

SEDIMENTATION IN THE PRESENCE OF UPWARD CURRENTS

Sedimentation in the Presence of Upward Currents

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INTRODUCTION

The work presented here is concerned with separating an initially homogeneous suspension of solid particles in a liquid into two separate components. This can be accomplished by means of gravity settling either as batch sedimentation or continuous thickening. We restrict ourselves to the case of batch sedimentation only, although under most conditions what can be said about the batch sedimentation process can also be applied to the continuous thickening process.

Very successful analyses of the sedimentation processes are performed neglecting both the viscous and inertia forces in the bulk, e.g. Wallis (1969) and Schneider (1982). The assumption that the sedimentation process is gravity driven is well justified in the absence of wall effects, which is usually the case in large commercial sedimentation pools. On the other hand, it is known that in certain geometrical configurations wall effects are present and can, indeed, be very beneficial: Boycott (1920), Davis and Acrivos (1985). The Boycott effect of tilted vessels is utilized in the 'lamella thickener', where slanted walls are suspended into the sedimentation volume in order to simulate tilted containers.

A popular explanation of the achieved benefit of slanted sedimentation containers is given in terms of the increased vertically projected settling area, e.g. Kelly and Spottiswood (1982). The aim of the present work is to study the possibility of simulating the beneficial wall effects without actually tilting any walls or without using walls in the flow domain at all.

THEORETICAL BACKGROUND

The theoretical approach adopted here is a natural extension of the ideals developed by Wallis (1969) for sedimentation in large containers of constant cross section and by Schneider (1982) for sedimentation in containers with slanted walls of either constant or variable cross section.

Our analysis assumes incompressible solids (subscript s in the formulae) and liquids (subscript l in the formulae), where the volume fraction (concentration) of uniform solid spheres is acted upon by gravity. The particles are characterised by their Stokes' settling velocity in an infinite sea of liquid:

$$U = \frac{2}{9} \frac{(\rho_s - \rho_l) g r^2}{\nu_l \rho_l} \quad (1)$$

where ρ = density, ν = kinematic viscosity, r = radius of the particles and g = gravitational acceleration.

In the hindered settling process the normalized volume flux densities are introduced as:

$$j_s = \alpha v_s \quad (2)$$

for solids,

$$j_l = (1 - \alpha) v_l \quad (3)$$

for liquids and

$$j = j_s + j_l \quad (4)$$

for the mixture, where v = velocity of the single phase, normalised with U .

Non-dimensional parameters which characterize the sedimentation process are the Reynolds number

$$Re = \frac{UH}{\nu_l} \quad (5)$$

and the Grashof number

$$Gr = \frac{H^3 g \alpha_0 (\rho_s - \rho_l)}{\rho_l \nu_l^2} \quad (6)$$

where H = height of the sedimentation bath and α_0 = initial concentration of the solids.

The equations governing the sedimentation process are the conservation equations for mass and momentum. The mass continuity equations for the solid phase and for the mixture are as follows:

$$\partial \alpha / \partial t + \text{div } j_s = 0 \quad \text{and} \quad (7)$$

$$\text{div } j = 0. \quad (8)$$

The continuity equations are identical in their dimensional and non-dimensional form.

Under conditions where buoyancy effects dominate over both inertia and viscous effects, i.e. where the following conditions

$$Gr/Re^2 \gg 1 \quad \text{and} \quad Gr/Re \gg 1 \quad (9)$$

are simultaneously fulfilled, the momentum equation is reduced to the hydrostatic pressure equation only:

$$\text{grad } p = [\alpha \rho_s + (1 - \alpha) \rho_l] g. \quad (10)$$

The equations above are sufficient to characterize the bulk flow. Of additional importance is the flow of solids relative to the liquid phase. This is characterized by the drift flux j_{sl}

$$j_{sl} = j_s - \alpha j \quad (11)$$

and its relationship with the solids concentration

$$j_{sl} = f(\alpha) \cdot \tilde{g}/g. \quad (12)$$

For $f(\alpha)$ an empirical relationship

$$f(\alpha) = \alpha(1-\alpha)^{4.65} \quad (13)$$

is used (Richardson and Zaki (1954)). The last relationship is based on empirical arguments and confirmed by experimental data. The essential

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characteristics of this relationship are that there is no drift flux when the solids concentration is either 0 or 1 as shown in Figure 1.

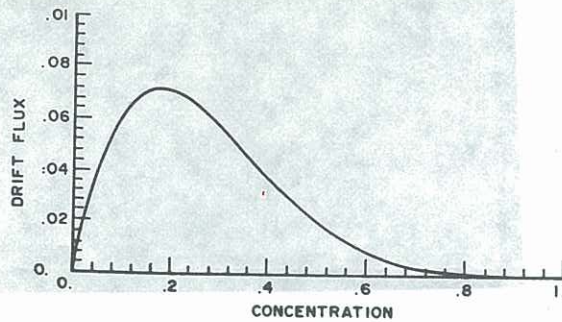


Figure 1: The drift flux relationship.

With the drift-flux relationships the continuity equation for the solid particulate phase becomes

$$\partial \alpha / \partial t + [j_z + f(\alpha)] \partial \alpha / \partial z = 0. \quad (14)$$

The last equation is an equation for a kinematic wave, where the velocity of the concentration wave V is given by the expression in the angular brackets.

The kinematic wave equation (14) has continuous and discontinuous (shock) solutions. The conditions for the existence of shocks have been worked out by Kluwick (1977) and by Schneider (1982). The discontinuities separate the clear liquid from the homogenous suspension and the suspension from the sediment.

The shock velocities are given by

$$W = j_z + f(\alpha_0) / \alpha_0 \quad (15)$$

for the clear/suspension interface and by

$$W = f(\alpha_0) / (\alpha_s - \alpha_0) \quad (16)$$

for the bottom interface of sediment of concentration α_s with the suspension, when the shock solutions are possible. The possibility of the existence of the shock solution depends upon the initial concentration in a manner explained with the help of Figure 1. When the initial concentration is such that a straight line can be drawn from the point $(\alpha_0, f(\alpha_0))$ to the origin and to the final sediment point without crossing the drift flux curve, both shocks are possible. In other cases continuous wave solutions have to appear too.

In order to determine the vertical component of the total volume flux j_z , and the velocity of the propagation of the clear/suspension interface the wave equation (14) has to be solved subject to boundary conditions.

BOUNDARY CONDITIONS

The solution to the kinematic wave equation (14) is straightforward in the case of a container with vertical walls. Schneider (1982) showed, how variations in the horizontal cross-section with height introduced additional flow components into the sedimentation process. Similar flow components are also observed in the case of constant cross-section containers with slanted walls (Acrivos and Herbolzheimer (1979), Schneider (1982)).

In the following analysis the a priori presence of a boundary layer of upward streaming clear liquid is assumed. This can be realized by a jet emerging from the bottom of the container, by heating the container walls or by releasing a buoyant plume from a source at the bottom of the container.

The sedimentation occurs in a volume between the two boundary layers with a characteristic thickness δ , a characteristic velocity u and a spacing of the order of the bath height H , with the ratio $\delta/H \ll 1$. In this case the bulk of the suspension is influenced by the boundary layer through continuity requirements only. The conditions are described in Figure 2.

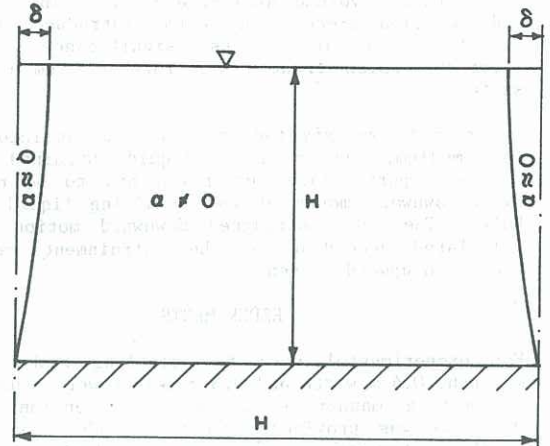


Figure 2: Settling volume enclosed by boundary layers.

Since the entrainment of solid particles into the upward moving liquid should be negligible, the viscous effects in the jet should be negligible against the buoyancy effects. This is expressed by the condition

$$Gr/Re_{\delta u} \gg 1, \quad (17)$$

where the Grashof and Reynolds numbers are formulated with the boundary layer properties.

At the same time we expect the vertical stream to have sufficient inertia to reach the free surface. This requirement is fulfilled when

$$Gr/Re_{u,H} = O(1) \quad (18)$$

The last conditions are coupled with the bulk flow requirement (9) by a continuity requirement

$$u\delta = kUH \quad (19)$$

where k is positive.

This requirement expresses the consideration that the boundary layers are not in an infinite sea of liquid. The amount of liquid dragged upward in the boundary layer flow has to return in the bulk flow region. The returning flow is expressed in terms of the volumetric flow dragged down by the settling particles.

The conditions (17) and (18) together with the conservation condition (19) mean that the upward velocity in the laminar upward flow has to stay in bounds given by the following expression

$$(Gr/Re^2)^{1/2} < u/U \ll (k^2 Gr/Re)^{1/3} \quad (20)$$

which also means that

$$Gr/Re^4 \ll k^4. \quad (21)$$

FLOW IN THE BULK

Given that the boundary conditions specified in the previous chapter are satisfied, the upward streaming boundary layer type of flow regions can be approximated as line sinks of liquid matter. The solids are rejected from these liquid boundaries in a manner analogous to the spouted bed process (cf. Soo(1967)) or due to wall effects (cf. Ho and Leal(1973)). In the present approximate analysis the assumption is made that the liquid entrained by the upward currents is released from it upon reaching the free surface and is immediately spread over the whole horizontal cross-section of the sedimentation volume available to it. In the batch sedimentation process the error introduced by this simplification loses its significance as the clear/suspension front moves further from the free surface.

In order to satisfy the continuity of an incompressible medium, any amount of liquid entrained out of the bulk particulate suspension has to be replaced by a downward motion of the remaining liquid in the bulk. The thus introduced downward motion can be calculated according to the entrainment rates of the given upward currents.

EXPERIMENTS

For experimental work two settling tanks of 0.1 m depth, 0.4 m width and 0.3 m height were constructed in such a manner as to give 2-dimensional flow. One tank was provided with two 'bubble generators' at the bottom and the other one with heated end walls drawn together as shown in Figure 3.

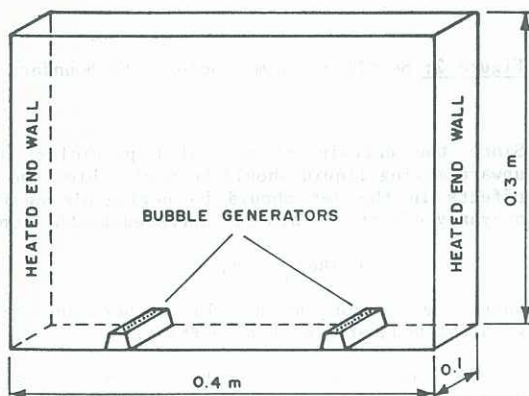


Figure 3: Settling tank.

Air bubble streams were used to create upward currents in the flow field far from walls. Air bubbles were used since they maintain their buoyancy along their path. Care had to be exercised in adjusting the bubble generators, since the enhancement of the sedimentation seemed to be beneficially influenced by the bubble velocity only until the bubble wakes became turbulent. The boundary layer velocity at the heated walls was limited by the experimental apparatus available. The wall temperature was maintained at approximately 5K above the liquid temperature.

The flow in the liquid phase without the heavy solid phase being present was visualised by light sheet technique. In order to achieve uniform illumination of the flow field, a thin laser beam was directed onto a rotating mirror, which then swept the beam through the whole volume. This laser beam illuminated small neutrally buoyant particles (crushed Vermiculite) in its path. By exposing the film to the scattered light for long exposure times (typically 30 seconds or more) streaklines, which in steady flow are identical in shape to streamlines, were recorded. The results are shown in Figures 4 and 5 for the bubble plume and heated wall baths respectively.

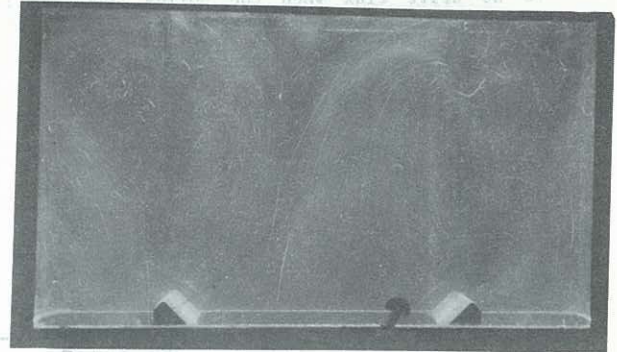


Figure 4: Streaklines of the liquid flow.

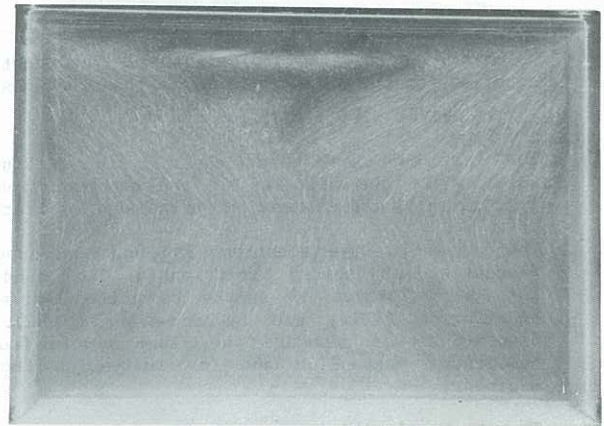


Figure 5: Streaklines of the liquid flow.

It is obvious, that particularly in the bubble plume case the flow patterns deviate from the idealized theoretical case considerably. One feature is still obvious: in the bulk regions the dominant flow component is directed downwards.

The next step was comparing the sedimentation rates in the presence of these upward currents with the rates when no upward currents were introduced. These experiments were carried out using silt particle suspensions in tap water. The particles were sized using a conventional stack of shaking sieves. The particle size distribution available for the experiments was such that the particle diameter d in the batch was $36 \mu\text{m} < d < 75 \mu\text{m}$. Unfortunately it was not possible to obtain a monodisperse particle size distribution for these experiments. The main effect of the presence of a wide distribution particle sizes is the spreading of shock fronts, which were observed in the experiments. An analysis of the influence of the presence of particles of different size has been performed by Schneider et al (1985).

Assuming the average particle diameter $d = 60 \mu\text{m}$, particle density $\rho_s \approx 3000 \text{ kg/m}^3$, water density $\rho_l \approx 1000 \text{ kg/m}^3$ and the kinematic viscosity of water $\nu_0 \approx 10^{-6} \text{ m}^2/\text{s}$, we can calculate the Stokes settling velocity $U \approx 5 \text{ mm/s}$. The volume fraction of solids in the experiments was $\alpha_0 \approx 0.01$. With this data the Reynolds number $Re \approx 1.25 \cdot 10^3$ and the Grashof number $Gr \approx 0.3 \cdot 10^{12}$. We can see that the conditions (9) and (21) are all fulfilled provided the constant $k > 0.2$.

A characterization of the speed of the settling process can be obtained by measuring the time from the start of the process until the clear/suspension

front passes a predetermined height in the sedimentation tank. For this purpose a light sensitive diode was connected to a power supply in such a way as to give output voltage inversely proportional to its illumination. This light sensitive diode was illuminated by a collimated laser beam horizontally through the settling tank and connected to a x-t recorder. Thus the recorded voltage was proportional to the solids concentration in the path of the light. The solids suspension was manually stirred until an equal distribution of the solids in the whole tank was achieved. As the stirring was stopped the recording of the inverse light intensities was started. The light intensities were recorded at a distance of half the bath height in the bubble plume channel and at 0.2 bath height in the heated wall bath. In the bubble plume bath the measurements were performed for different plenum pressures of the air supply and compared with the settling process without upward currents. A characteristic record of settling with and without buoyant bubbly plumes is shown in Figure 6.

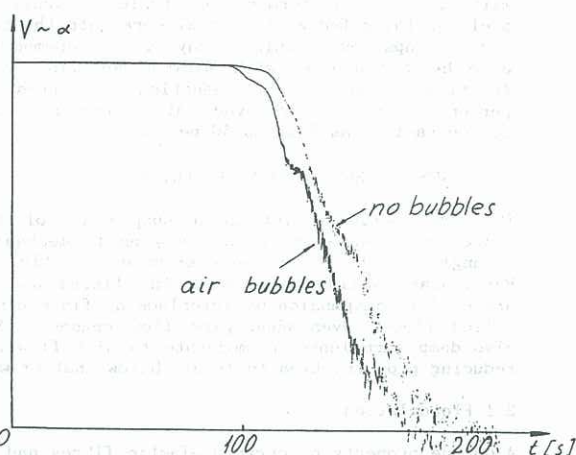


Figure 6: Output of light sensitive diode vs time.

A similar recording for sedimentation in the presence of heated walls is shown in Figure 7.

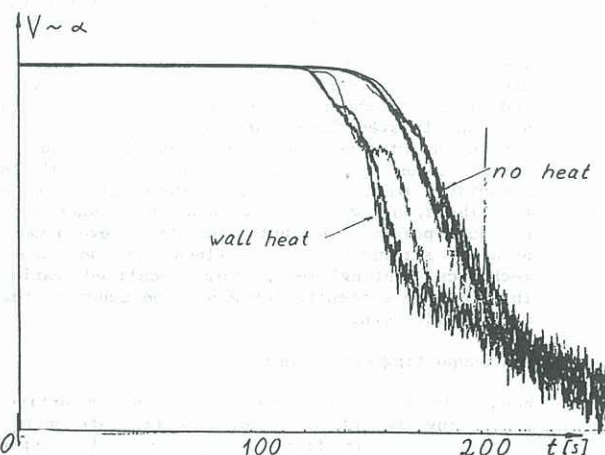


Figure 7: Output of light sensitive diode vs time.

In these recordings the initial plateau corresponds to the initial concentration. As the clear/suspension interface passes the light beam, the recorded voltage drops corresponding to the drop in the solids concentration at that height until it reaches its final level of clarified liquid value. This in a finite time due to the wide distribution of particle sizes in the suspension. The front passing time is reduced as the particle size distribution becomes

narrower. By measuring the times of mid-levels of the fronts passing the light beam the approximate gains in sedimentation rates can be assessed. In our experiments the gain was typically 15-20 per cent in the heated walls case and around 10 per cent in the bath with bubbly plumes.

A number of difficulties can be identified in the present experimental arrangement. The major one is due to the mechanical stirring of the suspension to achieve a homogeneous distribution of solids. The stirring introduces strong currents in the sedimentation tank which at the start of the process dominate over the currents introduced by the controlled buoyancy of bubbles or heated wall boundary layers.

CONCLUSIONS

The process of batch gravity settling in the presence of upward streaming currents was analysed. It was shown that the theoretical approach of Wallis (1969) and Schneider (1982) can be extended to cases where there are no downward facing walls present. The experiments in sedimentation were carried out using silt particles in water. Measurements of the travel time of the clear/suspension front show that there can be a definite increase in the settling rate when upward streaming currents are present. This contradicts the usual explanation of the Boycott effect, which only accounts for the increase in the vertically projected cross section of the settling area and requires a tilting of the walls. From the present work it seems that the enhancement of the settling rate is a flow effect, rather than a wall effect. In this sense the current experimental results are comparable with the results of Fessas and Weiland (1984), although the approaches have been somewhat different.

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