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Structural Similarity of Turbulence in Fully Developed Pipe Flow

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ABSTRACT

The structure of fully developed turbulence in a smooth pipe has been studied in detail for a Reynolds number of 134,000 (based on centre-line velocity and pipe radius) at two fixed distances from the wall (y^+=70 and 200). By taking Fourier transforms of the correlations of the longitudinal component of turbulence, three dimensional power spectral density functions were obtained with frequency ω and longitudinal and transverse wavenumbers k_x and k_z as independent variables. The data presented in this form shows the distribution of turbulence intensity among waves of different size and inclination and provides an estimate of the convection velocity as well as the lifetime of individual waves. The data reproted cover a wave size range of about 20 and substantially verify the similarity hypothesis.

INTRODUCTION

The understanding of the mechanism of turbulence is fundamental to predicting and improving the performance of heat and momentum transfer in everyday engineering situations. Fully developed flow in a smooth pipe, being one of the simplest cases of shear flow turbulence, has received much attention in the past, for example, Favre, Gaviglio & Dumas (1957), Sabot & Comte-Bellot (1973) and Perry & Abell (1975), and is the subject of this paper. The turbulent velocity field in a smooth circular pipe can be described by two-point space-time correlations which involve three turbulent velocity components (u,v,w) with six pair combinations. By assuming stationarity in x, z and t variables (defined in Figure 1), there are still five arguments in the correlation functions: x_1-x_2 z_1-z_2, t_1-t_2, and y_1 and y_2. If N is the number of points required to define a correlation function in any of the four coordinates (three space and time) then $6N^5$ data are necessary for a full description of the two-point correlations of three turbulent velocity components at each Reynolds number. For N=20, the total number of data points required amounts to 2×10^7. It can thus be seen that description of turbulence by multi-point space-time correlations would render the interpretation of these data almost impossible. Furthermore, the structural interpretation of time-delayed correlations and data in the untransformed variables ($\Delta x,\Delta z,\Delta t,y_1,y_2$) is difficult.

By performing measurements of the longitudinal $R_{uu}(\Delta x,\ \Delta z = 0,\ \omega,\ y_1 = y_2)$ and transverse $R_{uu}(\Delta x=0,\Delta z,\omega,y_1=y_2)$ correlations of the longitudinal component u of turbulence in narrow frequency bands and by taking Fourier transforms of the correlation functions so that power spectral densities are formed with frequency, ω, and longitudinal or transverse wave-numbers, k_x or k_z, as the independent variables, Morrison & Kronauer (1969) successfully demonstrated that their data may be interpreted in the light of a stochastic wave model with coordinates as specified in the wave schematic diagram in Figure 2. By introducing a similarity variable k^+y^+ based on the wave number k^+ and the distance y^+ from the wall, Morrison & Kronauer (1969) were able to collapse their turbulence data in wavenumber space. In particular, the two-dimensional power plots can be written as

$$P(\omega^+,k^+y^+) = f(k^+y^+)A(\omega^+,k_z^+) \quad (1)$$

where f is the 'wave intensity function'
A is the 'wave strength'
$k = [k_x^2 + k_z^2]^{1/2}$ $\quad k^+ = k\nu/U_\tau$
$y^+ = yU_\tau/\nu$ $\quad \omega^+ = \omega\nu/U_\tau^2$
$\alpha = \tan^{-1} k_x/k_z$
ν is the kinematic viscosity of the fluid
U_τ is the friction velocity

and superscript + refers to nondimensionalisation with respect to ν and U_τ.

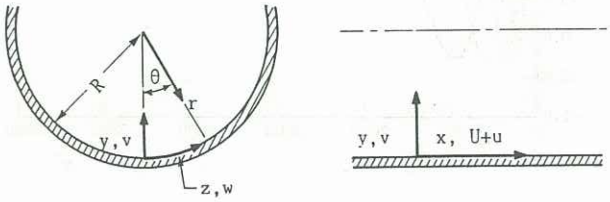

Fig.1: Pipe coordinates and velocity components.

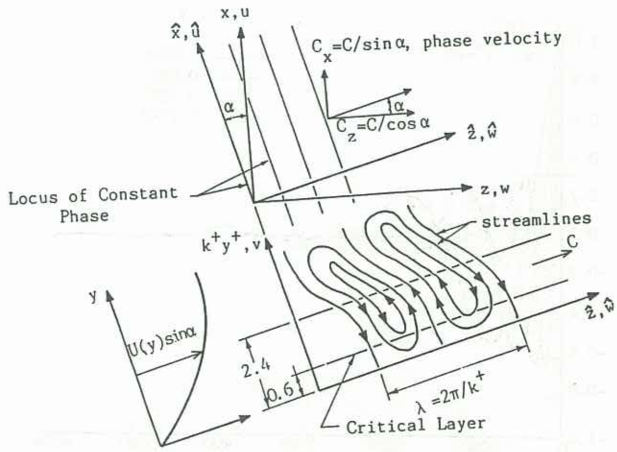

Fig. 2: Wave schematic diagram.

The wave description, if verified to be valid, will therefore, reduce the massive correlation data to one 'wave strength function' as a function of two variables (k_x^+ and k_z^+) and a few auxiliary functions of one variable (k^+y^+) only. This implies all the properties of turbulence can be summarized in the power distribution over the power sheet and the function of k^+y^+. The objectives of this study were to extend the measurements of Morrison & Kronauer (1969) by obtaining new three-dimensional spectra at $y^+=70$ and $y^+=200$, both for a friction velocity U_τ of 0.61 m/s and to examine evidence of the structural similarity of turbulence in pipe flow.

EXPERIMENTAL CONDITIONS

Correlations $R_{uu}(\Delta x^+, \Delta z^+ | \omega^+)$ of the longitudinal component of turbulence u in a smooth pipe of diameter 254 mm and length 14.675 m were generated simultaneously in seven narrow frequency bands at two fixed distances from the wall, namely, $y^+=70$ and 200, by using an automated data acquisition system which jointly varied the longitudinal and transverse separations of two hot-wire probes operated in constant temperature mode. For simplicity, $R_{uu}(\Delta x^+, \Delta z^+ | \omega^+)$ will be written as $R(x^+, z^+|\omega^+)$ hereafter. The pipe flow Reynolds number was 134,000 (based on centre-line velocity and pipe radius R) and the centre frequencies of the bandpass filters were 205,257,325,409,515,650 and 819 Hz corresponding to non-dimensional circular frequencies ω^+ of 0.0536, 0.0672, 0.0850, 0.1070, 0.1374, 0.1700, and 0.2142 respectively. The measurement stations were located between 52.75 and 56.75 pipe diameters downstream of the pipe entrance. A total of 1306 and 2551 spatial separations were used for generating the correlation $R(x^+,z^+| \omega^+)$ for $y^+=70$ and 200 respectively. The corresponding maximum longitudinal separations are $x^+=6500$ and 10500 and the maximum transverse separations are $z^+=572.28(6.65°)$ and 1382.22(16.5°). The longitudinal resolution x^+ for both $y^+=70$ and 200 is 40, corresponding to spatial Nyquist wavenumber $k_x^+=0.079$ whereas the respective transverse resolutions z^+ are 30.12 and 25.13, corresponding to $k_z^+=0.10$ and 0.125.

By taking Fourier transforms of the correlations, power spectral density functions $\Phi(k_x^+,k_z^+, \omega^+)$ were obtained with frequency ω^+ and longitudinal and transverse wavenumbers, k_x^+ and k_z^+, as independent variables.

CORRELATIONS AND SPECTRAL DENSITY RESULTS

The longitudinal correlation results $R(x^+,0|\omega^+)$ for four different frequencies ω^+ at $y^+=70$ and $y^+=200$ are displayed in Figure 3 (a) and (b) respectively. The results show the same trend as those reported by Morrison & Kronauer (1969). Contours for power spectral density functions are presented in the form $k_x^+ k_z^+ \Phi(k_x^+,k_z^+, \omega^+)$, hereafter denoted as $P(k_x^+,k_z^+,\omega^+)$, with typical plots being shown in Figure 4(a) and (b) for $y^+=70$ and 200, both being for $\omega^+=0.2142$.

It is obvious from Figure 4 that the longitudinal wavenumber k_x^+ is constrained by the frequency ω^+ which has virtually no effect on the transverse wavenumber k_z^+. In order to illustrate the general effects of frequency ω^+ and the distance of the wall y^+ on the power spectra, the spectral peaks for various ω^+ are plotted in Figure 5. Lines of constant wave size k^+ and constant wave angle α are also indicated. The lines of best fit to the data, with correlation coefficient better than 0.97, for $y^+=70$ and 200 are given respectively by

$$k_z^+ = 0.0052 + 1.47\, k_x^+ \tag{2a}$$

$$k_z^+ = 0.0029 + 1.16\, k_x^+ \tag{2b}$$

The slopes of the two lines are in the ratio of 1.27 which compares well with the ratio log 200/log 70. It is evident that at a given distance from the wall, the spectral peak moves from a smaller wave angle at lower frequency to a larger wave angle at higher frequency. Clearly there is a constraint with a wave model hypothesis on the minimum size of $k_z^+ = 2\pi/R^+ = 0.00126$ but there is no such constraint on k_x^+. Therefore it must imply that as the wave size component sampled at a particular ω^+ and y^+ is increased there is more $\hat{u}$ component in u (Fig. 2). For a given ω^+, the spectral peak at $y^+ = 200$ is associated with a wave of larger size and larger wave angle than at $y^+=70$. Furthermore, for a given ω^+, waves of the smaller size that exist at $y^+=70$ do not extend to $y^+=200$.

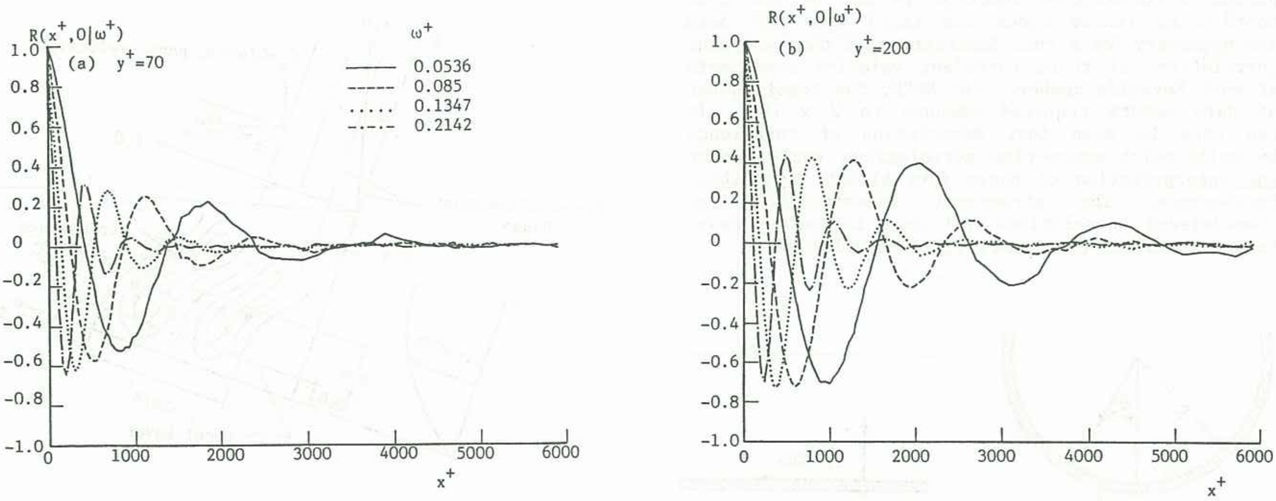

Fig. 3: Longitudinal correlation for (a) $y^+=70$ and (b) $y^+= 200$.

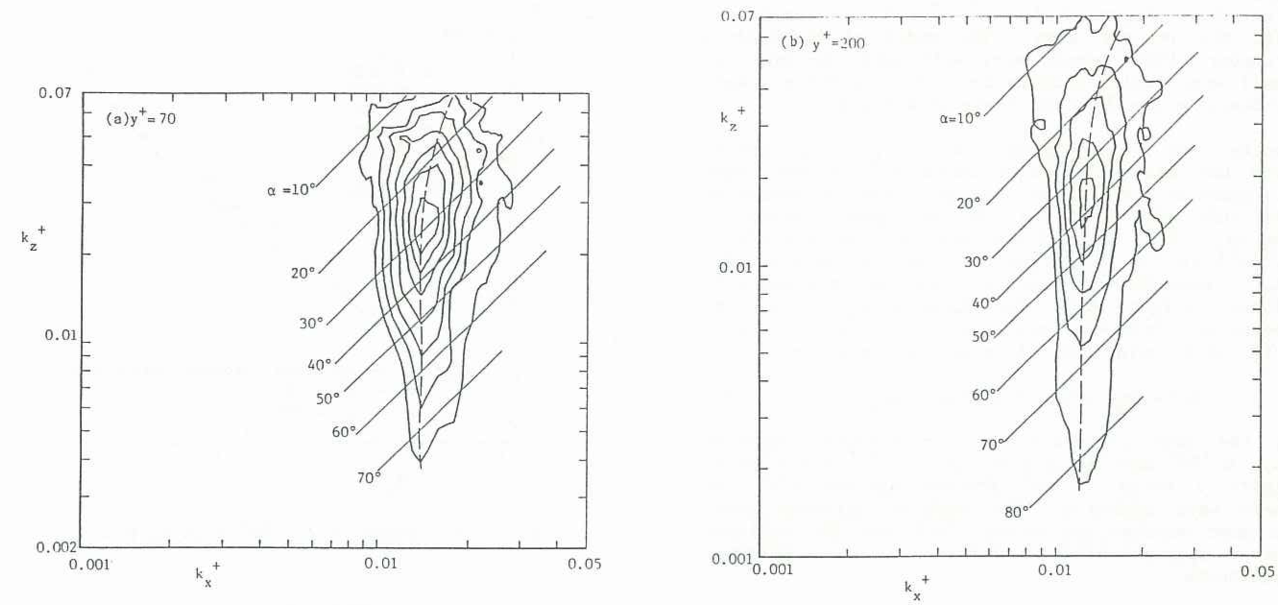

Fig. 4: Power spectral density contours for $\omega^+=0.2142$.

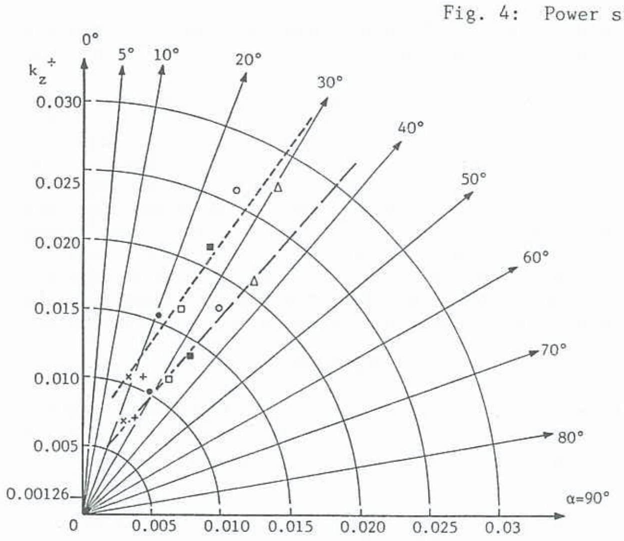

Fig. 5: Locus of spectral peaks. (Symbols same as in Fig. 8.)

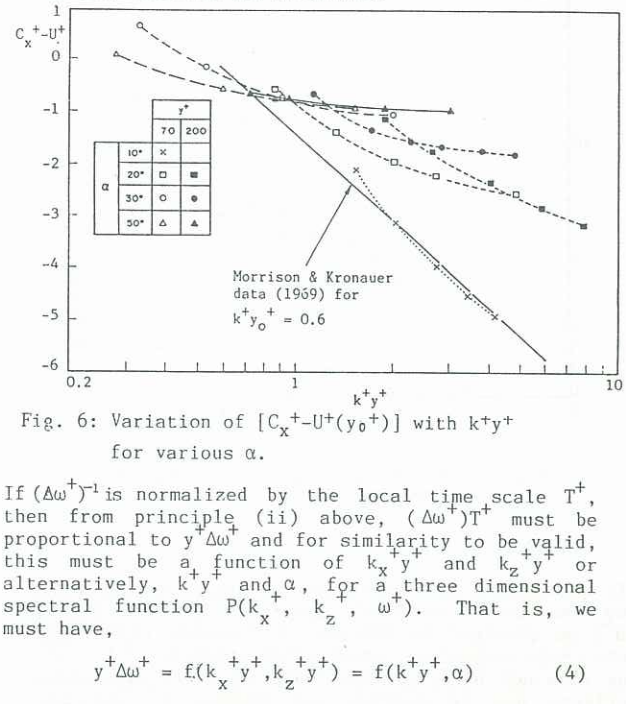

Fig. 6: Variation of $[C_x^+-U^+(y_0^+)]$ with k^+y^+ for various α.

STRUCTURAL SIMILARITY OF TURBULENCE

The basic principles of the similarity hypothesis are listed as follows:

(i) There is only one characteristic length scale, that is, distance from the wall, y^+.

(ii) There is only one characteristic time scale, T^+, that is $(dU^+/dy^+)^{-1}$. Because of the logarithmic distribution of the mean velocity U^+, T^+ is proportional to y^+.

(iii) Absolute velocity is irrelevant and only velocity relative to the local velocity U^+ is relevant.

As a result of principle (iii), the experimental data will be examined for structural similarity in a coordinate system with no relative motion, that is, a coordinate system moving with the average local speed $U^+(y^+)$. Consider a two dimensional power spectrum as a function of ω^+ and k_x^+. Then $(\Delta\omega^+)^{-1}$ associated with each k_x^+ can be interpreted as the typical lifetime for a disturbance of that size. In a coordinate system with no relative motion, this is expressed as

$$\Delta\omega^+ = \omega^+ - U^+k_x^+ \tag{3}$$

If $(\Delta\omega^+)^{-1}$ is normalized by the local time scale T^+, then from principle (ii) above, $(\Delta\omega^+)T^+$ must be proportional to $y^+\Delta\omega^+$ and for similarity to be valid, this must be a function of $k_x^+y^+$ and $k_z^+y^+$ or alternatively, k^+y^+ and α, for a three dimensional spectral function $P(k_x^+, k_z^+, \omega^+)$. That is, we must have,

$$y^+\Delta\omega^+ = f(k_x^+y^+, k_z^+y^+) = f(k^+y^+, \alpha) \tag{4}$$

Substituting equation (3) into equation (4), we get

$$C_x^+ - U^+ = g(k^+y^+, \alpha) \tag{5}$$

where $C_x^+ = \omega^+/k_x^+$.

Convection velocity C_x^+ as a function of α can be determined from the ridge line of the power spectral density contours as shown in Figure 4 and are plotted as $[C_x^+ - U^+]$ against k^+y^+ for various α as shown in Figure 6.

For small α, there is some scatter in the data; however, within the limits of experimental accuracy, the results are independent of y^+ and collapse very well for a given α, thus supporting the similarity hypothesis. According to the geometrically similar wave model of Morrison & Kronauer (1969), C_x^+ matches the mean fluid velocity at a certain y^+ so that

$$C_x^+(k^+y_0^+)-U^+(y_0^+)= 5.756 \log(k^+y_0^+/k^+y^+) \tag{6}$$

Here y_0^+ is the distance to the wall where the data are taken.

Equation (6) is a subset of equation (5) and is plotted in Figure 6 for $k^+y_0^+=0.6$ for comparison with the present data. The model of Morrison & Kronauer (1969) agrees very well with the data for small wave angles such as $\alpha=10°$ but is not adequate to describe the data for large wave angles.

Another way of examining similarity is to extact from the data information about $\Delta\omega^+$ in the light of equation (4). This can be found by measuring the full width between the half-power points in the (k_x^+, k_z^+) plot of the spectral function $P(k_x^+, k_z^+, \omega^+)$ in the k_x^+ direction, the half-width being Δk_x^+. Assuming the spectral sheet is thin and very closely aligned with the locus $\omega^+=k_x^+U^+$, we can deduce $\Delta\omega^+$ from the power spectral density contours (Fig. 4) by using the following relationship:

$$\Delta\omega^+ = \Delta k_x^+(\omega^+/k_x^+) \simeq U^+(y_0^+)\Delta k_x^+ \qquad (7)$$

By the same arguments used in deducing equation (4), $y_0^+\Delta\omega^+$ must be a function of k^+y^+ and α only. Figure 7 shows $y_0^+\Delta\omega^+$ plotted against k^+y^+ for three wave angles α. For each α, although there is some scatter, the data at $y^+=70$ and 200 collapse onto a straight line as predicted by the similarity hypothesis.

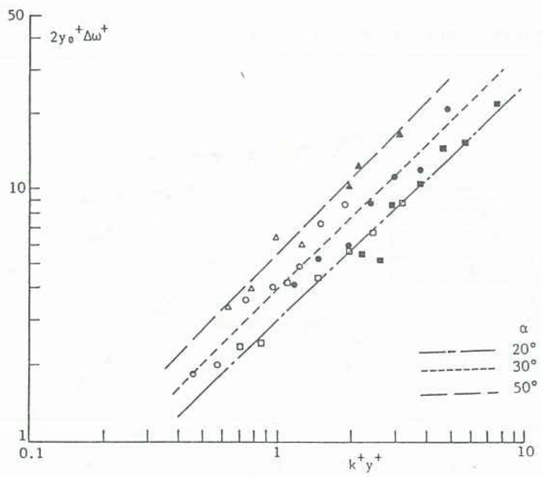

Fig. 7: Variation of $2y_0^+\Delta\omega^+$ with k^+y^+
(Symbols same as in Fig. 6.)

In principle, similarity should apply equally well to the two dimensional spectral function $P(k_x^+, \omega^+)$ as obtained by Morrison & Kronauer (1969) and the three dimensional spectral function $P(k_x^+,k_z^+,\omega^+)$ as obtained here, provided that spectral truncation effects due to minimum and maximum k^+ are not important, or that Reynolds number effects are not significant if data for different Reynolds numbers are compared. To enable a comparison with the data of Morrison & Kronauer (1969), the three dimensional spectral function data $P(k_x^+,k_z^+,\omega^+)$ obtained here are integrated over k_z^+ to give the two dimensional spectral function $P(k_x^+,\omega^+)$ from which $2y_0^+\Delta\omega^+$ is extracted and plotted for $y^+=70$ and $y^+=200$ against $k_x^+y^+$ in Fig. 8. The agreement with the data of Morrison & Kronauer (1969) which only extend up to $k_x^+y^+=1$ is encouraging.

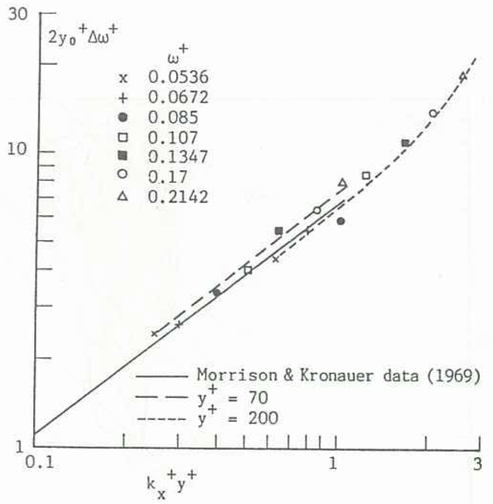

Fig. 8: Variation of $2y_0^+\Delta\omega^+$ with $k_x^+y^+$.

CONCLUSIONS

Three dimensional power spectral density contours have been presented for $y^+=70$ and 200 and the convection velocity results when reduced in a coordinate system moving with the average local speed have been shown to be a function of k^+y^+ and α only. The geometrically-similar wave model of Morrison & Kronauer (1969) has been shown to be valid for small wave angles only. An estimate of the structural lifetime $(\Delta\omega^+)^{-1}$ of turbulence has been obtained from the spectral sheet thickness and when normalised by the local time scale T^+ collapses onto a straight line in a plot against k^+y^+ as independent variable for a given α. All these results substantiate the similarity hypothesis.

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