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Nonlinear Dynamics of Inviscid Shear Layers

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ABSTRACT

We present the results of several numerical simulations of the incompressible Euler equations which describe the nonlinear dynamics of an infinite shear layer in an unbounded inviscid fluid. The flow is subject to streamwise periodic vorticity perturbations and its evolution is formulated within the vortex-dynamic framework as an initial value problem for the vorticity field. Specific examples illustrate the interaction of the first subharmonic with the fundamental perturbation in the infinite Reynolds number limit. In one example a background strain which stretches the vortex lines is added in order to simulate the dynamics of a stretched shear layer. This is found to inhibit the interaction of the subharmonic with the rolled-up vortex cores.

INTRODUCTION

In the past few years increased experimental and theoretical effort has been placed on the description of coherent structure of turbulent flows. Interpretations of phenomena observed in controlled laboratory experiments (e.g. Jimenez et al (1985)) and in detailed numerical simulations (e.g. Acton (1976)) are often cast in terms of typical vorticity distributions which interact through nonlinear instability mechanisms. In recent work Corcos, Lin and Sherman (Corcos & Sherman (1984), Corcos & Lin (1984), Lin & Corcos (1984)) propose a model of the shear layer in its early stages of transition to a fully three-dimensional turbulent flow. This model describes the layer evolution through a heirachy of primitive deterministic fluid motions with each level characterized by a specific vorticity distribution evolving within the ambient strain environment provided by the other scales of motion.

The first order motion is the much studied nominally two-dimensional nonlinear temporal Kelvin-Helmholtz instability of the streamwise periodic shear layer separating two streams of equal and opposite fluid velocity U/2. This instability leads to the well known shear layer rollup into spanwise vortex cores connected by nearly flat, thin braids of spanwise vorticity. The concentration of vorticity due to rollup is temporary. Experimental and numerical evidence (e.g. Winant and Broward (1974), Corcos & Sherman (1984)) indicates that the spanwise vortices further amalgamate into larger, more distant vortex cores and that this amalgamation is preferentially a two-dimensional process. A two-dimensional simulation may then provide a reasonable description of the first order flow.

The second order motion arises from the growth of a three-dimensional instability upon the time dependent base flow (i.e. first order motions). In the braid region this three-dimensional instability (possibly the "translative" instability of Pierrehumbert & Widnall (1982)) produces an array of flat, counter-rotating streamwise vortices whose axes are locally tangential to the braid. Lin & Corcos (1984) studied these secondary streamwise vortices for moderate Reynolds numbers using a quasi-two-dimensional prototype flow which included an imposed plane stretching strain to model the induced velocity midway between spanwise vortices.

Pullin & Jacobs (1986) used the method of Contour Dynamics (Zabusky et al (1979)) to calculate the evolution of the secondary streamwise vortices in the infinite Reynolds number ($Re = \infty$) limit. They found that initially elliptic distributions of streamwise vorticity collapse into compact, nearly axisymmetric cores surrounded by spiral arms. For ellipses of very high aspect ratio however, a Kelvin-Helmholtz type instability produced strings of small vortex cores embedded within the stretching strain field. This fine structure of the secondary vortices forms the basis for the higher order motions of the Corcos-Lin-Sherman model and suggests a possible dynamical mechanism for the cascade of turbulence energy to the smaller scales.

We will presently use the Contour Dynamic method to first study the purely two-dimensional shear layer evolution (i.e. first order motions). The simulations illustrate the sensitivity of the layer evolution to the relative phase of the fundamental perturbation and its first subharmonic in the nominal limit of $Re = \infty$. For $Re = O(100)$ this interaction has been studied by Patnaik, Corcos & Sherman (1976) using finite difference methods and Riley & Metcalfe (1980) who utilized a spectral technique. Results were also obtained by Acton (1976) using the point vortex technique. With the addition of a plane stretching strain field we obtain an inviscid model which is perhaps relevant to fine scale features of the mixing layer and other unbounded turbulent flows.

FORMULATION

We consider the evolution of a cylindrically-symmetric vorticity field in an inviscid, incompressible fluid of constant density subject to an externally applied, spatially uniform strain field of constant strength γ. Putting $\gamma = 0$ models the purely two-dimensional first order motions. Figure 1 shows a conceptual view of the flow model. A nonuniform vorticity field with only one component of vorticity, ω_z, is approximated by a piecewise constant distribution in which regions

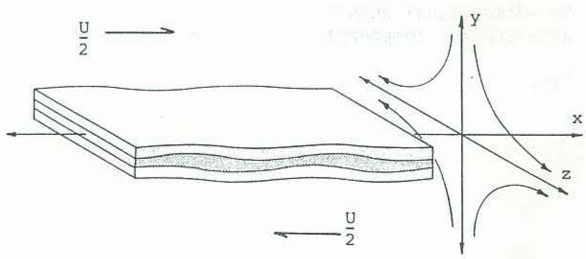


Figure 1. Sectional view of a periodic shear layer embedded in a plane stretching strain field.

R_j, $j = 1 \dots M$, contain uniform vorticity ω_j. The region R_j, bounded by contours C_j and C_{j+1}, extends to $x = \pm\infty$, and is periodic in x. Each C_j is described parametrically by the complex function $\zeta_j(e,t)$. The four curved streamlines in the (y,z)-plane represent the externally applied strain aligned with the z-axis so as to stretch the vortex lines.

The vortex dynamics maintains the piecewise constant distribution so that the C_j retain their identity as material curves defining the ω_z discontinuities. In the Contour Dynamic technique, the Eulerian velocity field $[u_x - iu_y]$ is derived kinematically from ω_z and is then identified with the particle velocities $d\zeta^*/dt$ on the C_j. This gives an initial value problem for the evolution of the C_j, fully equivalent to the inviscid Euler equations (see Jacobs & Pullin (1985) and Pullin & Jacobs (1986)), as

$$\frac{d\zeta_j^*}{dt} = i\gamma y_j + \frac{1}{2\pi\lambda}\exp(\gamma t) \times \sum_{m=1}^{M+1} \Delta\omega_m \oint_{C_m} (y_j - y_m')\cot\left[\frac{\pi}{\lambda}(\zeta_j - \zeta_m')\right]\frac{d\zeta_m'}{de'}\,de'$$

$$j = 1 \dots M. \tag{1}$$

The first term on the right hand side of (1) is the velocity in the (x,y)-plane due to the applied strain field while the summation of integrals is the self-induction of the vorticity field consisting of M + 1 adjacent regions.

The evolution equation (1) can be nondimensionalized by setting the period of the layer in the x-direction $\lambda = 2$ and choosing the total circulation $\Gamma = 1$ in one period of the computational domain $0 \leqslant x \leqslant 2\pi$, (or equivalently setting the shear velocity, $U = 1/(2\pi)$). Numerical solutions reported here were obtained by defining the contours as sets of nodes joined by parabolic line segments and then approximating the integrals over the segments with a Gaussian quadrature. This resulted in a set of ordinary differential equations for the node coordinates which was integrated in time using a Runge-Kutta-Fehlberg method. Invariably the evolution of the vortex regions produced sufficient local distortion of the C_j to require rediscretization at regular time intervals. This was done so as to maintain the apriori accuracy of the contour descriptions. Full details of the numerical scheme are given in Pullin & Jacobs (1986).

INITIAL CONDITIONS

The initial conditions specify the initial shape and position of the C_j and the initial values of the ω_j. Presently we take M = 7 and use the unperturbed vorticity distribution (the ω_j and Y_j columns of table 1) chosen to model the tanh velocity profile studied by Michalke (1964). The scaling is such that the infinitesimal disturbance with the highest growth rate ($k\delta_\omega \simeq 0.875$) has a dimensionless wave number k = 2. This was determined by performing a linear stability analysis of the model shear layer. Vorticity preserving solutions to the Euler equations were sought in which the characteristic contour shapes were described by

$$\eta_j = Y_j + \alpha_j\, e^{ikx}\, e^{i\sigma t} \quad , \; j = 1 \dots M+1. \tag{2}$$

Use of (2) plus similar expressions for the perturbed flow potential in the equations of motion then gave an eigenvalue problem with eigenvalue σ. Figure 2 shows the normalized growth rates $(-\sigma_i/\omega_{max})$ for the piecewise constant M = 7 vorticity profile used here, a uniform vorticity layer M = 1 and the continuous profile $u_x = U/2\tanh(y)$ of Michalke. The M = 7 distribution provides a close approximation to the perturbation growth rates of the continuous profile.

The shape of the perturbation is calculated through the eigenvector but the amplitude is indeterminant.

Table 1 : Initial Condition.

j	ω_j	Y_j	k=2 α_r	k=2 α_i	k=1 α_r	k=1 α_i
0	0.0	-----	-----	-----	-----	-----
1	0.100	0.365	-0.248	-0.339	-0.194	-0.652
2	0.234	0.219	-0.331	-0.479	-0.211	-0.772
3	0.316	0.128	-0.393	-0.667	-0.206	-0.883
4	0.364	0.059	-0.341	-0.940	-0.137	-0.991
5	0.316	-0.059	0.341	-0.940	0.137	-0.991
6	0.234	-0.128	0.393	-0.667	0.206	-0.883
7	0.100	-0.219	0.331	-0.479	0.211	-0.772
8	0.0	-0.365	0.248	-0.339	0.194	-0.652

Table 1 contains the values of α defining the perturbation shapes for the most unstable wave ($k\delta_\omega = 0.875$, k = 2) and its first subharmonic. All simulations reported presently employ this purely two-mode disturbance, that is, no small scale "turbulent" perturbation has been introduced. The contours with the largest perturbation amplitude $|\alpha|$ correspond to j = 4, 5 and are scaled to give $|\alpha| = 1.0$. For the nonlinear simulations presented in the next section $|\alpha|_{max} = 0.05$ for both k = 1 and k = 2 so that the simulations commence close to the time that the perturbations start to interact nonlinearly.

As the infinitesimally small perturbations grow independently, they may be combined with any relative phase angle. We choose the fundamental (k = 2) to have a phase $\phi = 0$ and allow the subharmonic to take values $\phi = 0$, $\pi/4$, and $\pi/2$. These are labelled as the pure pairing mode, the pairing/tearing mode and the pure tearing mode. The fundamental redistributes the vorticity of the layer producing two slight accumulations (later to become the spanwise vortex cores) of vorticity while maintaining $\Gamma = 1$ over the computational domain. The effect of superposing the subharmonic depends upon the value of ϕ. For $\phi = 0$ the subharmonic modulates the y-position of the centroids of the vortex concentrations while keeping their strengths equal. $\phi = \pi/2$ modulates the strengths of the vortex concentrations while maintaining the centriods undisturbed, and $\phi = \pi/4$ alters both the strengths and the centroid positions.

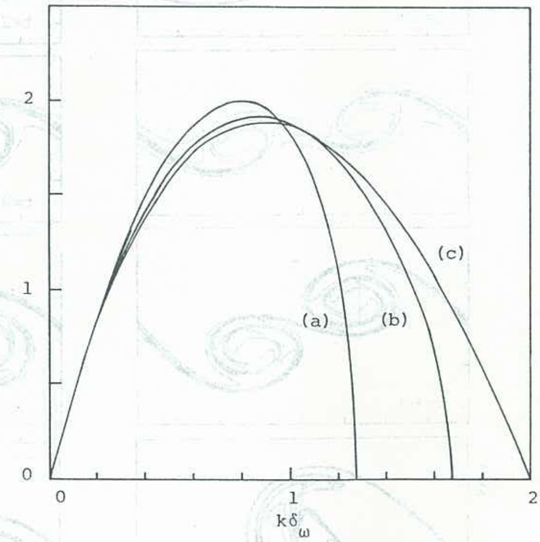


Figure 2. Normalized growth rate for three vorticity distributions;
(a) single region M=1 of uniform vorticity,
(b) M=7 regions as defined in table 1,
(c) hyperbolic-tangent velocity profile.

RESULTS AND DISCUSSION

Four specific simulations of the shear layer are shown here. Figure 3 (a-c) shows simulations of the purely two-dimensional first order motions of the mixing layer while figure 4 shows a calculation that includes a background strain.

Two-dimensional shear layer

Figure 3(a) (ϕ = 0) shows the shear layer rollup on the wavelength of the fundamental perturbation and the subsequent coalescence of pairs of vortex cores into larger, more distantly spaced cores. The similarity of the large scale features in this figure to those of figure 7 in Corcos & Sherman (1984) confirms that the large scale dynamics of the mixing layer are only weakly dependent upon Re. The small scale motions will however depend upon Re. For example, the very thin braid at t = 90 is almost nonexistent in the Re = O(100) simulations of Corcos & Sherman (1984) and Riley & Metcalfe (1980).

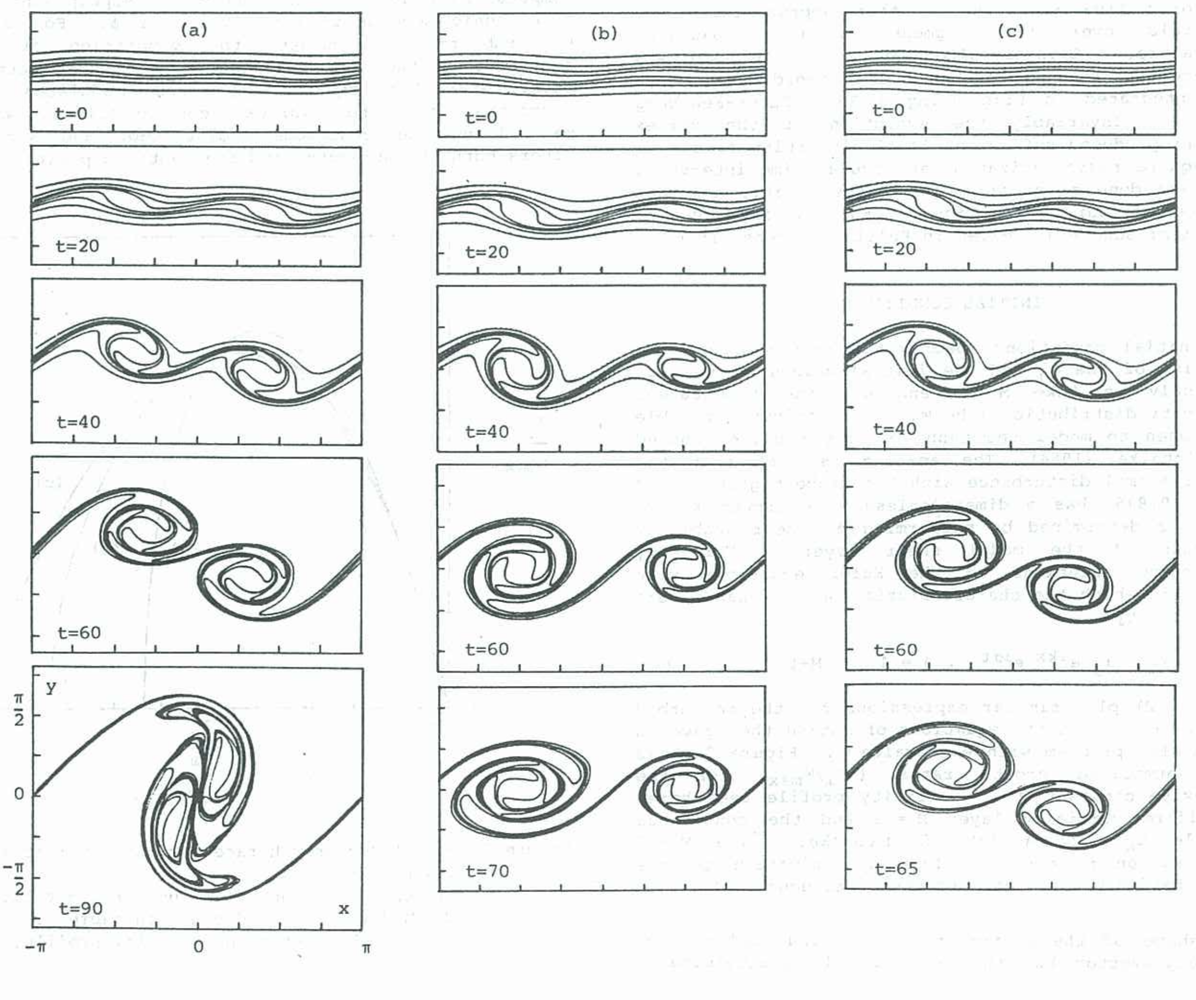
Modulation of the size of the initial vortex concentrations ($\phi = \pi/2$) dramatically changes the subharmonic interaction. The rollup of the vortex cores still occurs but simultaneously the smaller vorticity concentration is "shredded" or "torn" by the induced strain of the larger vortices located either side (Moore and Saffman (1975)). Vorticity migrates along the braids and although the process is much slower than pairing the simulation indicates that the interaction tends to redistribute the vorticity into fewer but stronger vortex cores with larger separation. The inviscid results of figure 3(b) differ from the moderate Re simulations of Riley & Metcalfe (1980) in that the roll-up of the fundamental concentrations here produces very thin braids and effectively halts the migration of vorticity to the larger cores. For longer simulation times, outside the range of the present results, the strain of the larger cores may elongate the smaller cores and restart the tearing process anew.

A combination of the two modes of interaction ($\phi = \pi/4$) is illustrated in figure 3(c). As the layer rolls up, shredding induces vorticity to migrate out of the smaller cores while the vortex centroids begin to rotate about each other. Although the pairing is proceeding at a slower rate than in figure 3(a) it is accelerating (as the distance between the vortex centriods decreases) and we expect that by t = 90 (out of the range of present results) the cores will have coalesced. This dominance of the pairing interaction over the tearing interaction has previously been noted in the point vortex simulations of Acton (1976) and and the spectral calculations of Riley & Metcalfe (1980).

Stretched shear layer

Introduction of a background stretching strain radically alters the long time evolution as shown in figure 4. This calculation has the same initial conditions as that in figure 3(a) but with the addition of the plane stretching strain we have a flow configuration possibly relevant to the tertiary motions of the mixing layer model. A nondimensional value of Υ = 0.015 corresponds roughly to the local conditions of the secondary vortex evolution of figure 17 in Pullin & Jacobs (1986), a single frame of which is reproduced here as figure 5.

Figure 3. Evolution of a periodic two-dimensional shear layer, M=7. Case (a) pure pairing interaction ϕ=0, (b) pure tearing $\phi=\pi/2$, (c) pairing/tearing $\phi=\pi/4$. Times t as shown.



The stretching strain accelerates the initial rollup by intensifying the vorticity but retards the pairing interaction by firstly inhibiting the vortex cores from rotating about each other and secondly by reducing the core diameters. A similar result was obtained for initial conditions $\phi = \pi/4$ (not shown). This inhibition of pairing may be interpreted in terms of the inviscid simulations of an isolated pair of stretched vortex cores (Jacobs & Pullin (1985)). The flow in the presence of the (y,z)-plane stretching strain can be shown to be equivalent to a strictly two-dimensional flow with an applied (x,y)-plane strain field that, with this relative vortex orientation, separates the vortex centres and inhibits vortex pairing and/or coalescence.

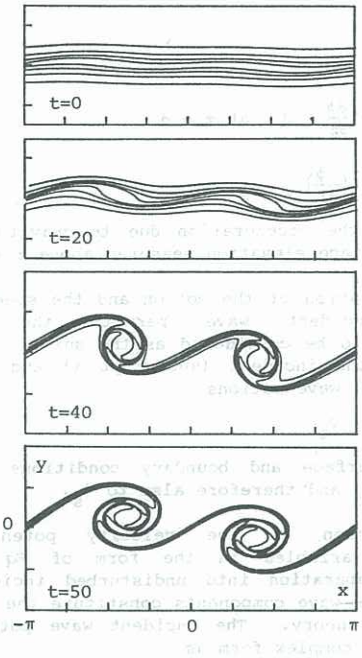


Figure 4. Evolution of a periodic shear layer, $\gamma=0.015$, $\phi=0$.

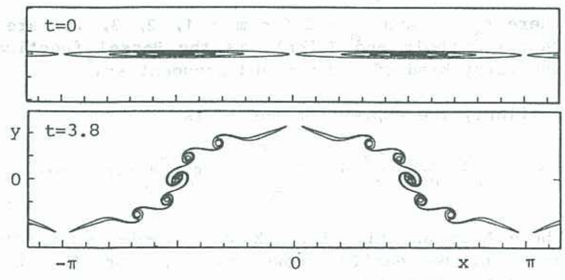


Figure 5. Evolution of an array of stretched vortices with alternating circulation. Initial aspect ratio of ellipse, $\pi/\delta_\omega=51.7$, $\gamma=0.1$.

CONCLUSIONS

We have used the method of Contour Dynamics to study a perturbed free shear layer in the Re = ∞ limit. The vortex dynamics of the large scale motions of the mixing layer appears to be only weakly dependent upon Reynolds number, at least for small times, as simulations illustrating the interaction of the first subharmonic with the fundamental perturbation agree qualitatively with previous simulations at moderate Re. Addition of a stretching strain inhibits the interaction of the subharmonic with the rolled-up vortex cores.

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SYMBOLS

U	shear velocity
u	Eulerian velocity
γ	stretching strain strength
t	time
ω	vorticity
Γ	circulation in the computational domain
δ_ω	vorticity thickness, U/ω_{max}
Re	Reynolds number, $U.\delta_\omega/\nu$ (ν = kinematic viscosity)
x,y,z	cartesian coordinates
i	$\sqrt{(-1)}$
ζ	complex coordinate, x + iy
R	region of uniform ω_z in the (x,y)-plane
C	contour delineating a discontinuity in ω_z
e	contour parameter
$\Delta\omega_m$	vorticity jump across C_m, $\omega_m(0) - \omega_{m-1}(0)$
λ	wavelength
k	wave number, $2\pi/\lambda$
Y	y-ordinate of the unperturbed contour
η	y-ordinate of the perturbed contour
σ	complex growth rate, $\sigma_r + i\sigma_i$
α	complex amplitude, $\alpha_r + i\alpha_i$
′	primed quantities are integration variables
*	superscript indicates complex conjugate