho U^2}\left[(1 - x)\ln(1 - x) + x\ln x\right. = 0 \tag{13}$$

Writing $e = \frac{E}{2\pi\rho U^2}$ and $t = \frac{T}{2\pi\rho U^2} = \frac{C_T}{4\pi}$

the required thickness distribution of the batten is given by

$$a^3e = 12x(1 - x)\left[\frac{x(1 - x)}{2} + t[(1 - x)\ln(1 - x) + x\ln x]\right] \tag{14}$$

As the tension is increased a becomes unrealistically negative near both the leading and trailing edges. To avoid this near the trailing edge when x = .999 for example t must be less than 0.06317, i.e. C_T must be less than 0.794.

The shape of camber line is given by ε from equation (5)

$$\varepsilon = \frac{1}{2}\left[\frac{\Delta p}{2\pi\rho U^2}\right]^2 \int_0^1 \left[\ln(1 - x) - \ln x\right]^2 dx$$

$$= \frac{1}{2}\left[\frac{\Delta p}{2\pi\rho U^2}\right]^2 (\frac{\pi^2}{3}) = \frac{1}{96}(\frac{\Delta p}{\frac{1}{2}\rho U^2})$$

For uniform loading $$C_L = \frac{\Delta p}{\frac{1}{2}\rho U^2} = (96\varepsilon)^{1/2} \tag{15}$$

and C_m about the leading edge = $\frac{1}{2}C_L$ (Stack, 1944).

Wong (1985) has applied Nielsen's procedure to solve equation (2) for the off-design condition and for any other more general case. Details will be given shortly in Ugolini & Newman (1986).

EXPERIMENT

Preliminary experiments have been conducted in the McGill 762 mm x 432 mm Blower Cascade Wind Tunnel using the same apparatus cited in Newman & Low (1984), (for details concerning the following description, see the above paper). A further contraction provides a 762 mm x 154 mm working section with the same balance arrangement and wall suction for boundary layer control used by Newman & Low (1984).

The experimental battens were evenly spaced idealized two-dimensional battens and hence closely resembled real battens.

The battens are modelled from equation (14) rewritten as

$$\frac{ba^3e}{s} = 12\,x\,(1 - x)\left[\frac{x(1 - x)}{2} + t[(1 - x)\ln(1 - x) + x\ln x]\right] \tag{16}$$

where s is the spacing between the experimental battens and b is the lateral width of sail battens which simulates the two-dimensional stiffened sail. (Fig. 3).

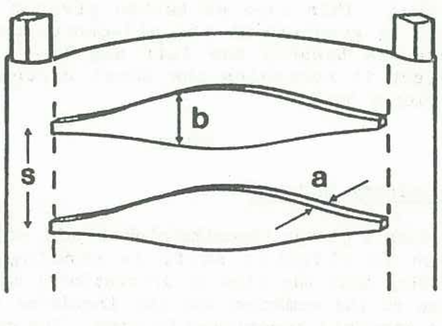

Fig. 3: Sail with idealized two-dimensional battens, showing cross-section of luff and leach wedge supports.

Here a = constant and b = b(x); consequently the experiment is not ideally two-dimensional. The values of a, s, q and e are fixed by the desired Reynolds number, constraining dimensions and properties of the batten material, in this case, brass. **But it remains for a unique distribution of b(x) to be defined.** To find a t which bears some physical significance the following approach was taken. We determine t as that value for which the tension and stiffness terms in equation (2) are of equal magnitude at the 1/4 chord point. Since tension is thought to be dominant near the leading and trailing edges, whereas the stiffness is more important near midchord, the 1/4 chord point is a sensible location to apply this criteria. **Thus a reasonable compromise between tension and stiffness was established. Using equations (14) and (2) the desired t is found to be 3/32.**

The battens used had maximum mid-chord width of 5.7 mm, a = 0.2 mm, and were spaced 19 mm apart along the sail. Battens were taped to the sail, the added flexural rigidity of the tape being approximately 0.02% that of the brass battens. The wedge supports did not permit full-chord battens, rather battens were cut a short distance from the ends to fit just within the supports. This is acceptable given that the b(x) distribution dictated by equation (16) tapers off rapidly near the ends. The portions omitted were therefore of relatively small lateral width and were considered to have relatively insignificant stiffness.

RESULTS AND DISCUSSION

C_T is plotted in Fig. 4 as a function of $\alpha/\varepsilon^{1/2}$ for experiment and theory. Some results of Newman & Low (1984) for an unbattened sail are included for completeness.

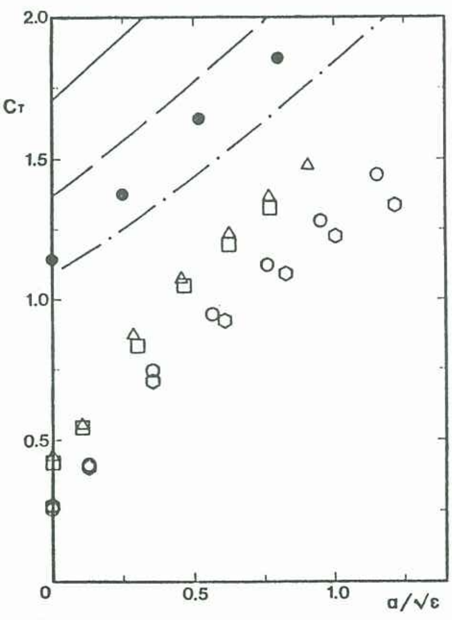

Fig. 4: C_T vs $\alpha/\varepsilon^{1/2}$ (radians): Newman & Low (1984) sail results; ——— theory, ● experiment: battened sail E/q = 1.94 x 10^8 — — theory, experiment without suction applied (see text) □ , with suction △ : battened sail E/q = 4.18 x 10^8 —·— theory, experiment without suction ⬡ , with suction ○ .

Note that applying suction to the walls of the working section gives some improvement in the right direction.

The results for C_L are presented in Fig. 5 and also show poor agreement, notably worse for the battened sail. The ideal constant loading case imposes a large pressure gradient near the trailing edge which would conceivably cause separation. Hence the Kutta condition would no longer apply and significant losses in lift would result. This phenomenon was indeed observed using tufts and can also be found in the results of Stack (1944). At zero incidence the sail shapes had large values of θ_T but levelled off quickly at which point separation occurs. The measured C_L is 4 times lower than that predicted by equation (15).

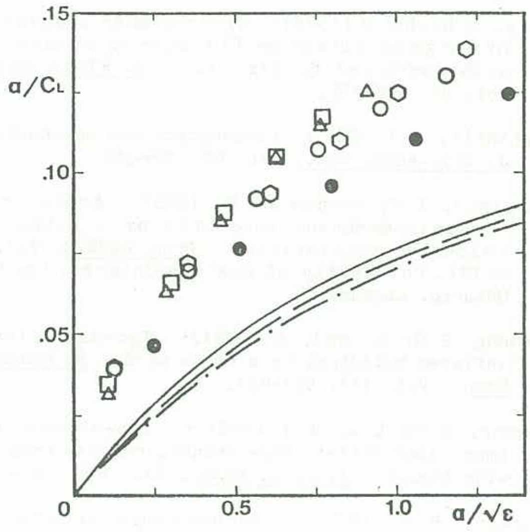

Fig. 5: α/C_L (rad) vs $\alpha/\varepsilon^{1/2}$(rad), legend as in Fig. 4.

There is however **greatly improved agreement between** C_T and $C_L/\varepsilon^{1/2}$, Fig. 6. Tuft observations show the presence of separation bubbles near the leading edge similar to those found by Newman & Low (1984). As mentioned, these bubbles counteract the trailing edge separation somewhat by altering the effective camber and incidence. (See Tse, M.-C. & Newman, B.G. (1987) where calculations of the effect are in order). One would therefore expect better agreement with theory for two parameters not involving incidence. The balancing effect of the bubbles is not complete however as most of the experimental points fall to the left of the theory.

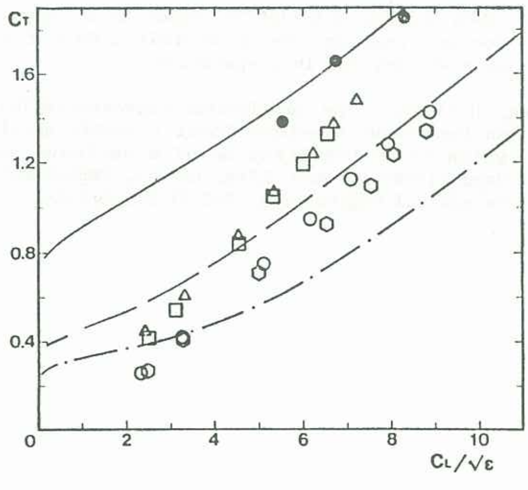

Fig. 6: C_T vs $C_L/\varepsilon^{1/2}$, legend as in Fig. 4.

Once again the shortcomings of the theory can be attributed to viscous effects which are not accounted for. Further experiments are planned with battens designed for a loading which tapers off gradually to zero at the trailing edge in order to avoid separation.

This work was supported by NSERC under grant number A7096 (B.G. Newman).

REFERENCES

Datta, S K; Gottenberg, W G (1975): Instability of an elastic strip hanging in an airstream. J. Appl. Mech. Trans. ASME., Vol. 42, No. 1, 195-198.

Glauert, H (1926): The elements of aerofoil and airscrew theory. (Second edition), Cambridge University Press, p. 87.

Katz, J; Weihs, D (1978): Hydrodynamic propulsion by large amplitude oscillation of an airofoil with chordwise flexibility. J. Fluid Mech., Vol. 88, 485-497.

Lighthill, M J (1960): Mathematics and aeronautics. J. Roy. Aero. Soc., Vol. 64, 375-394.

Mateescu, D F; Newman, B G (1985): Analysis of flexible-membrane aerofoils by a method of velocity singularities. Proc. CANCAM, Vol. 2, B-187, University of Western Ontario, London, Ontario, Canada.

Newman, B G; Goland, D (1981): Two-dimensional inflated buildings in a cross wind. J. Fluid Mech., Vol. 117, 507-530.

Newman, B G; Low, H T (1984): Two-dimensional impervious sails: experimental results compared with theory. J. Fluid Mech., Vol. 144, 445-462.

Newman, B G (1986): Aerodynamic theory for membranes and sails. (To be published) Progress in Aerospace Sciences,.

Nielsen, J N (1963): Theory of flexible aerodynamic surfaces. ASME, J. App. Mech., Vol. 30, 435-442.

Stack, J (1944): Tests of airfoils designed to delay the compressibility burble. NACA TN-976.

Thwaites, B (1961): The aerodynamic theory of sails. I Two-dimensional sails. Proc. Roy. Soc. A, Vol. 261, 402-422.

Ugolini, B; Newman, B G (1986): Theory and Experiments on idealized battened sails. To be submitted for publication.

Tse, M-C; Newman, B G (1987): Paper to be submitted for publication. See also McGill Ph.D. thesis of M.-C. Tse now in preparation.

Wong, H (1985): The aerodynamic characteristics of an impervious two-dimensional flexible aerofoil which is an idealization of a battened sail. Unpublished honours B.Eng. thesis, Department of Mechanical Engineering, McGill University.