

An Axisymmetric Jet in a Moving Fluid

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ABSTRACT

Data are presented here for axisymmetric pure jets discharged into a coflowing fluid or at an angle to an ambient flow.

A theory based on Patel's growth of width equation is described and compared with the data.

INTRODUCTION

In the analysis of a jet-like flow, the only equation available is a form of momentum equation and to obtain a solution a further equation (a closure equation is necessary. Most investigators have in the past followed Morton, Taylor and Turner (1956) and used the volume conservation equation along with an entrainment assumption. This may be written as:

$$\frac{d}{ds} [I_1 u_{em}^2] = 2\pi \alpha b u_{em} \quad (1)$$

where s : distance from the port along the centre line

I_1 : cross sectional constant

u_{em} : maximum excess velocity in the cross section

α : entrainment coefficient

b : radial distance from the centre line in which $u_e = e^{-1} u_{em}$

This equation is satisfactory for a jet in still ambient fluid. A second form of closure equation involves assuming the rate of spread of the jet, i.e.

$$\frac{db}{ds} = k \quad (2)$$

where k is a constant. This assumption is equivalent to the entrainment assumption but has been extended to the case of a jet in a coflow by Patel (1971) and Antonia & Bilger (1974). For this case it becomes

$$\frac{db}{ds} = k \frac{u_{em}}{u_{em} + U_\infty} \quad (3)$$

where U_∞ is the ambient velocity. For reasons which will become apparent in the next section this closure has been used in this paper and the analysis is extended to the case of a jet ejected at an angle to the coflow.

THEORY

Axisymmetric Pure Jet in Coflow

For the flow illustrated in fig. 1 the integral mass conservation equation is:

$$\frac{d}{dx} \int_0^R (u_e + U_\infty) 2\pi r dr - Q_{inflow} = 0 \quad (4)$$

where R is some large distance from the centre line which we will let $\rightarrow \infty$

u_e is the local excess velocity

u is the local velocity

x is a horizontal coordinate

Q_{inflow} is the amount of water that flows in through the sides of the control volume.

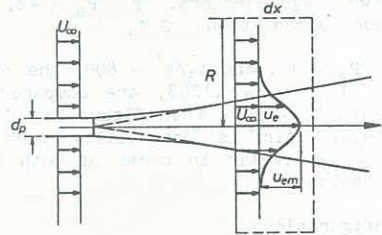


Figure 1 - An axisymmetric pure jet in an ambient coflow.

At sufficiently large Reynolds numbers the viscous stresses can be neglected. For the case of zero pressure gradient and applying a boundary layer type assumption the x-component of the integral momentum equation is

$$\int_0^R (u_e + U_\infty)^2 2\pi r dr - U_\infty Q_{inflow} = 0 \quad (5)$$

(4) and (5) are combined and letting $R \rightarrow \infty$ yields the required form of the momentum equation

$$\frac{d}{dx} \int_0^\infty (u^2 + u U_\infty) 2\pi r dr = 0 \quad (6)$$

or

$$\frac{d}{dx} \int_0^\infty (\bar{u}^2 + \bar{u}'^2 + \bar{u} U_\infty) 2\pi r dr = 0 \quad (7)$$

where bars denote the mean values and dashes denote fluctuating components. Assuming that the mean velocity and turbulent fluctuations are self-similar then

$$u_{em}^2 b^2 I_1 + U_\infty u_{em}^2 I_2 = M \quad (8)$$

$$I_1 = \frac{1}{u_{em}^2 b^2} \int_0^\infty \left(\frac{u_e}{u_{em}} \right)^2 + \frac{\bar{u}'^2}{u_{em}^2} 2\pi \frac{r}{b} \frac{dr}{b}$$

$$I_2 = \frac{1}{u_{em}^2 b^2} \int_0^\infty \left(\frac{u_e}{u_{em}} \right) 2\pi \frac{r}{b} \frac{dr}{b}$$

M = the total momentum flux.

Equation (3) with $k = 0.107$ [the spreading rate for pure jets in a stagnant fluid] is used as the closure equation. Non-dimensional variables are defined as $u_* = u_{em}/U_\infty$, $b_* = b/M^{1/2}/U_\infty$, and $x_* = x/(M^{1/2}/U_\infty)$. Then combining (3) and (8) and integrating gives an analytical solution for u_*

$$-\frac{4I_1 + 6I_2}{3I_2^2} k (2I_1 u_* + I_2) (I_1 u_*^2 + I_2 u_*)^{-1/2} - \frac{2}{3} k u_*^{-1} (I_1 u_*^2 + I_2 u_*)^{-1/2} + \frac{4I_1}{I_2} k u_* (I_1 u_*^2 + I_2 u_*)^{-1/2} = x + x_0 \quad (9)$$

Combining (3) and (8) also yields an expression for b_*

$$k^{-1} \left(b_* + \frac{1}{6} b_*^3 + \frac{1}{2} (I_2^2 b_*^2 + 4I_1) \right)^{3/2} = x - x_0 \quad (10)$$

Neglecting the turbulent fluctuation term u'^2 in (7) and assuming the velocity distribution is Gaussian ($u_e = u_{em} \exp(-r^2/b^2)$). Then I_1 and I_2 can be computed*. Further, if dye is added to the jet as a tracer and if its spread is also assumed to be Gaussian ($c = c_m \exp(-r/\lambda b)^2$, where c_m is the centre line concentration) then the centre line dilution of tracer is given by

$$S_D = \lambda^2 \pi^2 b_*^2 \left(\frac{U_0}{U_{\infty}} - 1 \right) \left(\frac{1}{1+\lambda} u_* + 1 \right) \quad (11)$$

where λ is a dispersion coefficient
 U_0 is the jet exit velocity
 S_D is the centre line velocity

Axisymmetric Pure Jet at an Angle to an Ambient Flow

Figure 2 - Definition sketch for an axisymmetric pure jet discharging at an angle to an ambient flow.

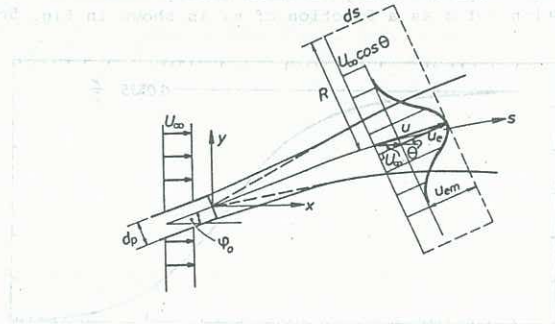


Figure 2 - Definition sketch for an axisymmetric pure jet discharging at an angle to an ambient flow.

For the flow illustrated in fig. 2 - an axisymmetric pure jet at an angle to an ambient flow - where the end sections of the control volume have been chosen perpendicular to the excess velocity u_e , the integral mass conservation equation becomes:

$$\frac{d}{ds} \int_0^R (U_{\infty} \cos \theta + u_e) 2\pi r dr - Q_{inflow} = 0 \quad (12)$$

where θ is the angle between u_e and U_{∞} .

In the case of zero pressure gradient, the integral x-component of the momentum conservation equation is:

$$\frac{d}{ds} \int_0^R (U_{\infty} + u_e \cos \theta)^2 2\pi r dr - Q_{inflow} U_{\infty} = 0 \quad (13)$$

and similarly, the y-component of the momentum conservation equation is:

$$\frac{d}{ds} \int_0^R (U_{\infty} + u_e \sin \theta)^2 2\pi r dr - 0 = 0 \quad (14)$$

Combining (12) and (13) with (14), letting $R \rightarrow \infty$ yields

$$I_1 u_{em}^2 b^2 \cos \theta + I_2 U_{\infty} u_{em} b^2 \cos^2 \theta = M_{x0} \equiv M_0 \cos \theta \quad (15)$$

where I_1 and I_2 are cross sectional constants. M_{x0} is the constant x-component of the momentum which we also write as $M_0 \cos \theta_0$. Performing similar operations on the y-momentum equation gives:

$$I_1 u_{em}^2 b^2 \sin \theta + I_2 U_{\infty} u_{em} b^2 \sin \theta \cos \theta = M_{y0} \equiv M_0 \sin \theta_0 \quad (16)$$

where M_{y0} is the constant y-component of the momentum $M_{y0} = M_0 \sin \theta_0$. Equation (15) divided by (16) shows that $\theta = \theta_0$ and this highlights the advantage of using cross section perpendicular to u_e rather than the more

* The neglected normal stress term \bar{u}^2 would have led to an increase in I_1 by 13.5% (Papanicolaou, 1983) but this makes small difference to the theoretical curves.

normal cross sections which are perpendicular to s . Equations (15) and (16) then both become:

$$I_1 u_{em}^2 b^2 + I_2 U_{\infty} u_{em} b^2 \cos \theta = M_0 \quad (17)$$

Introducing a modification of Patel's equation

$$\frac{db}{ds} = \frac{k u_{em}}{u_{em} + U_{\infty} \cos \theta} \quad (18)$$

(18) is non-dimensionalised (see below) and combined with (17). After some manipulation and introduction of the variable

$$X = \left[\frac{I_1 u_{em}^2 b^2}{M_0} \right]^{1/2} \quad (19)$$

where

$$u_* = u_{em}/U_{\infty}$$

$$b_* = b/b_c$$

$$b_c = \sqrt{M_0/U_{\infty}}$$

neglecting the turbulent fluctuation term and assuming a Gaussian velocity distribution the following differential equations are obtained for the trajectory:

$$\frac{dX}{d\theta} = \frac{1}{2^{3/2} k \pi^{1/2} \cos \theta} \frac{[X^2 - 1 - 2X^2 \cos^2 \theta](X^2 + 1)}{(X^2 - 1 - 2X^2 \cos^2 \theta)^2 + (2X^2 \cos \theta \sin \theta)^2} \quad (20)$$

$$\frac{dy_*}{dx_*} = \frac{-2X^2 \cos \theta \sin \theta}{(X^2 - 1) - 2X^2 \cos^2 \theta} \quad (21)$$

$$b_* = \frac{1-X^2}{X} \frac{2}{\pi^{1/2} \cos \theta} \quad (22)$$

$$u_* = \frac{-X^2 \cos \theta}{2(X^2 - 1)} \quad (23)$$

$$\text{and } S_D = \frac{4b^2 \lambda^2 (1+\lambda^2 u_* + \cos \theta)}{dp^2 \frac{U_0}{U_{\infty}}} \quad (24)$$

The Experiments

The pure jet in an ambient flow is modelled by discharging dyed fresh water from a constant head tank through a port into a 6 x 1.5 x 1 metre glass tank containing quiescent water (fig. 3). The port is attached to a trolley and to model a moving environment the port is towed through the stationary ambient fluid. This implies zero ambient turbulence. By altering the angle between the port and horizontal, the direction at which the ambient flow approaches the port can be changed. The flows were recorded on video (Hitachi VT-8E recorder and Hitachi VK-C 1600E camera). By tracing the boundaries off a still frame picture, trajectories could be obtained.

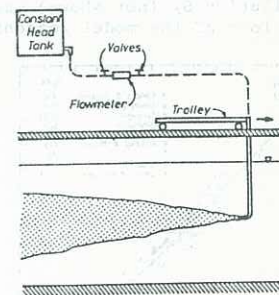


Figure 3 - Sketch of experimental setup.

Concentration profiles were measured with a set of 40 probes that were positioned so that they were perpendicular to the excess velocity (\approx port angle) in

the jet cross section concerned (fig. 4). The probe lead via fine tubes to a sealed box on top of the trolley. Samples were taken when a vacuum was applied to the sealed box and the effluent was withdrawn. The absorbance of each sample was then measured by using a spectrophotometer Unicam SP600 series 2 and a concentration profile obtained.

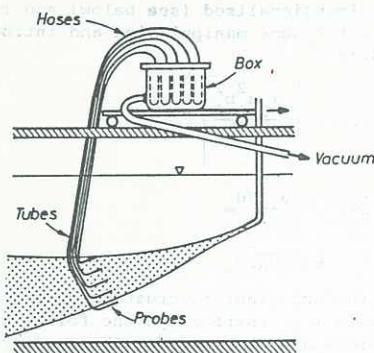


Figure 4 - Sketch of equipment for concentration profile measurements.

RESULTS - COMPARISON OF DATA AND MODEL

The distribution of dye in a cross section followed approximately the expression

$$c = c_m e^{-\left(\frac{r}{\lambda b}\right)^2}$$

where c is tracer concentration

c_m is centre line concentration

λ is dispersion coefficient

λb could be found from the concentration profiles. λ was assumed to be 1.16 and hence b was found. u_* could be found indirectly by inserting the obtained b_* value in the momentum conservation equation (7) (or alternatively in a dye conservation equation. The results from this method, however, showed much greater scatter). Finally, the centre line dilution $S_D = c_0/c_m$ where c_0 is the effluent concentration of tracer, was found.

In fig. 5a-b the u_* and b_* data for the coflow experiments are compared with other existing data and equations (9) and (10). The experiments were made for U_0/U_∞ values between 3 and 165, and with port diameter 0.2, 0.45 and 0.8 cms. The present data are seen to be consistent with existing data and the theory predicts the average of the b_* data within 15% and the average of the u_* data within 5%. The theory predicts very well in the limits $x_* \rightarrow \infty$ and $x \rightarrow 0$. The centre line dilution S_D (not shown) was found to fit the predictions of the model reasonably well.

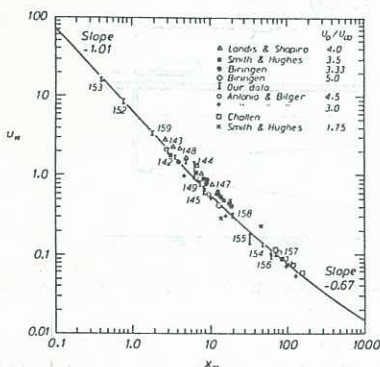


Figure 5a - Dimensionless excess velocity for a pure jet in a coflow.

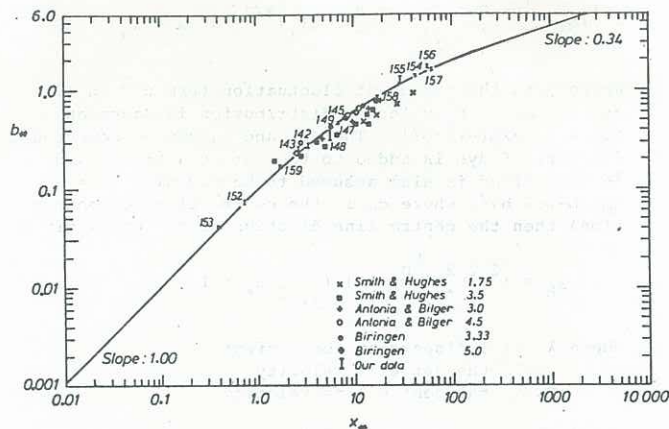


Figure 5b - Dimensionless width for a pure jet in a coflow.

It is of interest to obtain the variation of the entrainment coefficient α as defined in equation (1) from this experimentally verified theory. The variation of α as a function of s_* is shown in fig. 5c.

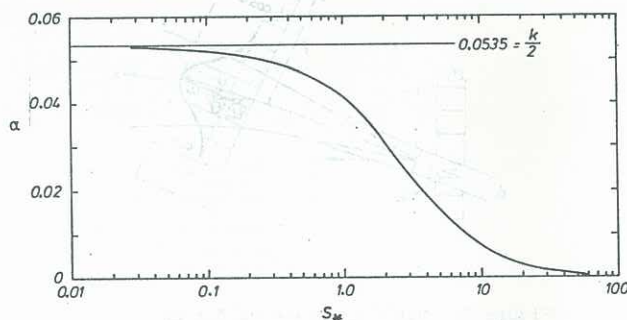


Figure 5c - Entrainment coefficient α as a function of dimensionless distance from the port.

For the case of a pure jet discharged at an angle to an ambient flow very little data is available, Margason (1968) and Platten & Keffer (1971), and the experiments have all been performed in wind tunnels and only the trajectory was recorded. Furthermore, only the near field region (up to 15-20 port diameters from the port) has been studied.

In this study measurements were obtained by towing the jet (hence with zero ambient turbulence) and were made up to 260 diameters from the port. The experiments were made for U_0/U_∞ values 50 and 75, with port diameter 0.24 cm. and angle 30° with horizontal.

In fig. 6a-b the measured trajectories and widths are compared with the theory. The trajectory data are consistent with the theory but 10% lower than the calculated trajectories. This could possibly be caused by the slight stratification caused by temperature differences. Also the width data were consistent but were 10% higher than the calculated values. The centre line dilution S_D (not shown) was again consistent with the theory but approximately 20% higher than the theory.

CONCLUSION

A theory has been presented for pure jets in a coflow and at various angles to an ambient flow. New data were presented for these cases. The coflow data were consistent with existing data and were predicted well by the theory.

The data for the jet at an angle were consistent but showed some 10% deviation from the theory. The reason for this discrepancy is considered to be that the counter-rotating vortices which are known to develop in jets in a cross flow (90°) to some extent develop in jets discharged at smaller angles to the ambient.

flow. The vortices increase the entrainment. An entrainment slightly higher than the theory was in fact observed in the data obtained from jets at an angle to the ambient flow.

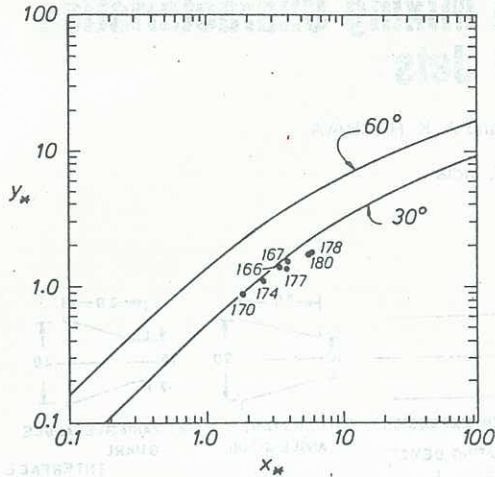


Figure 6a - Dimensionless trajectory for a pure jet at an angle to an ambient flow.

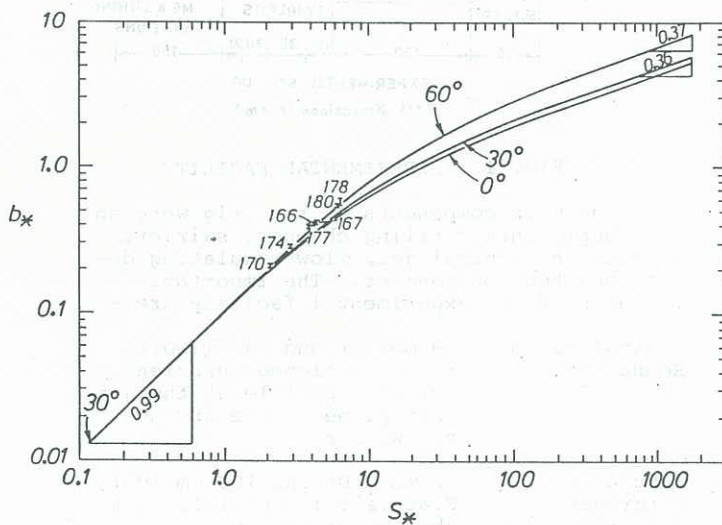


Figure 6b - Dimensionless width of a jet for a pure jet at an angle to an ambient flow as a function of the dimensionless distance from the port.

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