9th AUSTRALASIAN
FLUID MECHANICS CONFERENCE
AUCKLAND, 8-12 DECEMBER 1986

Laboratory Experiments on Convection in a Rotating Rectangular Cavity

S. A. CONDIE and G. N. IVEY

Research School of Earth Sciences, Australian National University, Canberra, Australia.

ABSTRACT

Many geophysical fluid flows are characterized by an imposed longitudinal density gradient and low aspect ratio, and are influenced by the earth's rotation. Flows with these characteristics have been produced in a rectangular cavity with differentially heated end-walls rotating about a vertical axis through its centre. This paper presents some preliminary results with emphasis on the initial intrusion.

INTRODUCTION

A convectively driven flow, moving with velocity $\underset{\sim}{u}$ in a system rotating with angular velocity $\underset{\sim}{\Omega} = \frac{1}{2}\underset{\sim}{f}$, will experience a Coriolis force $\underset{\sim}{f} \times \underset{\sim}{u}$. In the absence of boundaries, the flow will approach a state of equilibrium with a balance between Coriolis and buoyancy forces, and the buoyantly-driven spreading will be inhibited. In the presence of an impermeable wall, however, the component of the Coriolis force parallel to the boundary must vanish, and a gravity current flows along the wall.

Geophysical examples of these rotating gravity currents are numerous. Fresh water entering the sea from rivers is deflected to the left (in the Southern hemisphere) by Coriolis forces and moves along the coast as a boundary current. On a larger scale, ocean currents flowing along continental coastlines have a significant effect on the global circulation. The Leeuwin current consists of a low salinity flow moving southward from tropical regions, along the west coast of Australia, round Cape Leeuwin and finally east towards the Great Australian Bight. The flows in large scale confined water bodies such as the South Australian Gulfs are also affected by rotation. High evaporation rates at the shallow head of Spencer Gulf produce a strong salinity gradient. The resulting gravity currents form a net landward residual flow (Nunes et al. 1985) and there is concern that this will result in an increased concentration of pollutents at the head of the Gulf.

These geophysical flows are characterized by three common features. In each case the flow is driven by a longitudinal density gradient. Secondly, the vertical scale is much less than the horizontal scale, resulting in a small aspect ratio. Finally, the length scales are large enough for the dynamics to be affected by the earth's rotation. The aim of this study is to use a laboratory model to examine the nature of the circulation and mixing mechanisms in these flows. This paper presents some preliminary results of the laboratory work.

EXPERIMENTS

The experiments were conducted in a rectangular perspex cavity of height H = 15cm, length L = 200cm and width B = 60cm (Figure 1). The two end-wall heat-exchangers (x = 0,200cm) were differentially heated and cooled to produce a longitudinal density gradient. The base of the tank was insulated with expanded polystyrene. Perspex air cavities on the sides and top provided insulation and visual access. Heat losses constituted approximately 6% of the heat transfer within the tank. This assembly was mounted on a one meter diameter rotating table and hot and cold fluid pumped on and off the table via fluid slip rings.

The tank was filled with water and spun up to solid body rotation. The experiment was initiated by commencing the pumping of the hot and cold fluid through the heat exchangers. As the flow is symmetric it is sufficient to describe the kinematics of the flow originating from the heated boundary. The current was traced by dye injected at this surface. Adjacent to the hot end-wall a thermal boundary layer rapidly forms, in which buoyant fluid (as yet unaffected by rotation) rises vertically. This flow is turned into the interior by the upper horizontal lid. Coriolis forces push this current to the right (anticlockwise rotation), resulting in a boundary current propagating along the right-hand wall (see Figure 1). The nose of the current takes the form of a wedge, rather than the steeper bore structure observed in dam-break experiments (Stern et al. 1982, Griffiths & Hopfinger 1983). When f $\geq$ 0.5 rotational instabilities develop well behind the nose, approximately ten rotation periods after the boundary current is first formed. These eventually mix the hot fluid over the width of the cavity. When steady state is achieved there is very little horizontal temperature structure remaining; however, a substantial vertical gradient is still present. The steady circulation is quite complex and strongly influenced by the rotationally induced eddy motions.

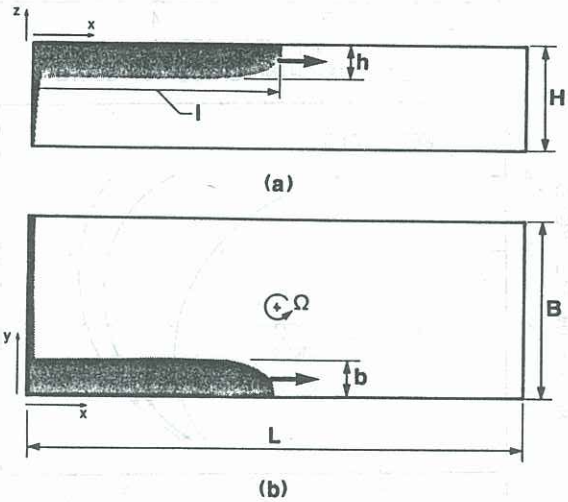

Fig 1: Schematic of a gravity current emerging from the hot wall ((a) side view, (b) top view) with coordinate definitions.

ANALYSIS OF THE INITIAL INTRUSION

The dynamics of the convective flow in a stationary cavity can be described in terms of three dimensionless external parameters (Batchelor 1954). The first is the aspect ratio which defines the geometry of the cavity

$$A = \frac{H}{L} \quad (1)$$

The Prandtl number is defined

$$Pr = \frac{\nu}{\kappa} \quad (2)$$

where ν and κ are the kinematic viscosity and thermal diffusivity of the fluid respectively. Finally the Rayleigh number incl·· es the imposed temperatue difference ΔT between the end-walls and the ambient fluid.

$$Ra = \frac{g\alpha\Delta TH^3}{\nu\kappa} \tag{3}$$

g is the acceleration due to gravity and α is the thermal expansivity of the fluid. In order to include rotational effects an additional parameter is required. This is the Ekman number, defined by the relation

$$E = \frac{\nu}{H^2 f} \tag{4}$$

Some simple scaling arguments can be used to make estimates of the depth, width and velocity of the boundary current in terms of these dimensionless numbers. The flow is assumed to be parallel to the side boundary (i.e. $v \sim 0$, $w \sim 0$). Initially inertia will dominate over frictional forces and the components of the Navier-Stokes equation of motion in the interior take the form of the geostrophic and hydrostatic approximations. That is

$$u\frac{\partial u}{\partial x} = -\frac{1}{\rho_0}\frac{\partial p}{\partial x} \tag{5}$$

$$fu = -\frac{1}{\rho_0}\frac{\partial p}{\partial y} \tag{6}$$

$$0 = -\frac{1}{\rho_0}\frac{\partial p}{\partial z} + g' \tag{7}$$

Here $g' = g\alpha(T-T_0)$ is the reduced gravity and ρ_0, T_0 are the density and temperature of the isothermal fluid before start-up. Using scaled estimates of the current dimensions, $x \sim \ell$, $y \sim b$, $z \sim h$ (Figure 1), and combining equations (1) and (3), we find that the downstream velocity is equal to the internal wave velocity

$$u \sim (g'h)^{\frac{1}{2}} \tag{8}$$

From (6) and (7)

$$fu \sim g'\frac{h}{b} \tag{9}$$

and eliminating u,

$$b \sim \frac{(g'h)^{\frac{1}{2}}}{f} = Ro \tag{10}$$

This states that the width of the current scales with the internal Rossby radius Ro. Since the thickness of the thermal boundary layers $\delta_T \sim H/Ra^{\frac{1}{4}}$ (Patterson & Imberger 1980) is small compared to the Rossby radius, we assume that the volume flux Q of heated (or cooled) fluid out of the boundary layer is not significantly affected by rotation. Therefore

$$Q \sim \kappa B Ra^{\frac{1}{4}} \tag{11}$$

(Patterson & Imberger 1980). The work of Bejan et al. (1981) and Ivey & Hamblin (1982) indicate that (11) can be written as the approximate equality

$$Q = (0.7\pm0.1)\kappa B Ra^{\frac{1}{4}} \tag{12}$$

By continuity the volume flux is related to the boundary current parameters by

$$Q \sim bhu \tag{13}$$

Combining (13) with (9), (10) and (8) yields expressions for the depth, width and velocity respectively.

$$h \sim \left(\frac{Qf}{g'}\right)^{\frac{1}{2}} \tag{14}$$

$$b \sim \left(\frac{Qg'}{f^3}\right)^{\frac{1}{4}} \tag{15}$$

$$u \sim (Qg'f)^{\frac{1}{4}} \tag{16}$$

It is useful to express these scales in terms of external parameters. Using (11) and scaling the reduced gravity with the applied temperature difference

$$g' \sim g\alpha\Delta T \tag{17}$$

yields

$$h \sim (BH)^{\frac{1}{2}} Ra^{-3/8} E^{-\frac{1}{2}} \tag{18}$$

$$b \sim (BH^3)^{\frac{1}{4}} Pr^{-\frac{1}{2}} Ra^{5/16} E^{\frac{3}{4}} \tag{19}$$

$$u \sim \left(\frac{B}{H^5}\right)^{\frac{1}{4}} (\nu\kappa)^{\frac{1}{2}} Ra^{5/16} E^{-\frac{1}{4}} \tag{20}$$

Alternative (20) can be expressed as a spreading law

$$\ell(t) \sim \beta t \tag{21}$$

where β is defined by

$$\beta = \left(\frac{B}{H^5}\right)^{\frac{1}{4}} (\nu\kappa)^{\frac{1}{2}} Ra^{5/16} E^{-\frac{1}{4}} \tag{22}$$

RESULTS

The distance the intrusion travelled along the tank ℓ was measured as a function of time for a range of rotation periods. This length was plotted as a function of time t after the commencement of the experiment for each rotation rate and a virtual time origin t_0 defined such that $t' = t - t_0 = 0$ when $\ell = 0$. ℓ/β is graphed as a function of t' in Figure 2. The small relative displacement of the data for each run suggests the dependence on rotation rate may be slightly weaker than the $f^{\frac{1}{4}}$ predicted by the scaling arguments. This effect is thought to be related to the slight reduction in heat transfer and hence Qg' as f increases (see Figure 4 below). However, as predicted by (21), all runs clearly exhibit an initial slope of one and the data are consistent with a best fit

$$u = (0.28\pm0.03)\left(\frac{B}{H^5}\right)^{\frac{1}{4}} (\nu\kappa)^{\frac{1}{2}} Ra^{5/16} E^{-\frac{1}{4}} \tag{23}$$

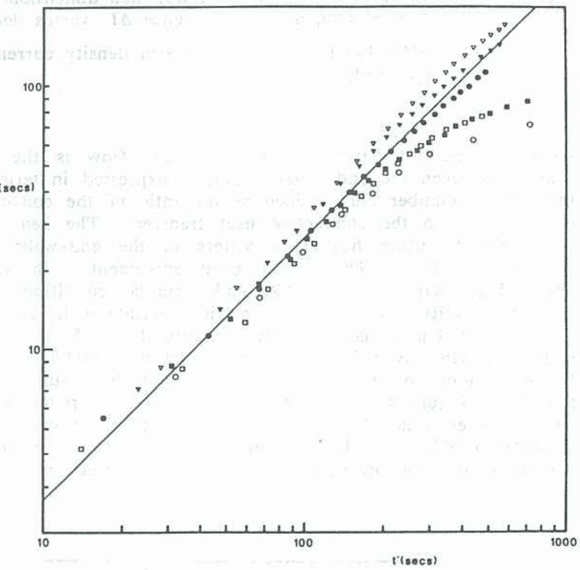

Fig 2: Current length divided by β (equation (22)) versus time for several runs. $Ra = 4 \times 10^8$ for all runs and the rotation rates are:

Run	▽	▼	●	□	■	○
f	0.10	0.20	0.5	1.5	1.5	2.0

In applying this to geophysical flows, the equality corresponding to expression (16) may often be more relevant. From temperature profiles taken during this series of experiments (Figure 3), we can write

$$g' = 0.2g\alpha\Delta T \tag{24}$$

Using (24) and (12), (23) may equally be written as

$$u = (0.46\pm0.05)(Qg'f)^{\frac{1}{4}} \tag{25}$$

Figure 2 also reveals a decay of the velocity near the far end-wall. This affect is more acute and occurs earlier at high rotation rates. Dye streaks released ahead of the intrusion revealed no communication with the approaching end-wall. It seems likely that the phenomena represents a transition to flow influenced by viscous and drag forces and is the subject of future study.

Preliminary studies of the steady-state flow have also been conducted. Those flows which were initially unstable ($f \gtrsim 0.5$) are later dominated by eddies, with the highest rotation rates characterized by geostrophic turbulence filling the cavity. For all rotation rates temperature profiles were linear with depth (Figure 3). As expected there is little lateral temperature variation at low rotation rates, however as f increases the isotherms tilt slightly away from the horizontal. In the unstable regime the lateral variation again disappears due to the lateral mixing by the eddy motions.

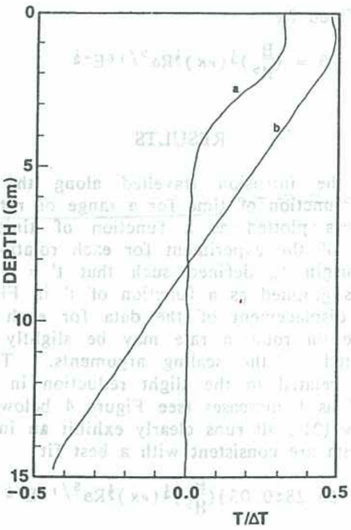

Fig 3: Typical profiles of temperature, non-dimensionalized by imposed temperature difference ΔT, versus depth.
a. After the passage of the warm density current.
b. At steady state.

Another important parameter of the steady flow is the heat transfer between the end-plates. This is expressed in terms of the Nusselt number Nu, defined as the ratio of the convective heat transfer to the conductive heat transfer. The heat flow was measured using heat flux meters in the end-walls, and typically $Nu \approx 400$. This is in good agreement with values obtained by Wirtz et al. (1982) under similar conditions in a stationary cavity. To account for small variations in end-wall temperatures Nu is conveniently normalized by $Ra^{\frac{1}{4}}/2A$ (the scaled estimate given by Patterson & Imberger (1980) for the Nusselt number of a stationary cavity), and the result plotted against f (Figure 4). The plot reveals a small reduction of heat transfer (and hence Qg') for increased rotation rate. Centrifugal effects at high rotation rates produce a parabolic curvature to the isotherms and, as dye injection studies

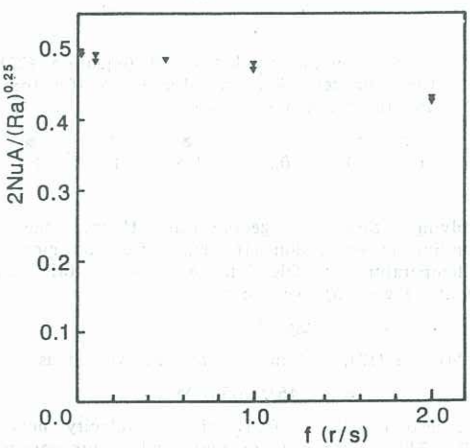

Fig 4: Longitudinal heat transfer as a function of rotation rate.

revealed, this produces a relatively stagnant region of fluid at the base of each end-wall which, in turn, reduces the heat transfer efficiency as f increases.

CONCLUSIONS

The paper presents some preliminary results from an experiment on convection in a rotating rectangular cavity. The sudden initiation of the convection produces gravity currents which flow along the sidewalls under the influence of rotation. Observations of these currents are in close agreement with theoretical predictions based on an assumed geostrophic balance. It is clear that the currents are ultimately influenced by drag effects, and the study of this motion and the subsequent evolution of the entire cavity to a steady state is the subject of ongoing research.

ACKNOWLEDGEMENTS

The authors would like to thank Pat Travers and Derek Corrigan for their efforts in the construction of the experimental facility.

REFERENCES

Batchelor, G K (1954): Heat transfer by free convection across a closed cavity between vertical boundaries of different temperatures. Quart. J. Appl. Math. 12, 209.

Bejan, A; Al-Homoud, A A; Imberger, J (1981): Experimental study of high-Rayleigh-number convection in a horizontal cavity with different end temperatures. J. Fluid Mech. 109, 283-299.

Griffiths, R W; Hopfinger, E J (1983): Gravity currents moving a lateral boundary in a rotating fluid. J. Fluid Mech. 134, 357-399.

Ivey, G N; Hamblin, P F (1982): Convection near the temperature of maximum density due to horizontal temperature differences. Part 2: Experimental results. Environmental Dynamics Report No. ED-82-034, Dept. of Civil Engineering, University of Western Australia.

Nunes, R A; Lennon, G W (1985): Episodic density flows and stratification in Spencer Gulf, South Australia. IUTAM Symposium on Mixing in Stratified Fluids. Margaret River, Western Australia.

Patterson, J; Imberger, J (1980): Unsteady natural convection in a rectangular cavity. J. Fluid Mech. 100, 65-86.

Stern, M E; Whitehead, J A; Hua, B-L (1982): The intrusion of a density current along the coast of a rotating fluid. J. Fluid Mech. 123, 237-265.

Wirtz, R A; Righi, J; Zirilli, F (1982): Measurements of natural convection across tilted rectangular enclosures of aspect ratio 0.1 and 0.2. J. Heat Transfer 104, 521-526.