

Numerical Investigation of Axisymmetric Vortex Breakdown

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ABSTRACT

The Navier-Stokes equations for the laminar, incompressible, axisymmetric evolution of the breakdown of a columnar vortex in a cylindrical tube are solved using the streamfunction-vorticity formulation by an explicit finite difference technique. The use of high spatial and temporal resolution enables a detailed study of the evolution and internal structure of the recirculation zone when breakdown occurs. Preliminary results reveal the growth of a mean two-celled internal structure similar to that observed experimentally when the breakdown takes the form of a 'bubble'.

INTRODUCTION

Vortex breakdown in swirling flows has been the subject of much attention in the literature since it was first recognized in the tip vortices of delta winged aircraft (Peckham and Atkinson 1957). The term vortex breakdown refers to an abrupt change at some axial station in the character of a columnar vortex. This is usually observed as a sudden widening of the vortex core, and is often followed by a stagnation point on the axis and a region of recirculation. Sarpaka (1971) and Faler and Leibovich (1977) observed a number of distinctive types of vortex breakdown, however our attention will be directed to the axisymmetric form, apparently the only form for which measurements of the flow inside the recirculation zone are available (Faler and Leibovich 1977, Escudier and Keller 1983).

The results of flow visualization experiments of vortex breakdown in tubes has lead to a general consensus (Leibovich 1978) that to describe vortex breakdown a three-dimensional, time-dependent Navier-Stokes model, possibly with turbulent parameterization, is needed. Recently however, Escudier (1984) observed vortex breakdown of swirling flows in a cylindrical container with a rotating endwall to be axisymmetric and steady over a large range of the governing parameters. Even when the flow is oscillatory, it was observed to be axisymmetric for a range of the parameters, and for all the cases reported the flow was observed to be laminar - turbulence only setting in at extreme values of the parameters. A feeling was expressed in that report, as well as in Escudier and Keller (1983) as a result of experiments in cylindrical tubes, that vortex breakdown is inherently axisymmetric and that departures from axial symmetry result from instabilities not directly associated with the breakdown process. Hence, in this preliminary investigation, a laminar, time-dependent, axisymmetric model has been constructed to examine the structure of the recirculation zone of a vortex breakdown in a cylindrical tube and compare it with available experimental observations.

EQUATIONS AND METHOD OF SOLUTION

Vortex breakdown in a tube is modelled by considering a cylindrical tube of radius R and length Z . The fluid is incompressible with a constant kinematic viscosity ν , density ρ and maintained at a constant temperature. The axisymmetric form of the time-dependent Navier-Stokes equations in cylindrical co-ordinates (r, ϕ, z) with corresponding velocity components (u, v, w) are employed to describe the evolution of the flow. These are non-dimensionalized by scaling the lengths with the core radius of the vortex, δ , which Faler and Leibovich (1977) have indicated is the natural unit of length for the problem; velocities by the free-stream axial velocity W and pressure by ρW^2 after subtraction of a reference pressure. Time is scaled by δ/W . The core Reynolds number is defined as $Re = W\delta/\nu$, in contrast to the tube Reynolds number which is usually quoted in experimental

investigations where the diameter of the tube is employed as the length scale.

The streamfunction-vorticity formulation introduces a streamfunction ψ , where

$$u = -\frac{1}{r} \frac{\partial \psi}{\partial z} \quad \text{and} \quad w = \frac{1}{r} \frac{\partial \psi}{\partial r},$$

automatically satisfy the conservation of mass. The azimuthal component of vorticity is given by

$$\eta = -\frac{1}{r} \frac{\partial^2 \psi}{\partial z^2} - \frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial \psi}{\partial r} \right).$$

This formulation leads to the prediction equations for the azimuthal components of velocity and vorticity together with the prognostic equation for the streamfunction from the Navier-Stokes equations. In conservative form, these are:

$$\frac{\partial v}{\partial t} = \frac{1}{r} J(v) + \frac{1}{r} \frac{\partial v}{\partial z} + \frac{1}{Re} \left[\frac{\partial^2 v}{\partial z^2} + \frac{\partial^2 v}{\partial r^2} + \frac{1}{r} \frac{\partial v}{\partial r} - \frac{v}{r^2} \right], \quad (1)$$

$$\frac{\partial \eta}{\partial t} = J\left(\frac{\eta}{r}\right) + 2\frac{v}{r} \frac{\partial v}{\partial z} + \frac{1}{Re} \left[\frac{\partial^2 \eta}{\partial z^2} + \frac{\partial^2 \eta}{\partial r^2} + \frac{1}{r} \frac{\partial \eta}{\partial r} - \frac{\eta}{r^2} \right], \quad (2)$$

and

$$\frac{\partial^2 \psi}{\partial z^2} + \frac{\partial^2 \psi}{\partial r^2} - \frac{1}{r} \frac{\partial \psi}{\partial r} = -r\eta, \quad (3)$$

where

$$J = \frac{\partial \psi}{\partial r} \frac{\partial}{\partial z} - \frac{\partial \psi}{\partial z} \frac{\partial}{\partial r}.$$

The upstream boundary condition is chosen so as to mimic the flow field found experimentally in vortex cores. The model chosen for this study is that of Mager (1972), with which he conducted an integral analysis of vortex breakdown.

Specifically, for the upstream boundary ($z = 0$) we have:

$$\begin{aligned} u(r) &= 0, \quad \text{for } 0 \leq r \leq R, \\ v(r) &= \begin{cases} Vr(2-r^2), & \text{for } 0 \leq r \leq 1; \\ V/r, & \text{for } 1 \leq r \leq R, \end{cases} \\ w(r) &= 1, \quad \text{for } 0 \leq r \leq R. \end{aligned}$$

From these, we have

$$\psi = \frac{r^2}{2} \quad \text{and} \quad \eta = 0, \quad \text{for } 0 \leq r \leq R.$$

V is the specified azimuthal velocity at the core edge, and is equal to the circulation around the core after non-dimensionalization by $2\pi\delta W$. The cubic form of $v(r)$ allows for solid-body type rotation near the core centre and a smooth transition to irrotational flow at the core edge.

On the axis of symmetry ($r = 0$), there are no radial motions or shear stresses and together with continuity we have

$$v = \psi = \eta = 0, \quad \text{for } 0 \leq z \leq Z.$$

Artificial boundary conditions have to be used at the downstream boundary ($z = Z$). It will be assumed that this boundary will be placed far enough downstream so as to have negligible effect on the evolution of the flow further upstream. To

this end the condition that axial gradients vanish is imposed. This is less restrictive than specifying *a priori* the flow at the boundary. Care needs to be taken in having a long enough tube as 'eddies' are advected downstream in some cases and the effects of their flow past the downstream boundary are reflected upstream to a certain extent. The boundary conditions used are :

$$\frac{\partial v}{\partial z} = \frac{\partial \eta}{\partial z} = \frac{\partial^2 \psi}{\partial z^2} = 0, \text{ for } 0 \leq r \leq R.$$

At the radial boundary ($r = R$), for this preliminary study, frictional effects have been ignored, with the following boundary conditions being implemented :

$$v = \frac{V}{R}, \psi = \frac{R^2}{2} \text{ and } \eta = 0, \text{ for } 0 \leq z \leq Z.$$

As an initial condition, the upstream boundary condition is implemented over $0 \leq z \leq Z$. This then gives a uniform columnar vortex and satisfies the other boundary conditions.

The system of equations (1)–(3) together with the corresponding boundary conditions has been solved by means of a finite-difference technique using second-order accurate central differencing for the differential equations and second-order one-sided differencing for the derivative boundary conditions. The time-marching procedure used is due to Miller and Pearce (1974), and the generalized cyclic reduction method of Sweet (1974) is employed to solve the prognostic equation for the streamfunction.

After a number of trials, it was found that a tube length of about 40 vortex core radii was needed to ensure that the outflow boundary conditions did not have any noticeable effects on the evolution of the recirculation zone. A 5 : 1 tube to core radius ratio was used in order to keep blockage effects down. Experimental investigations use similar or smaller ratios. Typically, the numerical model has shown that as this ratio is increased, the radius of the recirculation bubble grows larger. The results presented here are for $R = 5$ and $Z = 40$ with 81 grid points in r and 321 in z , giving a spatial resolution of 26,001 mesh points on a regular grid. Fine temporal resolution is ensured by using $\Delta t = 0.01$ whereas the C.F.L. condition for the system is satisfied by $\Delta t = 0.085$ in the worst case considered.

RESULTS AND DISCUSSION

A completely consistent picture of the flow inside the recirculation zone of an axisymmetric breakdown bubble is lacking. Previous numerical simulations (eg. Kopecky and Torrance 1973, Grabowski and Berger 1976) failed to simulate significant features which are generally agreed upon by flow visualization experiments – the most probable reason for this is poor spatial resolution together with the use of the steady state form of the governing equations or poor time-marching techniques. To date, the only other simulation which reveals the 'two-celled' structure of the bubble is that of Krause, Shi and Hartwich (1982). Their calculations however were not carried out for long enough times to observe the vortex ring shedding phenomenon illustrated in figure 1. Krause *et al.* (1982) employed a free-stream radial boundary condition, which was also employed while testing this model. It was found that the initial evolution of the recirculation zone was not affected by the radial boundary conditions, and our results agreed quite well with those of Krause *et al.* (1982), who employed an Alternating Direction Implicit method of solution.

The difference between the evolutions in a free-stream and a tube begin to show when, in a free-stream, the streamlines near the radial boundary are no longer parallel to that boundary, in which case the recirculation zone tends to grow beyond the computational domain. The resulting evolution is then no longer physically valid. The effect of a tube boundary condition is to limit this radial expansion of the recirculation zone and thus keep the streamlines near the boundary parallel to it. This then leads to a periodic shedding of vortex rings from the downstream end of the bubble, which mimics the periodic emptying and filling process which is often observed experimentally (Sarpkaya 1971, Faler and Leibovich 1977, Escudier and Keller 1983). An example of this phenomenon is given in figure 1 where a close-up of the evolution of the breakdown bubble and the near wake is given. A number of significant features, which are reminiscent of descriptions of flow visualizations and laser-Doppler anemometer (LDA) measurements emerge.

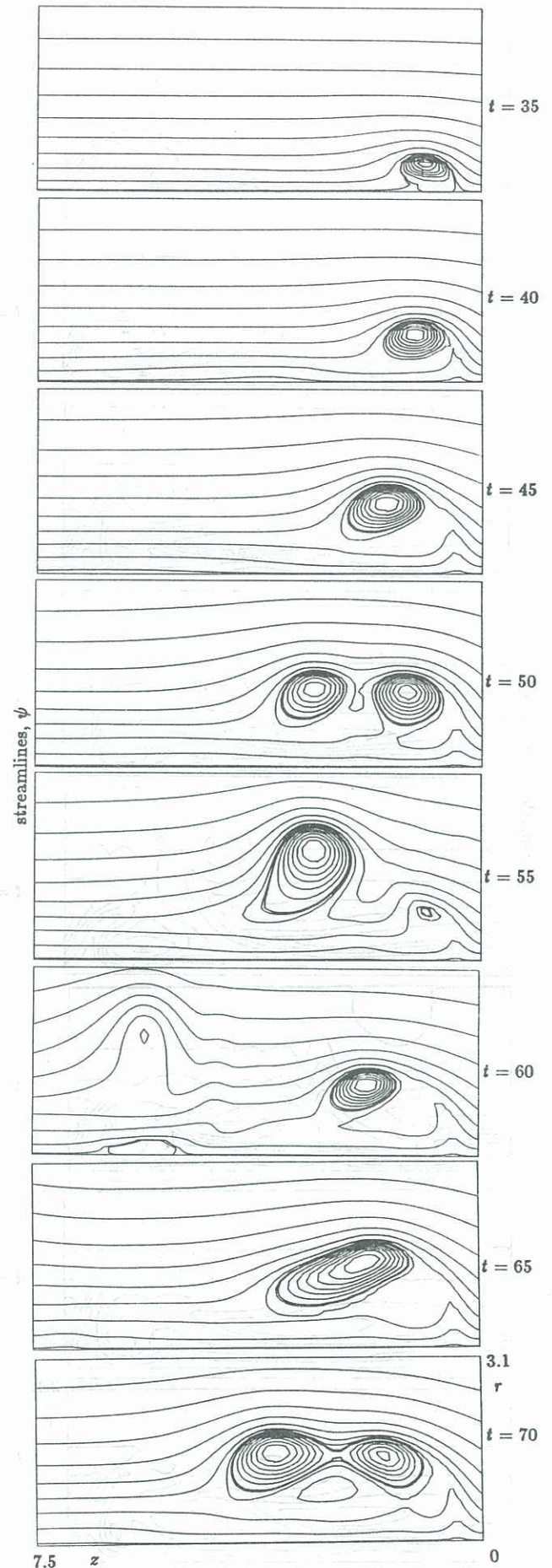


Fig 1: Isotachs of ψ , v and η at indicated times for $Re = 240$ and $V = 1.0$ – close-up of the breakdown and near wake region.

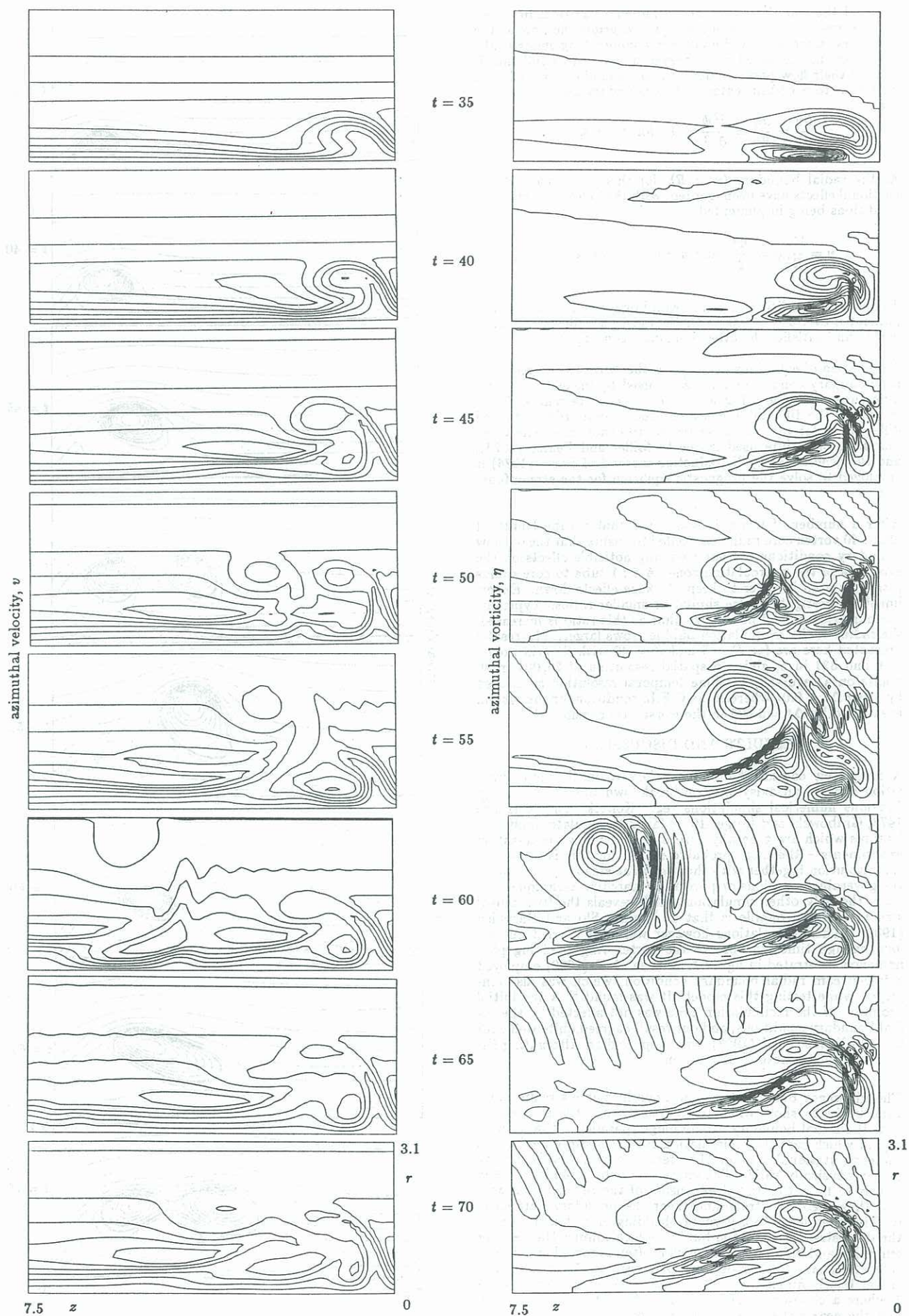


Fig 1: (continued.)

The measurements of Faler and Leibovich (1977) are claimed to show four stagnation points on the axis – however, the position of the axis was redefined to achieve this. What they are probably seeing are saddle points just off the axis with very slow positive axial flow along the axis. This would then be consistent with the LDA measurements and flow visualizations of Escudier and Keller (1983) where they observe the fluid entering the bubble to emanate from a region much smaller in radius than that of the vortex core and the bubble itself, and the axial flow on the axis to be positive. Their streamline map also shows the ‘two-celled’ structure. Bellamy-Knights (1976) concludes from his flow visualization experiments that there is axial backflow somewhere in the bubble but not along the axis, and hence that the backflow must occur over an annular region of the bubble. Uchida, Nakamura and Ohsawa (1985) also use LDA and flow visualization to examine axisymmetric breakdown bubbles and conclude that a hole in the head part of the bubble is often observed, that some of the smoke (used to visualize the flow) is entrained from the rear part of the bubble and reaches the neighbourhood of this hole and then flows back downstream, while other smoke particles are sucked directly from this hole into the interior of the bubble. Further, they observe a dark region in the center of the bubble which, from LDA measurements, has a positive axial velocity and leads to the hole at the head part. Faler and Leibovich (1977) observe that the head of the bubble is relatively steady, whereas the downstream portion is subject to large variations in both axial and azimuthal velocities.

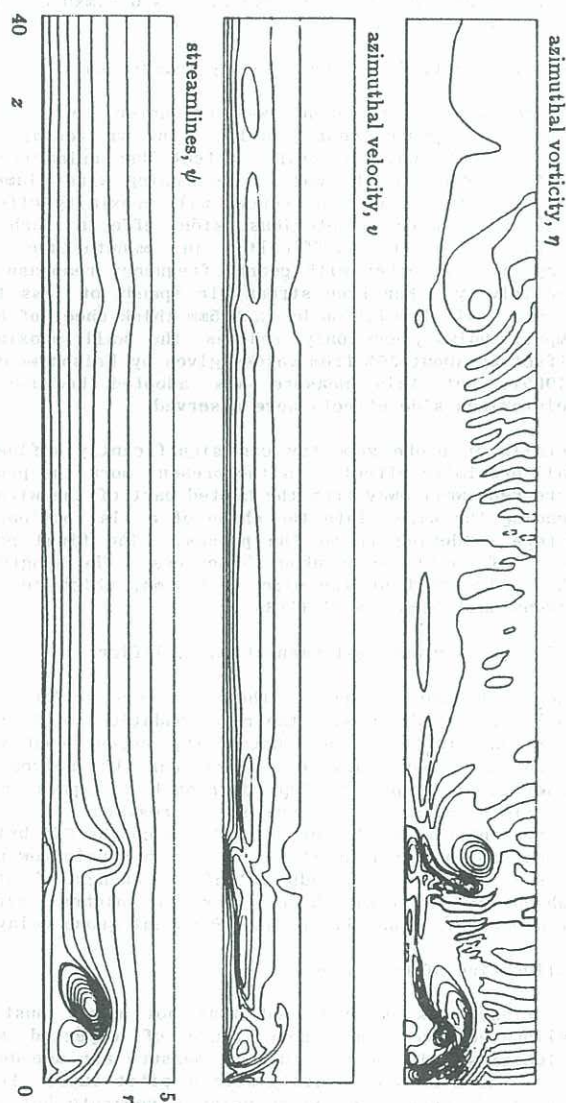


Fig 2: Isotachs of ψ , v and η at $t = 90$ for $Re = 240$ and $V = 1.0$ – full computational domain.

All these salient features, from numerous experimental observations, are well reproduced by the numerical model. The upstream portion of the bubble, whether we focus our attention on the streamlines (the contour levels for the recirculation zones are linearly spaced, whereas the other contour levels are quadratically spaced from zero on the axis), the azimuthal velocity or the azimuthal vorticity from figure 1, clearly has minimal temporal variations, whereas the rest of the bubble undergoes large variations as a result of the shedding process. From examining the plots of the complete computational domain, an example of which is given in Figure 2 at time $t = 90$, we find that once the vortex ring is completely shed, it is advected downstream at approximately the mean axial velocity. Also, from figure 1, the head of the bubble has very little azimuthal velocity since it primarily consists of fluid which originates in the axial region which has low angular momentum. Just inside the bubble the flow is nearly stagnant.

The cyclic behaviour of the shedding process is also evident from Figure 1, and in Figure 2 at a later time $t = 90$ the system has returned essentially to the configuration of $t = 60$. A simplified schematic of the shedding process is given in Figure 3. The ‘hole’ referred to by Uchida *et al.* (1985) is clearly seen, as is the manner in which the bubble is fed. Clearly, the particle paths will need to be computed for such an unsteady flow before a more definitive description of the flow inside the bubble can be arrived at. However, the schematic diagram summarizes the main features of the experimental observations.

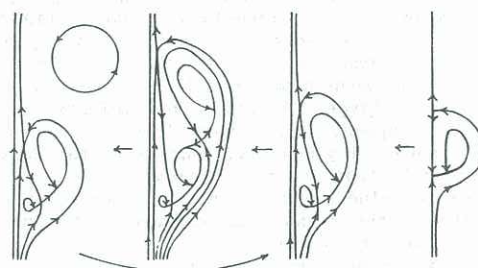


Fig 3: Simplified schematic of the shedding process.

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