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Laminar Fluid Convection of Varying Prandtl Number in the Annuli of Rotating Cylinders

T. S. LEE

Department of Mechanical & Production Engineering, National University of Singapore, Singapore 0511.

ABSTRACT

Convective heat transfer in the annular space between horizontal cylinders when the inner cylinder is heated and rotating, has been investigated numerically for fluids of various Prandtl numbers (Pr). Experimental verification for Prandtl number of 0.7 (air) is based on a calorimetric technique. Comparison is made between numerical and experimental results for moderate radius ratios with different eccentricities. The overall equivalent thermal conductivity (K_{eq}) is obtained for Rayleigh numbers (Ra) up to 10^6 with rotational Reynolds number (Re) varies from 0 to the order of 10^3. Investigation shows that for $PrRa/(Re)^2 > 1$, the numerical model shows promising results when compared with the experimental results. For $Pr \ll 1$ interesting isotherms and streamlines were also observed. The above investigations are for a radius ratio of 2.6.

INTRODUCTION

A comprehensive review of the work involving concentric and eccentric cylindrical annuli have been collated by Kuehn & Goldstein [1,2]. The reported studies, however, were concerned with stationary cylinders and few consider the rotation of the inner cylinder with varying Prandtl number. These parameters were first studied for the concentric case by Gardiner & Sabersky [3]. The primary objective of this paper is thus to study the fluid motion and heat transfer in concentric and eccentric annular spaces of moderate radius ratio when the inner cylinder is heated and rotating at low Reynolds number with varying Prandtl numbers.

MATHEMATICAL MODEL

A schematic configuration of the annulus is shown in Fig. 1. Fluid motion in the annular space is assumed to be laminar and two dimensional. End effects of the rotational cylinder are assumed negligible and that the Boussinesq approximation is valid. The governing equations in dimensionless form are:

$$\frac{\partial \zeta}{\partial t} + \nabla \cdot (\zeta \underline{U}) = - Pr\, Ra\, \nabla x(T\hat{g}) + Pr \nabla^2 \zeta \qquad (1)$$

$$\nabla^2 \psi = -\zeta \qquad (2)$$

$$\frac{\partial T}{\partial t} + \nabla \cdot (T\underline{U}) = \nabla^2 T \qquad (3)$$

Velocities in dimensionless form are

$$U_x = \frac{\partial \psi}{\partial y} \text{ and } U_y = - \frac{\partial \psi}{\partial x} \qquad (4)$$

The transformation between the Cartesian x-y coordinate system and the bipolar ξ-η coordinate system is given by

$$x = c \sinh \eta/(\cosh \eta - \cos \xi) \qquad (-\infty < \eta < \infty) \qquad (5)$$

$$y = c \sin \xi/(\cosh \eta - \cos \xi) \qquad (0 \leq \xi \leq 2\pi)$$

where c is a scaling factor of the transformation related to the eccentricity ratio e and the radius ratio of the two cylinders.

NUMERICAL METHOD

The finite difference solutions of equations (1) - (4), with their boundary conditions, are obtained at the nodal points of the solution region as shown in Figure 1(b). A second order finite differing approach and an up-wind differencing method were used. A convergence criterion of the form $(\Phi - \Phi_{ref})/\Phi_{ref} \leq 0.001$ is used for the temperature field and the overall thermal conductivity K_{eq} to indicate steady state convergence. The stream function ψ_o on the outer cylinder wall is arbitrarily set to zero. At the inner rotating cylinder wall, ψ_i cannot be pre-assigned. The use of $\frac{\partial \psi}{\partial n}$ = wall velocity, gives solution for which $\frac{\partial \psi}{\partial s} \neq 0$ along the wall of the inner cylinder. This implies that fluid was numerically 'leaked' through the moving inner cylinder wall. In the present study ψ_i is determined using the criterion that the pressure distribution in the solution region is a single-valued function. Mathematically, this criterion implies that the line integral of the pressure gradient $\frac{\partial P}{\partial s}$ along any closed loop circumscribing the inner cylinder is zero i.e. $\int \frac{\partial P}{\partial s} ds = 0$. $\frac{\partial P}{\partial s}$ can be evaluated from the momentum conservation equations. From the stream function solution, the vorticity along the solid walls is then evaluated from

$$\zeta_{wall} = - \frac{1}{h^2} \frac{\partial^2 \psi}{\partial \eta^2} \Big|_{wall} \qquad (6)$$

where $h = c/(\cosh \eta - \cos \xi)$

The temperatures are $T_i = 0$ at the inner wall and $T_o = 1.0$ at the outer wall. Heat transferred per unit length of the annulus is computed by numerical integration of the surface temperature gradient around the surface concerned. Heat transfer is characterized by the overall equivalent thermal conductivity K_{eq}:

$$K_{eq} = \frac{\text{Heat transferred through the annulus per unit length}}{2\pi k \Delta T'/\ln(S)} \qquad (7)$$

where S is a scale constant.

The denominator of the expression for K_{eq} is the rate of heat transfer per unit length by pure conduction in a motionless medium having the same thermal conductivity as the fluid. The local equivalent thermal conductivity K_{eql} is the corresponding point value of K_{eq} over a unit surface area. The Nusselt

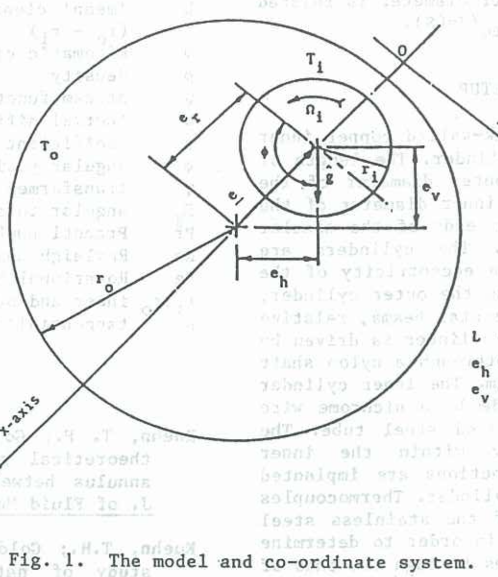

Fig. 1. The model and co-ordinate system.

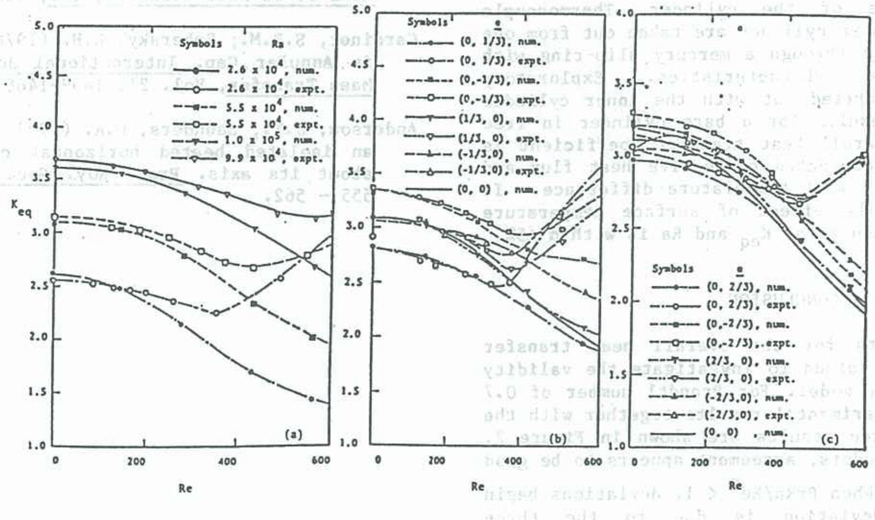

Fig. 2. Comparison of experimental and numerical results for air. Pr = 0.7, Radius ratio = 2.6 with (a) e = 0; (b) e = 1/3 and (c) e = 2/3.

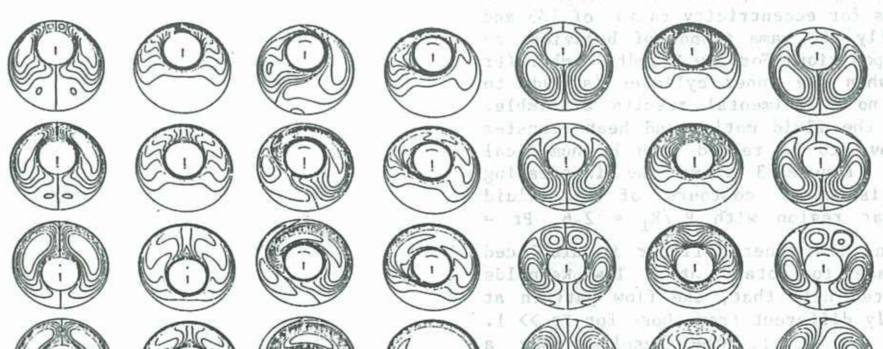

Fig. 3. Streamlines and isotherms. Vertically displaced downwards. Ra = 10^5, Radius ratio = 2.6. (a) Pr = 1.0, Re = 0; (b) Pr = 1.0, Re = 560; (c) Pr = 0.01, Re = 0 and (d) Pr = 0.01, Re = 560.

number based on the inner cylinder diameter is related to K_{eq} by the relation $Nu_i = 2\ K_{eq}/\ln(S)$.

EXPERIMENTAL SETUP

The test cell consists of a thick-walled copper inner cylinder, and a copper outer cylinder. The length of the cylinders is 262 mm. The outer diameter of the inner cylinder is 37 mm and the inner diameter of the outer cylinder is 97 mm. The two ends of the annular space are thermally insulated. The cylinders are mounted on a steel frame and the eccentricity of the annular space is varied by moving the outer cylinder, which is suspended from two horizontal beams, relative to the inner cylinder. The inner cylinder is driven by a variable speed DC shunt motor through a nylon shaft and a belt speed-reduction system. The inner cylinder is heated radiantly on the inside by a nichrome wire wound on an asbestos cloth covered steel tube. The steel tube is held centrally within the inner cylinder. Three thermocouple junctions are implanted evenly along the length of the cylinder. Thermocouples are also placed at both ends of the stainless steel shaft and in the end insulation in order to determine the correction for the heat losses through the ends of the shaft and tube. Eight thermocouples junctions are also evenly and axially located within about 0.5 mm of the inner surface of the cylinder. Thermocouple signals from the inner cylinder are taken out from one end of the cylinder through a mercury slip-ring with very low noise characteristics. Exploratory experiments were carried out with the inner cylinder to verify known results for a bare cylinder in free convection. The overall heat transfer coefficient is computed from the corrected convective heat flux and the inner and outer wall temperature difference. It is expected that the effect of surface temperature variation on the results of K_{eq} and Ra is within ±5%.

CONCLUSION

Experimental results for the overall heat transfer coefficient were obtained to investigate the validity of the mathematical model. For Prandtl number of 0.7 (i.e. air), the experimental results together with the numerically predicted results are shown in Figure 2. For concentric cylinders, agreement appears to be good when $PrRa/Re^2 > 1$. When $PrRa/Re^2 < 1$, deviations begin to occur. The deviation is due to the three dimensional instability of the flow around the heated cylinder when the inner cylinders is rotated. This was observed by Anderson and Saunders [4] using smoke visualization technique. Experimental studies at eccentric positions for eccentricity ratios of 1/3 and 2/3 show essentially the same trends of behaviour as at the concentric position. For low Prandtl number (Pr = 0.01 say) and when the inner cylinder is made to rotate, there are no experimental results available. The prediction of the fluid motion and heat transfer characteristics have to be relied upon by numerical experimentations. Figure 3 shows the interesting streamlines and isotherms contours of the fluid between the annular region with $R_o/R_i = 2.6$, Pr = 0.01, Ra = 10^5 and the inner cylinder is displaced vertically and made to rotate at a low Reynolds number. It is noted here that, the flow pattern at low Pr is distinctly different from those for Pr >> 1. For fluids with Pr << 1, the results show a multicellular flow pattern which contrasts with the usual monocellular flow field obtained for Pr >> 1 fluids. As a consequence of this multicellular flow pattern, the heat transfer profiles for fluids with Pr << 1 were found to have points of maximum and minimum at the interior nodes, instead of the top and bottom nodes which is known for Pr >> 1. Therefore, the results for high Pr fluids should be used with care to predict those for low Pr fluids.

NOMENCLATURE

e	eccentricity ratio, $e = e_r/L$
L	'mean' clearance between the two cylinders, $L = (r_o - r_i)$
ν	kinematic viscosity
ρ	density
ψ	stream function
α	thermal diffusivity
β	coefficient of volumetric expansion
ϕ	angular position of gravity as defined in Fig. 1
γ	transformed ϕ
Ω_i	angular rotational rate of inner cylinder
Pr	Prandtl number, $Pr = \nu/\alpha$
Ra	Rayleigh number, $Ra = g\ \beta\ \Delta T' L^3/(\nu\alpha)$
Re	Rotational Reynolds number, $\rho Re = (r_i \Omega_i L)/\nu$
r_i, r_o	inner and outer cylinder radius respectively
s	tangential directional vector

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