9th AUSTRALASIAN
FLUID MECHANICS CONFERENCE
AUCKLAND, 8-12 DECEMBER 1986

Temperature Variance and Kinetic Energy Budgets in the Near-Wall Region of a Turbulent Boundary Layer

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ABSTRACT

After establishing that consistent temperature measurements can be obtained in the near-wall region of a turbulent boundary layer, an experimental budget of the temperature variance is constructed in this region. The turbulent diffusion term, obtained by difference, exhibits a plausible near-wall distribution. Using this distribution, published results for the near-wall turbulent kinetic energy budget are slightly modified to yield more plausible distributions of turbulent energy diffusion due to the velocity fluctuation normal to the wall and the pressure fluctuation.

INTRODUCTION

Although turbulence modelling in the vicinity of a wall has made extensive use of the wall function approach (e.g. Launder, 1982), current computational treatments (e.g. Shih and Lumley, 1986) of wall flows point to an abandonment of the wall function approach for dealing with the thin region adjacent to the wall where viscosity modifies the turbulence structure. Launder (1984) gives several examples where the abandonment of the wall function approach has resulted in the improved prediction of convective heat transfer rates. However, the development of reliable turbulence closure models for the near-wall region depends on the availability of accurate data in this region. For example, the extension of the k-ε model to the near-wall region would require measurements of the important terms in the transport equations of the turbulent kinetic energy and the turbulent energy dissipation. Several key terms, such as the turbulent diffusion and the dissipation, in the kinetic energy budget are not easily amenable to measurement. Further, the exact behaviour of the pressure diffusion term is not known and an objective assessment of the accuracy of this term is difficult. It is however possible to either measure or infer indirectly the corresponding terms in the transport equation for the temperature variance. After providing some evidence that consistent temperature data can be obtained in the near-wall region of a turbulent boundary layer and that a plausible experimental budget of the temperature variance can be constructed, we use the form of this budget to infer the behaviour of various terms in the transport equation for the turbulent kinetic energy.

EXPERIMENTAL ARRANGEMENT

The experiments were performed in a suction type wind tunnel at a nominal free stream velocity U_1 of 9 ms^{-1}. The boundary layer develops over the heated aluminium floor of the working section (width = 0.6 m, height = 0.12 m, length = 1.8 m). Details of the tunnel, heated plate and basic instrumentation are given in Krishnamoorthy and Antonia (1986). The temperature difference between the wall and the free stream (T_w-T_1) was approximately 9.6 K. At the measuring station (x = 1.4 m, x measured from the beginning of the working section) the thermal layer thickness δ_T and the boundary layer thickness δ_1 were approximately the same, equal to 28 mm. The Reynolds number based on the momentum thickness δ_2 is about 2000. Since the walls of the working section are parallel, the boundary layer develops with a small favourable pressure gradient.

A single cold wire was made from a 6 mm long Wollaston wire (Pt-10% Rh, core dia. d = 0.63 μm). Five wire lengths (ℓ/d = 300 to 1700) obtained by centrally etching the original wire, were used for assessing the effect of ℓ/d on statistics of the temperature fluctuation. The component $\alpha\overline{\theta_{,x}^2}$ (where α is the thermal diffusivity, $\theta_{,x} = \partial\theta/\partial x$) of the temperature dissipation $\overline{\varepsilon_\theta}$ was obtained from the temporal derivative and Taylor's hypothesis using a single cold wire (0.63 μm dia., ℓ/d = 300). The other components of $\overline{\varepsilon_\theta}$, $\alpha\overline{\theta_{,y}^2}$ and $\alpha\overline{\theta_{,z}^2}$ were obtained from the temperature autocorrelation function measured with a pair of parallel cold wires (0.63 μm dia., ℓ/d = 800) aligned in the z direction.

An in-house circuit supplied a constant current of 0.1 mA to the cold wires. The mean and rms temperature profiles were obtained by analogue measurements using 1076 digital voltmeter and a DISA 55D35 rms voltmeter respectively. The temporal derivatives were obtained by differentiating the signal with an analogue circuit designed to have unity gain at 1 kHz. Higher order moments of the temperature fluctuations were obtained by recording the output of the constant current anemometer on an FM tape (HP3968A) recorder at 95 mms^{-1}. The signals from the parallel wires and those from the analogue differentiator were recorded at 381 mms^{-1}. The signals were low-pass filtered (Krohn Hite model 3322) before digitising on a PDP 11/34 computer and processed on a VAX 780 computer. The 40 s duration of digital records was sufficient for moments and correlation coefficients to converge within ±0.5% of its final value.

TEMPERATURE STATISTICS NEAR THE WALL

As was pointed out by Yaglom (1979) the scatter in existing data on temperature fluctuations in turbulent wall flows is quite large. It is possible however that this scatter can be reduced by accounting for the various sources of error associated with the use of fine wires for measuring the temperature fluctuation. Lecordier et al (1984) noted three major sources : (a) the wire thermal inertia; (b) the conductive heat loss to the wire prongs; and (c) the spatial resolution of the wire. While (a) and (c) affect the high frequency part of the spectrum, (b) is associated mainly with low frequencies. Lecordier et al (1984) corrected the temperature spectrum for all three errors by using experimental transfer functions which characterise the wire and prong time constants and an analytical expression obtained by Wyngaard (1971) to account for (c). However, their procedure does not enable one to assess the effect of wire length on the probability density functions. An assessment on the higher order moments such as skewness S_θ [$\equiv \overline{\theta^3}/(\overline{\theta^2})^{3/2}$] and flatness F_θ [$\equiv \overline{\theta^4}/(\overline{\theta^2})^2$] of the temperature fluctuations can only be made experimentally and such an attempt is explored in the present study. For the present wires ℓ/d is in the range 300 to 1700. The lower bound corresponds to a 1.27 μm dia. (Pt-10% Rh Wollaston) wire.

It is well known that the mean velocity measured with a hot wire may be significantly affected by the proximity of the wall (e.g. Wills, 1962; Krishnamoorthy et al, 1984). By contrast, the cold wires, because of the extremely low overheats at which they are usually operated, can yield values of the mean temperature that are essentially unaffected by the wall. In the present experiments, the value of the wall temperature inferred by extrapolating the approximately linear temperature profiles, were in good agreement (±2%) with the values obtained from the surface transducers. The mean temperature $T^+ \equiv (T_W-\overline{T})/\theta_\tau$ (a superscript + denotes normalisation by either U_τ, the friction temperature θ_τ or the viscous length ν/U_τ) measured with different wire lengths fell within a scatter band of ±2% about the distribution shown in Figure 1, thus indicating that T^+ is approximately independent of ℓ/d.

The near-wall variation of θ'^+ ($\equiv \overline{\theta^2}^{\frac{1}{2}}/\theta_\tau$) is shown in Figure 1 for different wire lengths. Apart from a small

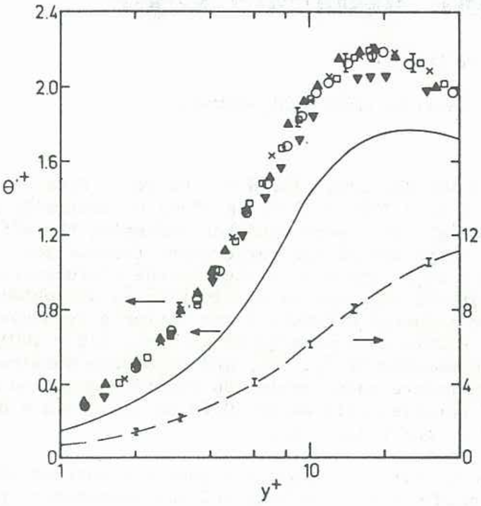

Figure 1 Near-wall distributions of T^+ and θ'^+ for different values of ℓ/d.
▼, ℓ/d = 300; ×, 550; ○, 900; ▲, 1100; □, 1700. I, uncertainty. —, Elena (1977).

departure (8% at y^+ = 20) for ℓ/d = 300, the distributions are generally independent of ℓ/d. This result is at variance with the observations of Lecordier et al (1984) who obtained a reduction (maximum of 6.5%) in the measured θ' behind a line source of temperature in a turbulent boundary layer when ℓ/d was reduced from 1600 to 600. The observed independence of the present distributions suggest that the reduction in the low frequency part of the temperature spectrum when ℓ/d becomes small may be compensated by the corresponding increase in the high frequency part due to the improved spatial resolution. At y^+ = 20, the spatial resolution ℓ/η, where η is the Kolmogorov length scale ($\equiv \nu^{3/4}/\overline{\varepsilon}^{1/4}$ where $\overline{\varepsilon}$ is the turbulent energy dissipation) is in the range 6 (ℓ/d = 300) to 18 (ℓ/d = 1700). The smaller values of θ'^+ at the smallest ℓ/d are due to the reduced frequency response of the larger diameter wire (the time constant of the 1.27 μm wire was estimated to be about one fourth of the 0.63 μm wire). The θ'^+ distribution increases linearly in the region $y^+ \le 5$ according to the relation $\theta'^+ = 0.23\, y^+$. We cannot explain the discrepancy between the present values and those of Elena (1975), also shown in Figure 1.

The skewness S_θ and flatness factor F_θ of temperature are independent of ℓ/d (≥ 550) for $y^+ \ge 3$ (Figure 2). For $y^+ < 3$, smaller values are obtained for ℓ/d = 300. Apparently due to the normalisation, there is good agreement between the present values of S_θ and F_θ and those of Elena. The limiting behaviour at $y^+ \to 0$ of S_θ and F_θ can be estimated from a Taylor series expansion. Approximate expressions for $\overline{\theta^{+2}}$, $\overline{\theta^{+3}}$ and $\overline{\theta^{+4}}$ at small y^+ are

$$\overline{\theta^{+2}} \simeq \overline{\left(\frac{\partial\theta^+}{\partial y^+}\right)_0^2}\, y^{+2} + \frac{1}{3}\,\overline{\left(\frac{\partial\theta^+}{\partial y^+}\right)_0\left(\frac{\partial^3\theta^+}{\partial y^{+3}}\right)_0}\, y^{+4} = \delta_1 y^{+2} + \delta_2 y^{+4} \quad (1)$$

$$\overline{\theta^{+3}} \simeq \overline{\left(\frac{\partial\theta^+}{\partial y^+}\right)_0^3}\, y^{+3} + \frac{1}{2}\,\overline{\left(\frac{\partial\theta^+}{\partial y^+}\right)_0^2\left(\frac{\partial^3\theta^+}{\partial y^{+3}}\right)_0}\, y^{+5} = \delta_3 y^{+3} + \delta_4 y^{+5} \quad (2)$$

$$\overline{\theta^{+4}} \simeq \overline{\left(\frac{\partial\theta^+}{\partial y^+}\right)_0^4}\, y^{+4} + \frac{2}{3}\,\overline{\left(\frac{\partial\theta^+}{\partial y^+}\right)_0^3\left(\frac{\partial^3\theta^+}{\partial y^{+3}}\right)_0}\, y^{+6} = \delta_5 y^{+4} + \delta_6 y^{+6} \quad (3)$$

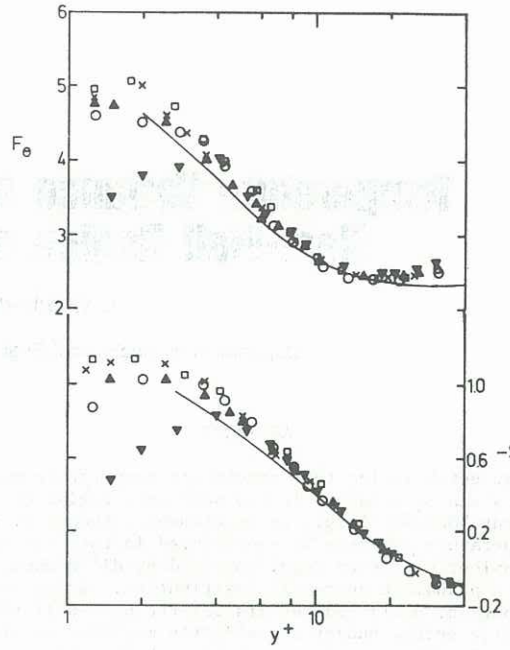

Figure 2 Near-wall distributions of S_θ and F_θ for different values of ℓ/d.
Symbols as in Figure 1. —, Elena (1977).

Least squares fits to equations (1), (2) and (3) using the data of Figures 1 and 2 in the region $y^+ \le 8$ yield $\delta_1 \simeq 0.056$, $\delta_3 \simeq \delta_5 = 0.012$. At $y^+ = 0$, the values for S_θ and F_θ correspond to those for skewness and flatness factors of the derivative $(\partial\theta/\partial y)$. Using the values of coefficients δ_1, δ_3, δ_5, we obtain $S_\theta = 0.9$ and $F_\theta = 3.8$ at $y^+ = 0$. By comparison, Eckelmann (1974) obtained values of about 0.8 and 3.7 for $(\partial u/\partial y)$ from measurements in a thick viscous sublayer.

NEAR-WALL BUDGET OF TEMPERATURE VARIANCE

The near-wall distribution of all three components of the average temperature dissipation $\overline{\varepsilon}_\theta$ is shown in Figure 3.

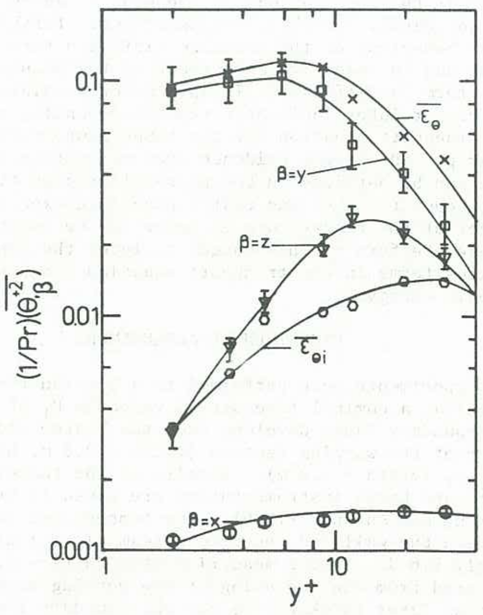

Figure 3 Averaged dissipation of temperature and its components. I, error bar.

The isotropic estimate $\overline{\varepsilon}_{\theta i}$ ($\equiv 3\alpha\overline{\theta^2_{,x}}$) is also plotted in

Figure 3. The relative magnitudes of the components of $\overline{\varepsilon}_\theta$ emphasise the increased departure from isotropy as the wall is approached. The ratio $\overline{\varepsilon}_\theta/\overline{\varepsilon}_{\theta i}$ increases from about 2 at $y^+ = 40$ to a value as large as 30 at $y^+ = 2$. A comparable increase was obtained by Laufer (1954) and Klebanoff (1954) for their approximations of the dissipation of $\overline{\varepsilon}$ of turbulent kinetic energy $(\overline{q^2}/2)$. For example, Klebanoff's estimate (only five of the nine terms of the total energy dissipation were measured and isotropy was assumed for the remaining four) is 4.5 times as large as a fully isotropic estimate at $y^+ = 28$ ($y/\delta = 0.01$).

The accuracy of the measured $\overline{\varepsilon}_\theta$ in the near-wall region can be obtained by evaluating the budget of $\overline{\theta^2}/2$. The transport equation for $\overline{\theta^2}/2$ in the near-wall region is, to a close approximation,

$$\underset{\substack{\text{I}\\ \text{Production}}}{\overline{v^+\theta^+}\,\frac{\partial T^+}{dy^+}} + \underset{\substack{\text{II}\\ \text{Turbulent}\\ \text{Diffusion}}}{\frac{1}{2}\,\frac{\partial(\overline{v^+\theta^{+2}})}{\partial y^+}} - \underset{\substack{\text{III}\\ \text{Molecular}\\ \text{Diffusion}}}{\frac{1}{2Pr}\,\frac{\partial^2\overline{\theta^{+2}}}{\partial y^{+2}}} + \underset{\substack{\text{IV}\\ \text{Dissipation}}}{\frac{1}{Pr}\,\overline{\varepsilon}^+_\theta} = 0. \tag{4}$$

All the terms, except II, were either measured directly or calculated by integrating the mean enthalpy equation. They are plotted in Figure 4. The turbulent diffusion

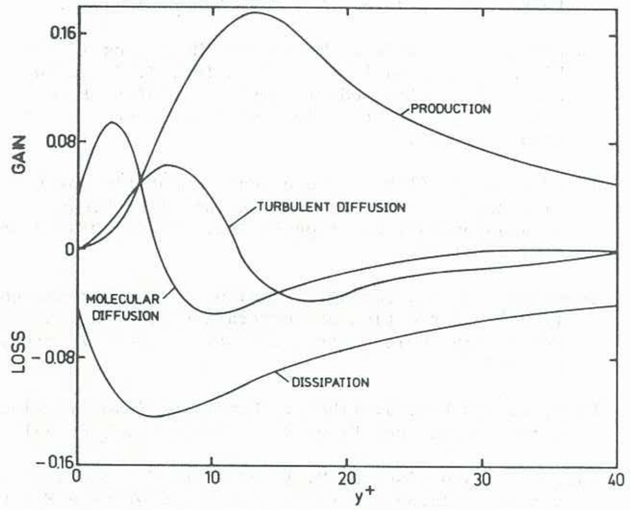

Figure 4 Budget of $\overline{\theta^2}/2$ in the near-wall region. All terms are normalised by $(\nu/u_\tau^2\theta_\tau^2)$.

term was obtained by difference. More details on the construction of the budget can be found in Krishnamoorthy and Antonia (1986). At the wall the molecular diffusion balances the dissipation since

$$\overline{\left(\frac{\partial\theta^+}{\partial y^+}\right)^2} = \frac{1}{2}\,\frac{\partial^2\overline{\theta^{+2}}}{\partial y^{+2}} \tag{5}$$

The contribution of the molecular diffusion to the budget is confined only to the region $y^+ \le 30$. Maxima for the molecular diffusion and production terms occur at $y^+ = 2.5$ and 12 respectively. Interestingly, the maxima for the corresponding terms of the near-wall turbulent energy budget (Laufer, 1958; Bernard and Berger, 1984) also occur at $y^+ \sim 2.5$ and 12. The distribution of the turbulent diffusion term indicates a gain of $\overline{\theta^2}/2$ in the region very close to the wall ($y^+ \le 12$) and a loss in the region $12 \le y^+ \le 40$. In the outer layer, the magnitude $\partial(\overline{\theta^2 r})/\partial y$ is small compared with that in the near-wall region. Since the gain and loss of $\overline{\theta^2}/2$, due to turbulent diffusion, are roughly in balance in the outer region, the requirement that the integration of $\partial(\overline{v\theta^2})/\partial y$ should be approximately zero when taken between $y = 0$ and $y = \delta$ amounts to checking the approximation

$$\int_0^{y^+\simeq 40} \frac{\partial(\overline{v^+\theta^{+2}})}{\partial y^+}\,dy^+ \simeq 0\,. \tag{6}$$

The present turbulent diffusion distribution provides reasonable confirmation of (6). A more detailed discussion is given in Krishnamoorthy and Antonia (1986) and suggests that the measurements of $\overline{\varepsilon}_\theta$ are reasonably accurate. The present diffusion compares favourably with that measured by Nagano and Hishida (1985) for $y^+ \ge 12$. For $y^+ < 12$, Nagano and Hishida's diffusion measurements also contribute to a gain of $\overline{\theta^2}/2$ but this gain is insufficient to satisfy (6).

NEAR-WALL BUDGET OF $\overline{q^2}/2$

In the light of the behaviour of the budget of $\overline{\theta^2}/2$ in the near-wall region, it seems reasonable to enquire into the possible form of the budget of the turbulent kinetic energy $(\overline{q^2}/2)$. Both q^2 and θ^2 are scalar quantities and transport equations for the scalar quantities have analogous terms, except for the presence of pressure in the q^2 equation. At the wall, both equations reduce to an equality between molecular diffusion and dissipation since in analogy to (5)

$$\overline{\left(\frac{\partial u^+}{\partial y^+}\right)^2} = \frac{1}{2}\,\frac{\partial^2\overline{u^{+2}}}{\partial y^{+2}} \tag{7}$$

The analogy between $\overline{\theta^2}$ and $\overline{q^2}$ has been well supported in a spectral sense by the boundary layer data of Fulachier and Dumas (1976). Also, the flow visualisation data of Iritani et al (1983) indicate that there is close similarity between the structures of the velocity and thermal fields in the near-wall region. In particular, good agreement was observed between low speed streaks and high temperature streaks as well as between high speed streaks and low temperature streaks.

In the inner region of the boundary layer, the major uncertainty in the $\overline{q^2}/2$ budget occurs in estimates for the dissipation, turbulent diffusion and pressure diffusion terms. The reasonable closure of the present $\overline{\theta^2}/2$ budget in the near-wall region should provide some qualitative if not quantitative insight into the behaviour of the budget of $\overline{q^2}/2$.

In the near-wall region, the only available budget of $\overline{q^2}/2$ was obtained by Laufer (1954) for turbulent pipe flow. Laufer measured five of the nine terms of the dissipation and assumed isotropy to estimate the remaining four. All terms, except pressure diffusion which was inferred by difference, were measured by Laufer. Townsend (1956) corrected Laufer's dissipation estimates in the region close to the wall to account for the influence of large gradients, with respect to y of u' and for the effect of the boundary on the diffusion of q^2 by v and p. Recently Bernard and Berger (1984) modified Townsend's energy budget in the region $y^+ \le 2$. Their modification indicated greater levels of dissipation and molecular diffusion than estimated by Townsend and a negative value for the pressure diffusion near the wall. A shortcoming of Townsend's budget is the failure of the $\overline{q^2 v}$ distribution to satisfy

$$\int_0^{y^+\simeq 40} \left(\frac{\partial(\overline{q^{+2}v^+})}{\partial y^+}\right) dy^+ \simeq 0 \tag{8}$$

A plausible distribution for the turbulent diffusion of $\overline{q^2}/2$ follows from the assumption that the diffusions, due to v, of $\overline{\theta^2}/2$ and $\overline{q^2}/2$ should exhibit similar behaviours. On this basis, a further revision of Townsend's budget in the near-wall region is shown in Figure 5. In Figure 5, two possible scalings for the turbulent diffusion are used. In the first, we arbitrarily assumed that the turbulent diffusion of $\overline{q^2}/2$, due to v, is in the same proportion to the turbulent diffusion of $\overline{\theta^2}/2$ as the production of $\overline{q^2}/2$ is to the production of $\overline{\theta^2}/2$. For the second, the scaling was based on the ratio of dissipations of $\overline{q^2}/2$ and $\overline{\theta^2}/2$. There is however little difference between the two distributions of $\partial(\overline{q^{+2}v^+})/\partial y^+$ and both satisfy (8). The dissipation, molecular diffusion and pressure diffusion (Figure 5) for $y^+ \le 2$ are those estimated by Bernard and Berger (1984) from channel flow measurements (Eckelmann, 1974) of u'/U, w'/U and $u(\partial y/\partial y)_0/u'(\partial u/\partial y)'_0$. The distributions of dissipation and molecular diffusion in the range $2 < y^+ \le 7$ were those suggested by Bernard and Berger. They provide a smooth transition between Townsend's distributions and those estimated by Bernard and Berger (1984) in the region $y^+ \le 2$.

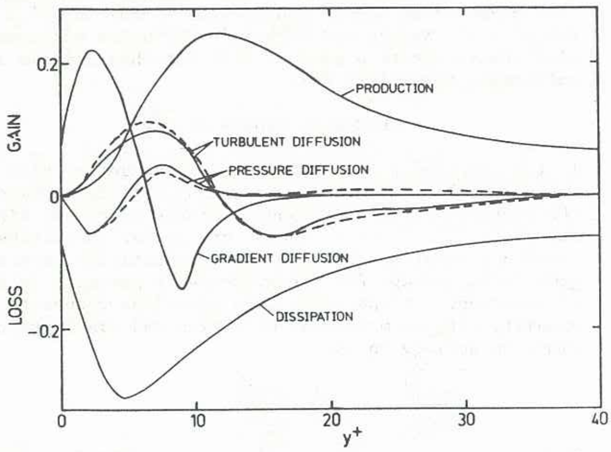

Figure 5 Budget of $\overline{q^2}/2$ in the near-wall region. Turbulent diffusion: ——, using a scaling based on the ratios of production of $\overline{\theta^2}/2$ and $\overline{q^2}/2$; — —, using a scaling based on the ratios of dissipation of $\overline{\theta^2}/2$ and $\overline{q^2}/2$. Pressure diffusion: ——, — — by difference using corresponding turbulent diffusion.

The pressure diffusion for $y^+ > 2$ is inferred by difference and, when it is integrated over the range $2 < y^+ < 40$, the resulting value is negligible. Note that the magnitude of the pressure diffusion is small compared with that of other terms in the budget. The significant difference between the $\overline{q^2}/2$ budget in Figure 5 and that of Townsend is the behaviour of turbulent diffusion in the near-wall region. Figure 5 indicates a gain of $\overline{q^2}/2$ for $y^+ \le 12$ whereas Townsend estimates a loss throughout the region $y^+ \le 40$. With the qualification that the measured turbulent diffusion in the region close to the wall may be of limited accuracy, Laufer's original measurements indicated a gain of $\overline{q^2}/2$ for $y^+ \le 6$. It is encouraging to note that in Moin and Kim's (1982) numerical calculation of turbulent channel flow, the turbulent diffusion contributes a gain of $\overline{q^2}/2$ in the region $y^+ \le 15$ and a loss at larger distances from the wall.

CONCLUSIONS

Consistent distributions are obtained for various temperature statistics, including the average dissipation of the temperature variance, in the near-wall region of a turbulent boundary layer. These measurements lead to a plausible budget of the temperature variance. Using the analogy between transport equations for the temperature variance and the kinetic energy and the observed similarity between velocity and thermal flow structures in the near-wall region, a plausible modification is made to Townsend's near-wall budget for the turbulent kinetic energy.

ACKNOWLEDGEMENT

The support of the Australian Research Grants Scheme is gratefully acknowledged

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