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Estimation of the Angular Distribution of Ocean Surface Waves by Means of Microseism-Spectra

A. C. KIBBLEWHITE[1] and C. Y. WU[2]

[1]Department of Physics, University of Auckland, N.Z.
[2]On leave from the Institute of Acoustics Academia Sinica, People's Republic of China.

ABSTRACT

Induced by non-linear interaction of ocean surface waves the underwater infrasonic noise pressure field has an energy spectrum depending directly on the angular distribution function, $G(\theta)$, of the wave energy. While Tayler's model, $G(\theta) = \cos^{2s}(\theta/2)$, is adopted, a new form of the spreading coefficient, $S(u_{10},f) = 2/(f/f_0 - 1)$ for $f > (1 + \delta)f_0$ and $S(u_{10},f) = 2/\delta$ for $f<(1 + \delta)f_0$, is proposed, where $f_0 = g/2\pi u_{10}$ is the "resonant frequency" of the surface wave corresponding to the wind speed u_{10} and δ is a parameter to be determined. Since the peak value of the spectrum of the acoustic pressure field appears to be rather sensitive to the change of the parameter δ, an experimental estimation of the angular distribution $G(\theta)$ can be obtained by matching the peak values of the measured and theoretical spectra of the microseism field, which is induced by the pressure field, directly through a spectrum transfer function $T_{Mp}(f)$. Data measured in Cook Strait and on the east coast of New Zealand have been used to provide an experimental estimate. The result shows that for low wind speeds (less than 10 m/s) an appropriate value of δ is 1.0, while for higher speeds the value is about 0.75.

INTRODUCTION

In a recent paper, Kibblewhite and Ewans (1985a), we reported a study which examined the correlation of the ocean-wave field with wave-induced seismic activity. The quality of the data, the long term nature of the observations and an unique property of the recording environment helped establish that the ocean ambient noise pressure field and seismic activity below 5 Hz are both controlled by nonlinear wave-wave interactions within the surface wave field. However certain characteristics of the noise spectra remained unexplained.

It was apparent that these unexplained features were related to the angular distribution of the energy in the ocean-wave field and the geoacoustic properties of the ocean/seabed system. A preliminary analysis of these two effects was reported subsequently (Kibblewhite and Ewans, 1985b) but the precise nature of the influences involved still remained unclear. The present analysis and a companion paper extends the investigation and provides additional clarification of the processes involved.

BASIC THEORETICAL PREDICTIONS

As first reported by Miche (1944) consideration of second order terms in the hydrodynamic equations leads to terms representing the generation of low frequency pressure fluctuations by the nonlinear interaction of opposing ocean waves. In contrast to the progressive waves producing them, the distinctive features of these second order waves are that the pressure signals they produce occur at twice the frequency of the interacting surface waves, are proportional to the amplitude product of these waves, and do not decrease with depth.

Miche's theory was developed by Longuet-Higgins (1950) to account for microseism generation and expanded further by Hasselmann (1963) in particular. Similar theoretical analyses in the context of underwater acoustics were carried out by Brekhovskikh (1966) and Hughes (1976). When minor errors are corrected (Lloyd, 1981), the derived pressure spectrum can be shown to be essentially the same in all treatments.

If a simple geoacoustic model is assumed, consisting of a water layer of constant depth, H, overlying an elastic half space, it can be shown that the spectrum of the source pressure field, induced by wave action and acting on the mean surface of the ocean, is given by

$$F_p(f) = F_p(2f_w) = \frac{32\pi^4 \rho_1{}^2 g^2}{\alpha_1{}^2} F_a{}^2(f_w) f_w{}^3 \; I \qquad (1)$$

or

$$F_p(f) = \frac{4\pi^4 \rho_1{}^2 g^2}{\alpha_1{}^2} F_a{}^2\left(\frac{f}{2}\right) f^3 \; I \qquad (1a)$$

The spectrum of the corresponding underwater noise field, $F_N(f)$, and that of the microseisms (the displacement of the sea-bed), $F_M(f)$, are given respectively by:

$$F_N(f) = F_p(f) \cdot T_{pN}(f) \qquad (2)$$

and

$$F_M(f) = F_p(f) \cdot T_{pM}(f) \qquad (3)$$

where $f_w \equiv f/2$ denotes the frequency of the ocean surface wave; f is the frequency of the wave-induced component of the ambient noise pressure field and its seismic equivalent; $\omega = 2\pi f$; ρ_1 is the density of sea water, and α_1 the sound velocity in water; $F_a(f_w)$ is the surface-wave spectral function; $T_{pN}(f)$ and $T_{pM}(f)$ are the transfer functions relating the pressure field on the sea surfaceto the underwater noise and seismic fields respectively.

The term in Eq.(1) represents an integral of the spreading function describing the angular distribution of the surface-wave field (Tyler *et al.*, 1974), viz.,

$$I \equiv \int_{-\pi}^{\pi} H(\theta)\, H(\theta + \pi)\, d\theta \qquad (4)$$

where $H(\theta)$ is the normalized spreading function defined as

$$H(\theta) = \frac{1}{H_0} G(\theta) \quad , \quad H_0 = \int_{-\pi}^{\pi} G(\theta)\, d\theta \qquad (5)$$

THE OCEAN WAVE SPECTRA

The data discussed in this paper were recorded either in the South Taranaki Bight (the Maui region) on the west coast of the North Island of New Zealand (Fig. 1) or off Great Barrier Island on the east coast (Ewans and Kibblewhite, In prep.).

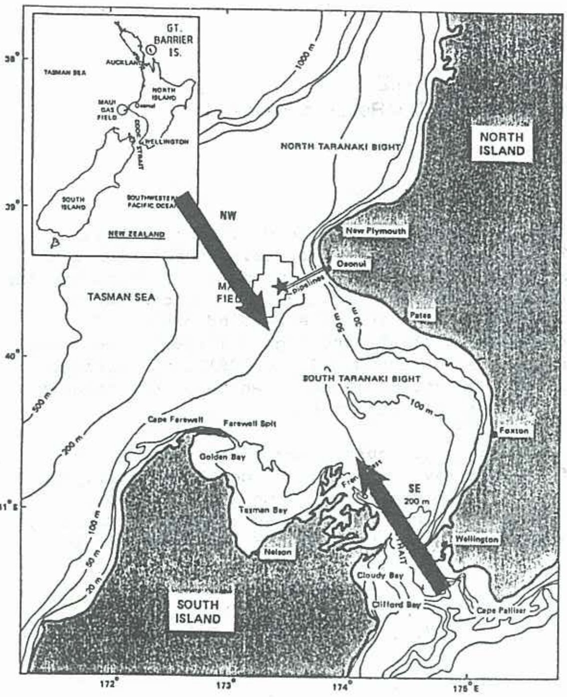

Figure 1. The Maui region of Cook Strait, the site of the west coast wave and seismic measurements. The position of Great Barrier Island, the site of the east coast measurements, is shown in the inset.

In the Maui region the wave field is essentially fetch limited at the Maui recording site for winds from the southeasterly quarter. Discounting the influence of a persistent swell from the southwest, the JONSWAP formula (Hasselmann *et al.*, 1973) has been shown to fit the measured fetch limited spectra very well (Ewans, 1984). Using this formula and paramaters appropriate to the Maui region it is possible to compute a theoretical Maui spectrum. Derived spectra for wind speed, u_{10}, ranging from 2.5 to 30 m/s are shown in Fig. 2.

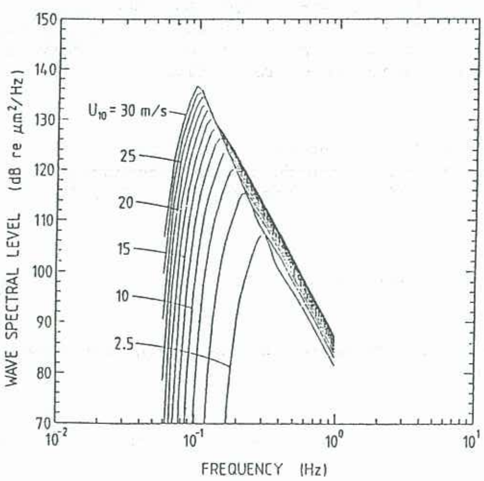

Figure 2. Maui ocean wave spectra, based on the JONSWAP formalism, as a function of wind speed.

THE INFLUENCE OF THE SPREADING COEFFICIENT ON THE PRESSURE FIELD

From Eqs. (4) and (5) we see that the term I is given by:

$$I = \frac{1}{H_0^{\,2}} \int_{-\pi}^{\pi} G(\theta)\, G(\theta + \pi)\, d\theta \tag{6}$$

In the case of a single sea (i.e., a sea generated by a steady wind, from a fixed direction, across an initially quiescent surface) a widely accepted form of the spreading function, $G(\theta)$, is that proposed by Tyler *et al.* (1974), viz.,

$$G(\theta) = \cos^{2s}\left[\frac{\theta}{2}\right] \qquad -\pi \le \theta \le \pi \tag{7}$$

with a spreading coefficient, s, which usually appears to be both frequency and wind speed dependent.

By substituting Eq. (7) into (6) we obtain an analytical form of I (Ewans, 1984):

$$I(s) = \frac{1}{2^{2s+1}\sqrt{\pi}} \; \frac{\Gamma(s+1)}{\Gamma(s+\frac{1}{2})} \tag{8}$$

in which $\Gamma(u)$ is the Gamma Function.

In the case of an omni-directional wavefield it follows that

$$s = 0 \quad \text{and} \quad I(0) = \frac{1}{2\pi} \tag{9}$$

$$\text{while} \quad \text{when } s \to \infty \ , \quad I(s) \to 0. \tag{10}$$

Stewart and Barnum (1975) found the following form of s to fit the measured data plotted in Tyler *et al.* (1974) very well:

$$\left.\begin{aligned} s &\approx \frac{0.2}{(\mu - 0.1)} && \mu \ge 0.1 \\ &= 2 && \mu < 0.1 \end{aligned}\right\} \tag{11}$$

Tyler *et al.* (1974) define the parameter μ as

$$\mu = u_*/(kc) \tag{12}$$

where $k = 0.4$ (the Karman's constant), c is the speed of the surface wave and u_* the friction velocity. Using the values $u_* = u_{10}\sqrt{c_{10}}$, where the drag coefficient $c_{10} = 1.5 \cdot 10^{-3}$ and noting that $c = g/2\pi f$ for gravity waves, we have

$$\mu \approx 0.062 \quad u_{10} f \tag{13}$$

whereupon

$$\left.\begin{aligned} s(f,u_{10}) &\approx \frac{2}{\frac{f}{f_0} - 1} && f > f_0 \\ &= 2 && f < f_0 \end{aligned}\right\} \tag{14}$$

where

$$f_o = \frac{1}{0.62u_{10}} \approx \frac{1}{6.08} \cdot \frac{g}{u_{10}} . \tag{15}$$

and u_{10} is the wind speed at 10 meters.

(Note this value of f_0 is close to the resonant frequency $g/2\pi u_{10}$.)

This model of s is shown in Fig. 3 by the dashed line from A to B to C (infinity) and then along the curve to D. The infinite discontinuity of the function s at point $f/f_0 = 1$ and a small constant value, s = 2, in the region $f/f_0 < 1$ suggests that the angular distribution of the wave energy becomes a δ-function (very narrow beam) at the "resonant" frequency for the given wind speed u_{10}, and becomes wider at lower values of f ($f < f_0$), even though, physically, this seems to be somewhat puzzling.

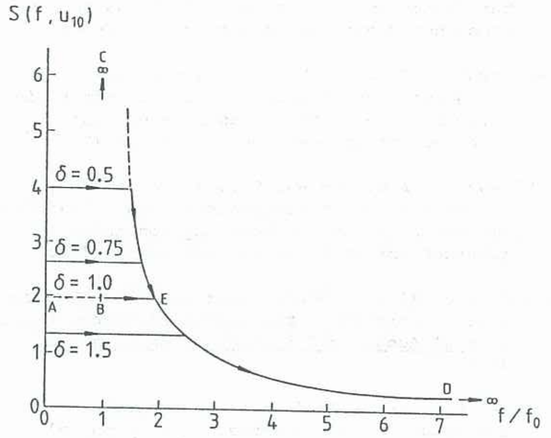

Figure 3. The dependence of the spreading coefficient on δ.

For ease of comparison with experimental data we have introduced an adjustable parameter, δ, and write

$$s = \begin{cases} \dfrac{2}{\dfrac{f}{f_0} - 1} & f \geq f_0(1 + \delta) \\ = \dfrac{2}{\delta} & f < f_0(1 + \delta) \end{cases} \tag{16}$$

Curves of s (as a function of f/f_0) for different values of δ are shown in Fig. 3. For a chosen δ (e.g., δ = 1.0), the spreading coefficient, s, is described by the horizontal line (A B E) and the curve (E D). With δ specified, the integral I defined by Eq.(6) can easily be calculated. The form of I so obtained is shown in Fig. 4.

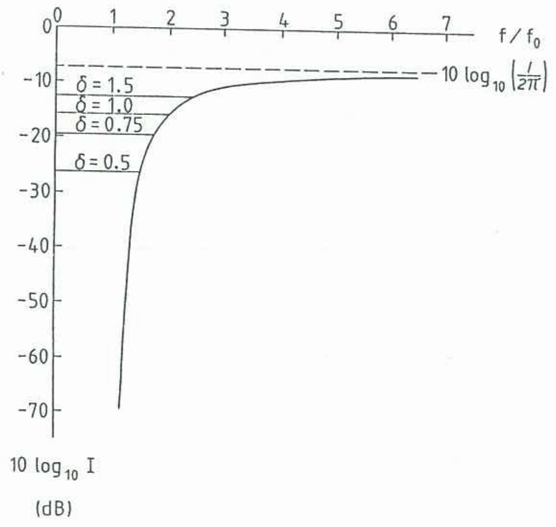

Figure 4. The dependence of the integral I on δ.

Since the spectrum of the source pressure field, $F_p(f)$, is (apart from a constant) given by the product of the integral, I, the square of the wave spectrum, $F_a^2(f/2)$, the term, f^3, (see Eq.(1)), the choice of δ will significantly change the value and position of the peak of the resulting spectrum, $F_p(f)$. An estimate of the spreading coefficient can thus be obtained by matching the peak values of the measured and theoretical spectra of the pressure fields.

Figure 5 (full lines) presents the calculated values of the peak of the pressure field spectra (based on the Maui spectra of Fig. 2) as a function of u_{10} for s = 1.0, 0.75 and 0.5. In this figure the data points represent experimental values of the pressure field determined indirectly from seismic spectra. The background to these measurements has been discussed fully by Kibblewhite and Ewans (1985a) and is not repeated here. Suffice it to say that the evaluation of the wave induced ambient noise pressure field from the seismic response it generates requires a detailed knowledge of the transfer function, $T_{pM}(f)$ - Eq.(3). Some earlier uncertainty over this parameter, which was discussed by Kibblewhite and Ewans (1985a), is now resolved (Kibblewhite and Wu, In prep.), and the experimental results in Fig. 5 are based on this revised transfer function.

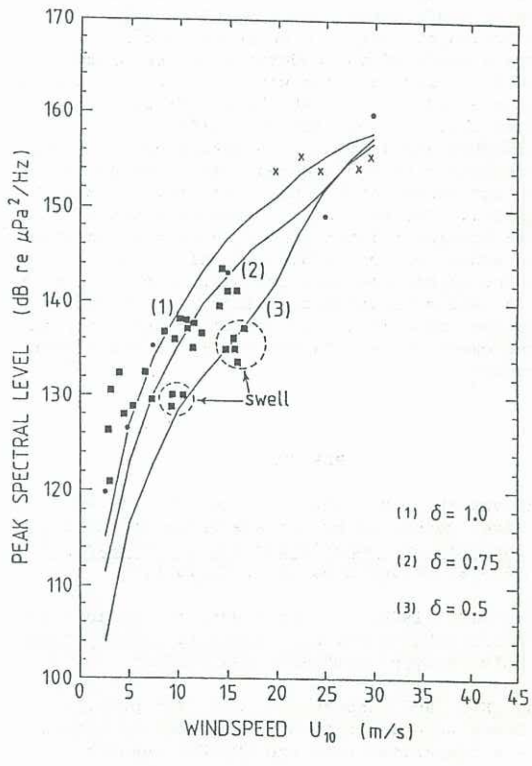

Figure 5. A comparison of the peak levels of the wave induced pressure field (full lines) derived from the Maui version of the JONSWAP function (for three values of δ), with experimental values of the ambient noisefield derived from microseism spectra relevant to the same wind speed. The dots represent west coast data involving interactions within a single-sea; the crosses represent west coast data involving two interacting seas; the squares represent east coast data involving a single-sea only.

While a detailed discussion is beyond the scope of this paper, it would appear from Fig. 5 that for low wind speeds (less than 10 m/s) a reasonable value of s is 1.0, especially when the circled points (which are based on swell rather than a local sea) are discarded. At higher wind speeds the value of 0.75 appears to be more appropriate. This modification is in accord with the general tendency for the beam defining the angular distribution of the surface-wave energy to become narrower (δ decreases) as the windspeed increases.

It is possible to test the form of I so established through a comparison of the spectra of the ambient noise field $F_N(f)$, determined through Eqs (1) and (2) from a knowledge of the ocean wave field $F_a(f/2)$, with those deduced from the seismic field, $F_M(f)$, using Eqs (2) and (3) and the transfer function relevant to the appropriate geoacoustical model. This comparison has been carried out and the analysis will be presented elsewhere (Kibblewhite and Wu, In prep.). The two sets of spectra show a very close correlation in terms of spectral amplitude and form, and give credibility to the shape of the spreading function described above.

CONCLUSIONS

The possibility of exploring the nature of the spreading coefficient governing the angular distribution of energy in an ocean wave field, through a study of the underwater noise produced by nonlinear interactions within it, was suggested by Tyler *et al.*(1974). At the low frequencies involved there are considerable difficulties, experimental and logistic, in making such acoustic measurements with hydrophones. The alternative of using land based seismometers has proved remarkably successful. Not only has it been possible to confirm nonlinear interactions between ocean waves as the source of low frequency ambient noise in the ocean and of microseisms, but also to confirm their role in wave generation and decay, and to obtain an independent measure of the spreading function, and so complement those obtained via radar measurement techniques.

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