

# Analytical and Experimental Study of Flow Characteristics in Open Channels of Compound Cross-Section

R. J. KELLER and W. T. WONG

Department of Civil Engineering, Monash University, Melbourne, Australia.

## ABSTRACT

The development of a two-dimensional mathematical model to calculate flow characteristics in channels of compound cross-section is described. The model utilizes a previously developed depth-averaged form of the  $k - \epsilon$  turbulence model. Experimental velocity data were obtained using laser-Doppler anemometry techniques in two geometries representing a main channel with an adjacent flood plain. Comparisons between these data and model predictions show good agreement. They indicate that the depth-averaged form of the  $k - \epsilon$  turbulence model can be utilized to predict the effects of interaction phenomena on velocity profiles in main channel/flood plain flows.

## INTRODUCTION

Current methods for two-dimensional numerical modelling of rivers in flood utilize the St. Venant equations of motion. These equations ignore the influence of turbulent shear stresses on the transfer of momentum. Although frequently permissible in practice, this assumption is invalid in flow regions at or close to sharp discontinuities in the bed geometry, such as the interface between main channel and flood plain flows.

The effect of turbulent shear stresses is to transfer streamwise momentum from fast moving parts of the stream to adjacent slow moving parts. This phenomenon can strongly affect the predicted water surface elevations, velocity distributions, and bed shear stress distributions with obvious consequences in such areas as flood plain modelling, positioning of water extraction offtakes, and detailed sediment transport studies.

The interface between main channel and flood plain flows is a particular example of the phenomenon and has been investigated in a number of previous experimental studies [e.g. Sellin (1964), Zheleznyakov (1971), Rajaratnam and Ahmadi (1979, 1981), and Knight et al. (1983)]. Analyses presented with these studies have been, in the main, empirically based with little application outside the experimental conditions studied. Nevertheless, the studies have confirmed the significant transfer of longitudinal momentum from the main channel to the flood plain. Despite the existence and apparent importance of the phenomenon, its influence is not accounted for in numerical models which are based on the St. Venant equations.

More recently attention has turned to the application of turbulence modelling techniques to the prediction of flow characteristics. Keller and Rodi (1984, 1986) have applied a depth-averaged numerical model to the prediction of existing experimental data from the earlier studies. Good quality data has, however, been sparse and limited in extent and for this reason the study reported herein has been undertaken.

The depth-averaged numerical model has been described elsewhere [Rodi et al. (1981), Keller and Rodi (1984)] and, for this reason, a brief review only is presented. The experimental rig is then described and the experiments and data acquisition system discussed. Finally, representative comparisons between numerical model predictions and the experimental data are presented and discussed.

## MATHEMATICAL MODEL

The development of the model commenced with the general elliptic form of the mean flow equations; however, the form of the data against which the model is tested is such that the simpler parabolic form of the equations can be utilized. The simplification arises from the assumption that downstream events cannot influence the flow upstream apart from an influence on the water level. Under these circumstances the flow is said to be of "boundary layer type" leading to the validity of the inequalities

$$\frac{\partial \bar{u}}{\partial x} \ll \frac{\partial \bar{v}}{\partial y} \quad \text{and} \quad \bar{v} \ll \bar{u} \quad (1)$$

where  $x$  and  $y$  are streamwise and transverse distances respectively, and  $\bar{u}$  and  $\bar{v}$  are the streamwise and transverse depth averaged velocities.

Application of the inequalities (1) to the full elliptic depth averaged equations yields [Keller and Rodi (1984)]

$$\frac{\partial h \bar{u}}{\partial x} + \frac{\partial h \bar{v}}{\partial y} = 0 \quad (2)$$

$$\bar{u} \frac{\partial \bar{u}}{\partial x} + \bar{v} \frac{\partial \bar{u}}{\partial y} = -g \frac{dh}{dx} + \frac{1}{\rho h} \frac{\partial}{\partial y} (\tau_{xy}^-) - \frac{\tau_{bx}}{\rho h} \quad (3)$$

where  $h$  is the depth,  $\rho$  and  $g$  are respectively density and gravitational acceleration,  $H$  is the water surface elevation above an arbitrary horizontal datum,  $\tau_{xy}^-$  is the depth averaged turbulent shear stress, and  $\tau_{bx}$  is the bed shear stress in the  $x$  direction.

The solution of Eqs. (2) and (3) requires prior knowledge of the streamwise variation of  $H$ . This is not a problem in the present study since the experimental flows are uniform and of known depth. In other cases the streamwise variation of the water surface needs to be calculated using a one-dimensional backwater-profile procedure.

The depth-averaged turbulent shear stress  $\tau_{xy}^-$  in Eq. (3) is evaluated using the Bousinesq eddy-viscosity concept and employing the  $k - \epsilon$  turbulence model to calculate the eddy viscosity.

The  $k - \epsilon$  model has been described in detail by Launder and Spalding (1974) and by Rodi (1980). Herein a depth averaged form of the model as developed by Rastogi and Rodi (1978) is employed. The parabolic form of the model is expressed by the following equations

$$\tau_{xy}^- = \rho \bar{v}_t \frac{\partial \bar{u}}{\partial y} \quad (4)$$

$$\bar{v}_t = c_\mu \frac{\bar{k}^2}{\epsilon} \quad (5)$$

$$\bar{u} \frac{\partial \bar{k}}{\partial x} + \bar{v} \frac{\partial \bar{k}}{\partial y} = \frac{\partial}{\partial y} \left[ \frac{\bar{v}_t}{\sigma_k} \frac{\partial \bar{k}}{\partial y} \right] + G + P_{kv} - \bar{\epsilon} \quad (6)$$

$$\bar{u} \frac{\partial \bar{\epsilon}}{\partial x} + \bar{v} \frac{\partial \bar{\epsilon}}{\partial y} = \frac{\partial}{\partial y} \left[ \frac{\bar{v}_t}{\sigma_\epsilon} \frac{\partial \bar{\epsilon}}{\partial y} \right] + c_1 \frac{\bar{\epsilon}}{k} G + P_{\epsilon v} - c_2 \frac{\bar{\epsilon}^2}{k} \quad (7)$$



$$G = \bar{v}_t \left( \frac{\partial \bar{U}}{\partial y} \right)^2 \quad (8)$$

In the above equations  $\bar{k}$  and  $\bar{\epsilon}$  are depth-averaged turbulence parameters,  $G$  is the production of the turbulent kinetic energy  $k$  due to the interaction of turbulent stresses with horizontal mean velocity gradients,  $P_{kv}$  and  $P_{ev}$  are source terms which absorb the effects of non-uniform vertical profiles and  $c_\mu$ ,  $c_k$ ,  $c_\epsilon$ ,  $c_1$  and  $c_2$  are empirical constants. Following the procedure of Rastogi and Rodi (1978),  $P_{kv}$  and  $P_{ev}$  are expressed by the equations

$$P_{kv} = \frac{1}{\sqrt{c_f}} \frac{U_*^3}{h} \quad (9)$$

$$P_{ev} = c_{e\Gamma} \frac{c_2}{c_f^{3/4}} \sqrt{c_\mu} \frac{U_*^4}{h^2} \quad (10)$$

where  $U_*$  is the friction velocity,  $c_f$  is the bed friction coefficient, and  $c_{e\Gamma}$  is a coefficient whose value depends on the dimensionless diffusivity  $e^* = \nu_t / (U_* h)$  [ $\sigma_t$  is the turbulent Prandtl number]. Following the recommendations of Fischer et al. (1979) a value for  $e^*$  of 0.15 was adopted, representing an average of experimental results for wide laboratory flumes.

The bed shear stress,  $\tau_{bx}$  in Eq. (3) is related to the depth-averaged velocity by the usual quadratic friction law

$$\tau_{bx} = c_f \rho \bar{U}^2 \quad (11)$$

The friction coefficient,  $c_f$ , is determined differently for the smooth main channel bed and the rough flood plain. For smooth beds the equation of Schlichting (1968) is used

$$c_f = 0.027 \left( \frac{\nu}{U_* h} \right)^{1/4} \quad (12)$$

where  $\nu$  is the laminar viscosity.

For rough beds,  $c_f$  is determined from the assumption of a logarithmic vertical velocity distribution. The standard equation is

$$u = 5.75 U_* \log \frac{30d}{k_s} \quad (13)$$

where  $u$  is the velocity at an elevation  $d$  above the bed,  $U_*$  is the shear velocity, and  $k_s$  is the Nikuradse sand grain roughness. Integration with depth of Eq. (13) and rearrangement yields

$$U_*^2 = \frac{\tau_{bx}}{\rho} = \left[ 6.25 + 5.75 \log (h/k_s) \right]^2 \quad (14)$$

from which, by direct comparison with Eq. (11)

$$c_f = [6.25 + 5.75 \log (h/k_s)]^{-2} \quad (15)$$

The parabolic equations (2), (3), (6) and (7) are solved using a modified version of the two-dimensional boundary layer procedure of Patankar and Spalding (1970). The procedure utilizes the assumption that conditions at a certain cross section do not depend on conditions downstream so the equations can be solved by marching downstream from cross-section to cross-section with prescribed initial conditions as the starting point.

When applied to channel flow, the method of Patankar and Spalding (1970) uses a streamline co-ordinate system. The transverse co-ordinate,  $y$ , is replaced by the dimensionless stream function  $\omega$  defined by

$$\omega = \frac{\psi}{\rho Q} = \frac{\int_0^y \rho \bar{U} h dy}{\int_0^B \rho \bar{U} h dy} \quad (16)$$

where  $\psi$ , the stream function, is by definition the mass flow between the bank with  $y = 0$  and a certain distance  $y$  from the bank, and  $Q$  is the total

discharge. Thus, the new lateral co-ordinate  $\omega$  varies between 0 at one bank and 1.0 at the other. The transformation of the equations in terms of coordinates  $x$  and  $\omega$  is described in detail by Rodi et al. (1981).

## EXPERIMENTAL

The experiments were conducted in a 12.0 m long and 1.2 m wide rectangular flume with glass walls and base. A slope of 0.00025 was adopted for all tests and two asymmetrical complex channel shapes were investigated. The geometries of the shapes tested are shown in Fig. 1. The model flood-plain sections were created from marine plywood coated with epoxy paint. This results in different roughnesses for the main channel (smooth) and the flood plain ( $k_s = 0.20$  mm).

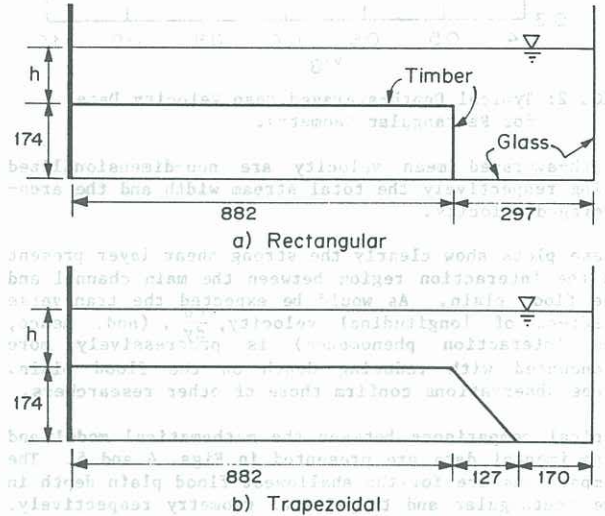


FIG. 1: Experimental Channel Cross-section Geometries.

Water was supplied to the flume from a constant head tank in a re-circulating system. Nominal uniform flow was established using an adjustable tailgate and checked by comparing mean velocity traverses upstream and downstream of the test section. Depths were measured using static pressure tappings connected to stilling wells. A magnetic flowmeter on the supply pipe was used to monitor the total discharge in the flume.

Instantaneous velocity measurements were made using a laser-Doppler anemometer operated in the backscatter mode. This instrument is non-intrusive and, hence, ideally suited to local measurements in regions of strong velocity gradient. A 2 watt Argon-Ion laser was used as the power source.

Detailed measurements were performed at the nodes of a closely spaced measurement grid with special emphasis on the interaction region. Measurements at a distance beyond 480 mm from the right hand wall were not possible due to the limitations of the particular laser system. This, however, was not of major importance since the measurement region extended beyond that part of the flood plain flow affected by the interaction phenomenon.

An on-line microcomputer was used for data collection and analysis. From the recorded fluctuating velocity data, time averaged mean velocities were determined at each node of the measurement grid and depth-averaged velocities subsequently calculated.

## RESULTS AND DISCUSSION

A total of sixteen sets of data have been obtained, eight with the rectangular geometry and eight with the trapezoidal geometry. Three representative sets of data for each geometry are presented in Figs. 2 and 3. The transverse distance from the left bank and the



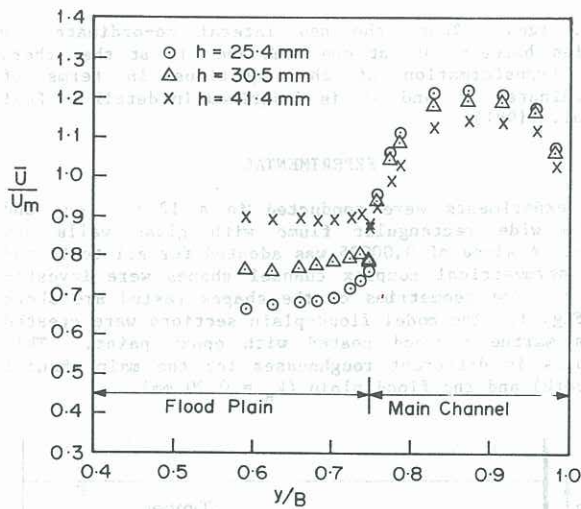


FIG. 2: Typical Depth-averaged Mean Velocity Data for Rectangular Geometry.

depth-averaged mean velocity are non-dimensionalized using respectively the total stream width and the area-averaged velocity.

These plots show clearly the strong shear layer present in the interaction region between the main channel and the flood plain. As would be expected the transverse gradient of longitudinal velocity,  $\frac{dU}{dy}$ , (and, hence, the interaction phenomenon) is progressively more pronounced with reducing depth on the flood plain. These observations confirm those of other researchers.

Typical comparisons between the mathematical model and experimental data are presented in Figs. 4 and 5. The comparisons are for the shallowest flood plain depth in the rectangular and trapezoidal geometry respectively. Comparisons for the other sets of test data have been carried out and are similar.

In general the agreement between prediction and data is within about 10% in the main channel and within closer tolerances on the flood plain. In Fig. 4 some discrepancy is apparent between prediction and data within the interaction region. This is probably due to the assumption of a logarithmic vertical velocity

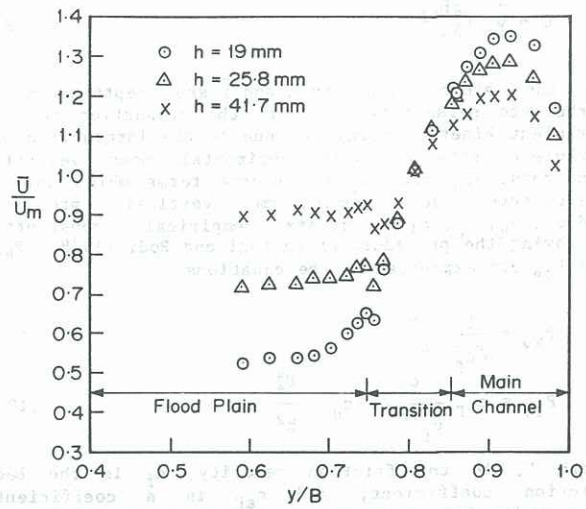


FIG. 3: Typical Depth-averaged Mean Velocity Data for Trapezoidal Geometry.

distribution to calculate the bed shear stress. Within the interaction region, the vertical velocity profiles are themselves strongly affected by the transverse velocity gradients. Furthermore, the transverse velocity gradient in this region varies with depth and the resulting strongly three-dimensional nature of the flow is not well represented by the two-dimensional model. The agreement in this region is better for the trapezoidal geometry which is a consequence of the less severe nature of the bed discontinuity.

#### CONCLUSIONS

The study reported in this paper indicates that the depth averaged form of the  $k - \epsilon$  turbulence model can be utilized to predict the effects of interaction phenomena on velocity profiles in main channel/flood plain flows. It is emphasized that in using the model none of the empirical coefficients were tuned to match the experimental results but were simply adopted from the literature.

Much of the experimental data previously obtained in studies of interaction phenomena cannot be used to check this mathematical model because either they are

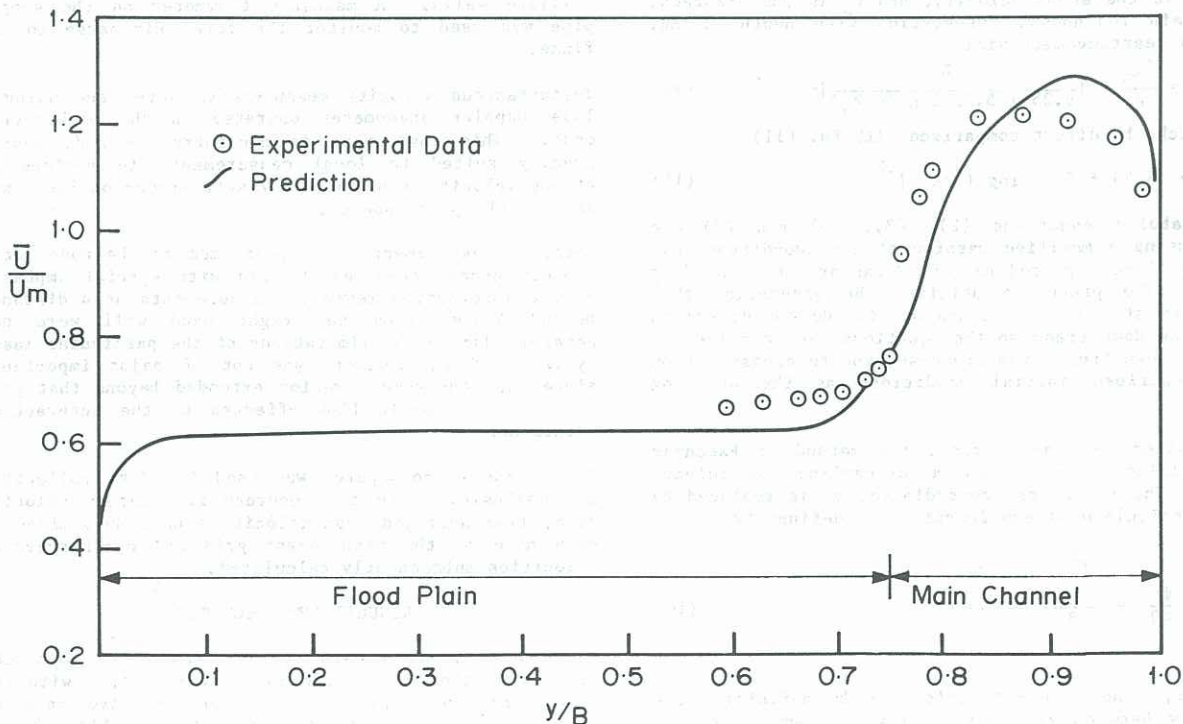


FIG. 4: Velocity Comparisons for Rectangular Geometry,  $h = 25.4$  mm.

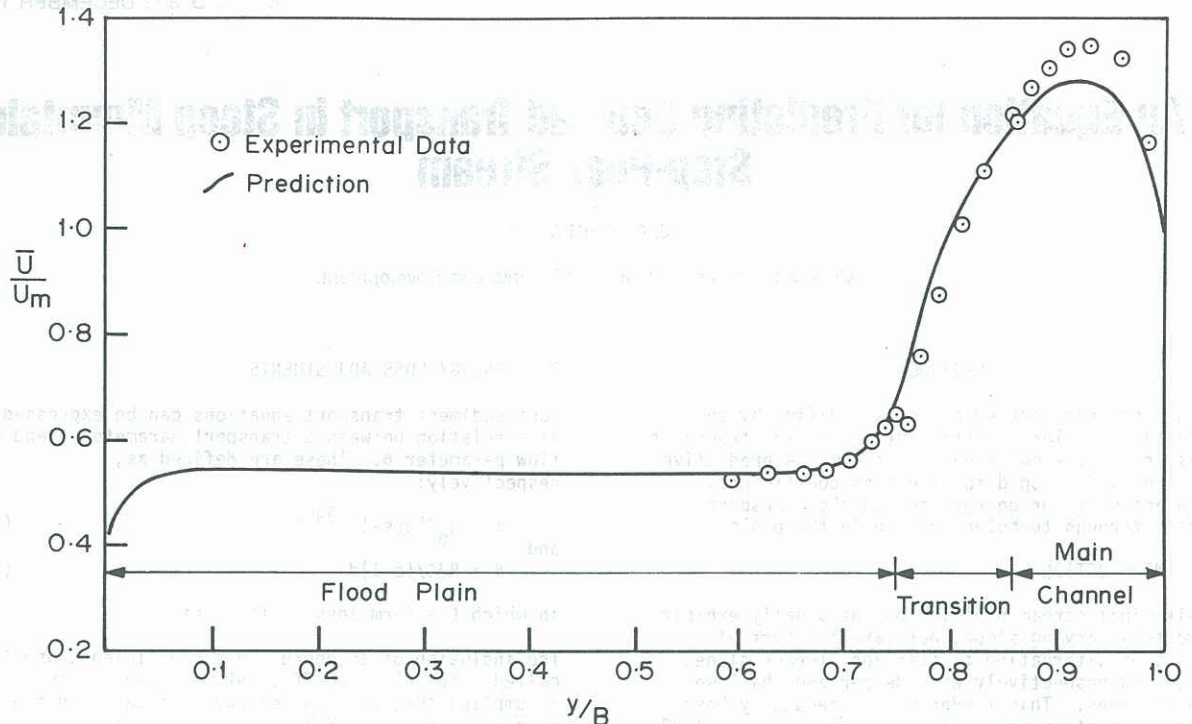


FIG. 5: Velocity Comparisons for Trapezoidal Geometry,  $h = 19$  mm.

not specific enough or they fail to meet acceptance criteria. There exists a need for further accurate experimental studies, both laboratory and field, to enable more extensive testing of the model.

#### ACKNOWLEDGEMENTS

The first author is grateful to the Alexander von Humboldt Foundation for their support in the form of a fellowship during the period when this research was initiated. The authors are grateful to Professor Franz Durst, University of Erlangen, for assistance during the experimental phase of this project.

#### REFERENCES

- Fischer, H.B.; List, E.J.; Koh, R.C.Y.; Imberger, J.; Brooks, N.H. (1979). *Mixing in Inland and Coastal Waters*, Academic Press, New York.
- Keller, R.J.; Rodi, W., (1984). Prediction of Two-dimensional Flow Characteristics in Complex Channel Cross-sections. *Proceedings, International Conference on Hydraulic Engineering Software*, Portonoz, Yugoslavia, September, 1984, Elsevier, pp. 3-3 to 3-14.
- Keller, R.J.; Rodi, W., (1986). Prediction of Flow Characteristics in Main channel/Flood plain flows, (in press).
- Knight, D.W.; Demetriou, J.D.; Hamed, M.E., (1983), *Hydraulic Analysis of Channels with Flood Plains*. *Proceedings of the International Conference on the Hydraulic Aspects of Floods and Flood Control*, London, England. BHRA, pp. 129-144.
- Lauder, B.E.; Spalding, D.B., (1974), *The Numerical Computation of Turbulent Flow*. *Computer Methods in Applied Mechanics and Engineering*. Vol. 3, pp. 269-289.
- Patankar, S.V.; Spalding, D.B., (1970), *Heat and Mass Transfer in Boundary Layers*. 2nd Ed., Intertext Books, London.
- Rajaratnam, N.; Ahmadi, R., (1979), Interaction between Main Channel and Flood Plain Flows. *Journal of the Hydraulics Division, ASCE*, Vol. 105, No. HY5, pp. 573-588.
- Rajaratnam, N.; Ahmadi, R., (1981), *Hydraulics of Channels with Flood-plains*. *Journal of Hydraulic Research*, Vol. 19, No. 1, pp. 43-60.
- Rastogi, A.K.; Rodi, W., (1978), Predictions of Heat and Mass Transfer in Open Channels. *Journal of the Hydraulics Division, ASCE*, Vol. 104, No. HY3, pp. 397-420.
- Rodi, W., (1980), *Turbulence Models and their Application in Hydraulics: A State-of-the-Art Review*, International Association for Hydraulic Research Book Publications, Delft.
- Rodi, W.; Pavlovic, R.N.; Srivatsa, S.K., (1981), Prediction of Flow and Pollutant Spreading in Rivers; in: *Transport Models for Inland and Coastal Waters*. Academic Press, Inc.
- Schlichting, H., (1968), *Boundary Layer Theory*, 6th ed., McGraw Hill, New York.
- Sellin, R.H., (1964), A Laboratory Investigation into the Interaction between the Flow in the Channel of a River and that over its Flood-plain. *La Houille Blanche*, Grenoble, France, No. 7, pp. 793-802.
- Zheleznyakov, G.V., (1971), Interaction of Channel and Flood-plain Streams. *Proceedings of IAHR Congress*, Paris, France, Vol. 5, pp. 144-148.