

Conditions for the Coalescing of Two-Dimensional Vortices, Obtained from Experiments with Rotating Flows

R. W. GRIFFITHS

Research School of Earth Sciences, Australian National University, Canberra, Australia.

ABSTRACT

The stability of pairs of like two-dimensional vortices were studied by observing the flow induced by two sources or two sinks in a rotating fluid. The sources and sinks generate vortices whose initial strengths, core sizes and relative positions can be controlled. Baroclinic geostrophic vortices are produced when the fluid contains a density stratification. For both homogeneous and two-layer fluids vortices coalesced into one larger vortex if they were generated sufficiently close together. The conditions for coalescing are outlined in terms of the core radius and internal Rossby radius of deformation. Merging of strongly baroclinic vortices occurs from large separations and appears to involve an increase in the potential energy of the flow.

INTRODUCTION

Vortex "pairing", the interaction and subsequent coalescing of neighbouring large vortices in a pair-wise manner, is responsible for growth of coherent vortex structures and for mixing in quasi-two-dimensional shear layers (e.g. Winant & Browand, 1974). Interactions of two-dimensional vortices have been studied through numerical and experimental investigations of such mixing layers (e.g. Aref, 1983; Thorpe, 1973). However, the properties and relative positions of vortices in mixing layers are beyond external control and pair-wise interactions are also influenced by other vortices in the layer. In order to understand vortex interactions it is helpful to consider a single pair of vortices of the same sign in an otherwise quiescent fluid. This problem is also relevant to the deterministic phenomena which contribute to the statistical properties of two-dimensional turbulence.

Numerical simulations of a pair of identical finite-core vortices (Christiansen & Zabusky, 1973; Overman & Zabusky, 1982) indicate that the flow configuration is stable when the two vortices lie farther than a critical distance apart, in which case the vortices orbit around their mutual centre of vorticity. When the vortex centres are separated by less than the critical distance, the vortices coalesce into a single vortex. The critical distance is found to be approximately 3.2 core radii.

The case of perfectly two-dimensional vortices is far simpler than the more general problem of interest in geophysics, where vortices occur in a fluid having a density stratification and a background (planetary) vorticity. Stratification implies a depth dependence in the flow while stratification and background vorticity together lead to an additional horizontal length scale (the Rossby radius, over which buoyancy and Coriolis forces are in balance). Flow is approximately geostrophic so long as velocities are not too large (Rossby number < 1), as is the case for oceanic and atmospheric eddies having scales of tens to hundreds of kilometres. Previous calculations (Gill & Griffiths, 1981; Hogg & Stommel, 1985) have shown that under the assumption of no dissipation, hence conservation of potential vorticity, merging of stratified geostrophic vortices requires an increase in the total potential energy associated with the density distribution in the fluid. Thus vortices must overcome an energy barrier if they are to coalesce. Here we summarise the results of laboratory experiments reported by Griffiths & Hopfinger (1986).

EXPERIMENTS

Pairs of vortices were generated in a container rotating at a constant angular velocity Ω by forcing flow through small sources (anticyclones) or sinks (cyclones) for a short time. When the forcing was turned off vortices proceeded to interact with each other. Sources and sinks were always at the free upper surface of the fluid, and the fluid was either homogeneous or consisted of two layers of different density but equal depth. A number of Rossby radii λ were used in order to cover both small and large values of the ratio λ/R . Here $\lambda = (g\Delta\rho H/\rho)/2\Omega$, where $\Delta\rho$ was the density difference between layers, H (≈ 20 cm) was the depth of each layer. In all cases $\Omega = 1.0 \text{ rads}^{-1}$, while $\lambda = 0, 1.5, 5, 10$ or 15 cm. Intensities and core radii for a number of vortices were measured using streak photographs and calibrated against forcing flowrate and duration.

With no density gradients, all effects of Coriolis forces in the interior of the fluid and associated with the background vorticity are removed by geostrophic pressure gradients so long as the flow is independent of distance parallel to the rotation axis. Rotation serves to create and maintain two-dimensionality, the vortices behaving otherwise as though in a non-rotating system. Only in the viscous Ekman layer on the base of the container is the rotation of significance: the magnitude and direction of the Ekman layer flux depends upon the magnitude of Ω and the sign of the vortex relative to Ω . In non-rotating systems, Ekman layers on boundaries perpendicular to vortex axes spin down vortices of both signs in the same manner as they do cyclones in the rotating case.

VORTEX STRUCTURE

A model for a two-layer vortex induced by forcing in the top layer is shown in Figure 1. The vortex is assumed to have a core of radius R , with a uniform but anomalous potential vorticity Π_0 in the top layer. The potential vorticity is defined as $\Pi = (2\Omega + \zeta)/\eta$, where $\zeta = \nabla \times \mathbf{u}$ is the relative vorticity and η is the local depth of the layer.

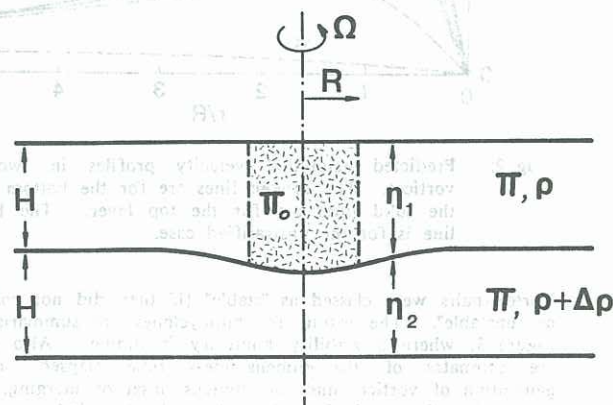


Fig 1: Diagram of the model vortex having piece-wise uniform potential vorticity.

Outside the core, and in the bottom layer, the potential vorticity is uniform at $\Pi = 2\Omega/H$. Griffiths & Hopfinger (1986) showed that the quasi-geostrophic potential vorticity equations lead to the following azimuthal velocities:

$$v = \frac{1}{2} \frac{Sr}{R^2} \left[1 \pm \frac{2R}{r} K_1 \left(\frac{R}{\lambda} \right) I_1 \left(\frac{r}{\lambda} \right) \right] \quad r < R, \quad (1)$$

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where s is the vortex intensity, the positive signs refer to the top layer and the negative signs to the bottom layer.

Examples of the velocity profiles (1) are plotted in Figure 2. The most important features are that the flow becomes independent of depth at large distances ($r \gg \lambda$) or for $\lambda/R \rightarrow 0$. A barotropic Rankine vortex with $v \sim r^{-1}$ is obtained in the limit $\lambda = 0$. For $\lambda > R$ and $r < \lambda$, on the other hand, motion is strongly baroclinic. The velocity in the top layer of a baroclinic vortex decays more rapidly with distance that it does in the barotropic case, and the rate of decay (at $R < r < \lambda$) is a maximum for $\lambda \approx R$. The above profiles were in very close agreement with our measured velocities for the laboratory vortices, the only deviation being some rounding of the peak in the upper layer velocity due to the absence of a discontinuity in vorticity at the edge of the core.

CONDITIONS FOR COALESCING

Vortex pairs coalesced, for all Rossby radii used, whenever their initial separation distance d was sufficiently small. Coalescing always began with the growth of a cusp on each vortex. Cusps then stretched around the opposite vortex until the vortices formed two elongated 's'-shapes adjacent to each other. Each of these collapsed to form a clump of 'coded' water at each end of an elliptical region of anomalous vorticity. The clumps proceeded to twist up to form two entwined spirals of water from the original vortices, the spirals making up a single circular vortex.

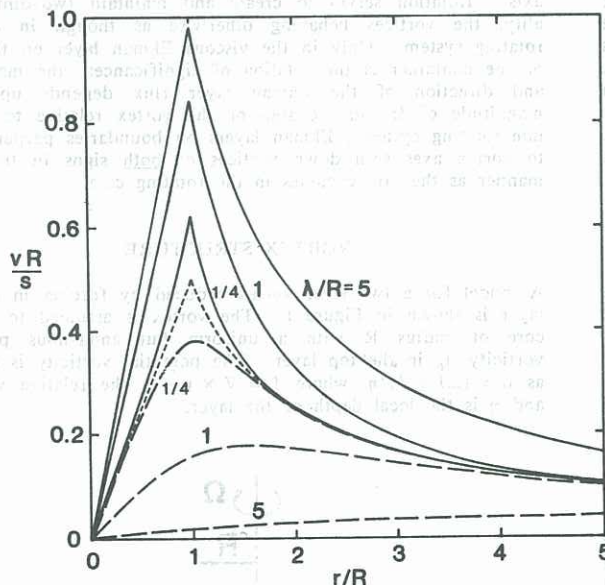


Fig 2: Predicted azimuthal velocity profiles in two-layer vortices. The broken lines are for the bottom layer, the solid lines are for the top layer. The broken line is for the unstratified case.

Vortex pairs were classed as "stable" (if they did not coalesce) or "unstable". The results for anticyclones are summarised in Figure 3, where a stability boundary is drawn. Also shown are estimates of the dimensionless time elapsed between generation of vortices and the obvious onset of merging. The normalising time scale is $2\pi d^2/s$, the orbital period for motion of two vortices separated by a distance d about the centre of vorticity. Unstratified anticyclones ($\lambda/R=0$) coalesced

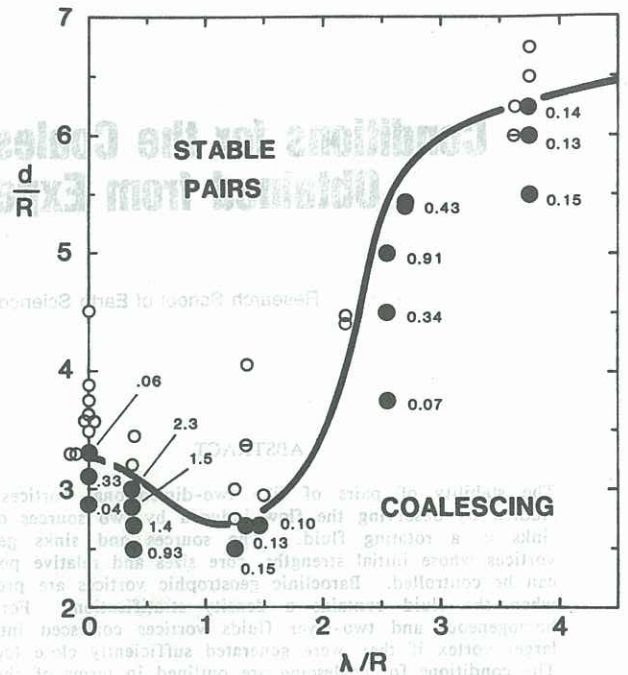


Fig 3: Observations of the stability of anticyclone pairs: O did not coalesce; ● coalesced; ◐ uncertain. Coalescing occurred after times of order $(0.1-1)2\pi d^2/s$.

whenever $d < (3.3 \pm 0.2)R$, in excellent agreement with the inviscid numerical simulations. The critical separation reached a minimum near $\lambda/R \approx 1$, where we expect the velocity field for each vortex to decay with distance most rapidly. However, for "strong stratification" ($\lambda/R > 2$) anticyclones coalesced from much greater distances. This surprising result reflects the dominant role played by the Rossby radius when this scale is much greater than the core radius.

Cyclones in the top layer with $\lambda > 0.3R$ gave a stability boundary very similar to that shown for anticyclones. However, unstratified cyclones coalesced from separation distances much greater than did unstratified anticyclones and no critical distance could be found. This result is attributed to the effects of bottom friction (which are small in the stratified case). Ekman pumping in cyclones generates divergence which tends to continuously increase the radius of the core of anomalous vorticity as the vortex spins down. The ratio d/R therefore decreases with time until coalescence occurs. This behaviour is relevant to pairs of vortices of either sign in non-rotating systems, where all vortices are spun down through divergence induced by Ekman pumping.

CONCLUSIONS

Coalescing of both unstratified and baroclinic vortices was observed when vortices were generated with centres sufficiently close together. Remarkably, vortices in a "strongly stratified" rotating fluid (Rossby radius \gg core radius) coalesced from distances much greater than the critical value for unstratified and "weakly stratified" vortices. No external energy supply was available after the initial generation of vortices and it must be concluded that either kinetic energy was converted into potential energy or the potential vorticity of the fluid in the vortices was altered during coalescence. The latter is unlikely as the rate of dissipation in Ekman boundary layers appears to provide insufficient alteration of potential vorticity during the relatively rapid merging events. The details of the merging process (including the time scale) are also identical to those found for unstratified vortices in inviscid numerical simulations. On the other hand, Ekman dissipation does cause those cyclones which are in contact with a rigid boundary to coalesce from much greater separations than do vortices isolated from boundaries by the effects of density gradients.

Oceanic eddies formed at density fronts and western boundary currents typically have radii of order 20-100km, with $\lambda/R \approx 0.3-0.6$. Their velocity fields are concentrated in the upper ocean, so that they are not likely to be greatly influenced by bottom friction. While their interactions must depend critically on advection by surrounding currents and eddies, our results indicate that eddies will tend to coalesce if their centres approach to within three radii. Coalescence provides one mechanism by which energy can be transferred to larger scales but, at the same time, increases the potential energy and carries energy toward scales at which baroclinic instability occurs. An opposite extreme is found in intense atmospheric cyclones (hurricanes). These have core radii of only 10-80km in an atmosphere whose Rossby radius is of order 800km. Hence $\lambda/R > 10$. Although the Rossby number is roughly four times greater than those of our laboratory vortices, we tentatively suggest that coalescence is likely (and rapid) for hurricane pairs with separations less than about 500km. Indeed, although hurricane pairs are regularly observed over the subtropical western Pacific and Atlantic Oceans, they are never found with separations of less than 400km. Pairs separated by less than 750km appear to attract each other, probably as a result of bottom friction which our experiments show can lead to eventual coalescence from greater distances.

ACKNOWLEDGEMENTS

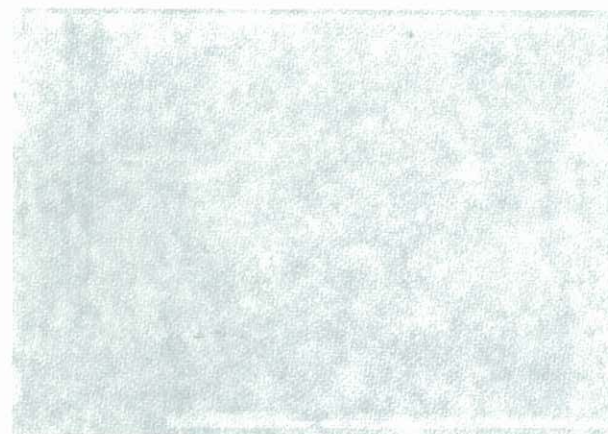
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REFERENCES

- Aref, H (1983): Integrable, chaotic, and turbulent vortex motion in two-dimensional flows. *Ann. Rev. Fluid Mech.* **15**, 345-389.
- Christiansen, J P; Zabusky, N J (1973): Instability, coalescence and fission of finite-area vortex structures. *J. Fluid Mech.* **61**, 219-243.
- Gill, A E; Griffiths, R W (1981): Why should two anticyclonic eddies merge? *Ocean Modelling* **41**, unpublished manuscript.
- Griffiths, R W; Hopfinger, E J (1986): Coalescing of geostrophic vortices. *J. Fluid Mech.*, submitted.
- Hogg, N G; Stommel, H M (1985): The heton, an elementary interaction between discrete baroclinic geostrophic vortices and its implications concerning eddy heat-flow. *Proc. Roy. Soc. London A* **397**, 1-20.
- Overman, E A II; Zabusky, N J (1982): Evolution and merger of isolated vortex structures. *Phys. Fluids* **25**, 1297-1305.
- Thorpe, S A (1973) Experiments on instability and turbulence in a stratified shear flow. *J. Fluid Mech.* **61**, 731-751.
- Winant, C D; Browand, F K (1974): Vortex pairing: a mechanism of turbulent mixing-layer growth at moderate Reynolds number. *J. Fluid Mech.* **63**, 237-255.

Fig. 1: The density of saturated solutions of KNO₃ as a function of composition and temperature.

of the viscosity of the two fluid layers are comparable, the lower layer gradually evolves towards a state where the density approaches that of the upper layer, but the interface remains sharp during this time, and there is little transfer of momentum between the layers. When the densities become equal, the interface becomes diffuse, and the two layers mix. This is shown in Fig. 1. The time taken for the interface to become diffuse is a function of the initial density difference and the viscosity of the fluids.



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