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The Phase and Time Dependence of the Wake of a Wind Turbine

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ABSTRACT

Phase-locked averaged measurements were obtained behind a model wind turbine to determinate the velocity field relative to the rotating blades. The accuracy of the phase-locked results was found to be excellent. An equation for the turbine power as a function of the average, phase-dependent, and turbulent components is derived. The phase-dependent and turbulent components are due mainly to viscous effects in the blade wake. Flow non-uniformities from slight differences between the blades were of greater significance than these viscous effects.

NOMENCLATURE

- r radius
- t time
- T torque
- U,u axial velocity components
- U_0 U measured upstream of the blades
- V,v radial velocity components
- W,w circumferential velocity components
- $\dot{W}$ power
- X = (tip radius).Ω/U_0, tip speed ratio
- x co-ordinate along the probe axis
- θ circumferential co-ordinate
- θ_p blade pitch angle
- ρ density of air
- Ω rotational speed of the turbine

INTRODUCTION

Clausen, Piddington and Wood (1986) [hereinafter cited as CPW] measured the mean axial and circumferential velocities at a fixed point in the wake of a model wind turbine. The turbine had two untwisted 58 mm constant chord blades attached to an 80 mm diameter centrebody. The turbine was held in and shrouded by a 260 mm diameter pipe.

Blade element theory (BET), which is commonly used to describe the flow through the turbine, agreed with the measurements only over a restricted range of operating conditions. Discrepancies occurred whenever the local angle of attack at any radius exceeded the angle for maximum lift/drag in two dimensional flow. In these cases BET under-predicted the power output of the turbine. BET evaluates the power contribution from each streamtube by using an angular momentum balance across the relevant blade element. A radial integration gives the total turbine power. By independently measuring the turbine power CPW showed that an angular momentum balance in the wake slightly under-predicted the power. As the axial profile is fixed by continuity, then BET under-predicted the circumferential velocity.

To investigate these discrepancies further, phase-locked averaged (PLA) measurements were obtained for nominally the same operating conditions and probe location, about one blade chord length downstream of the blades, as in CPW. The results enabled the determination of the velocity field relative to the rotating blades. Conventionally averaged results were also obtained.

The paper discusses the accuracy of the measurements and the contribution of the phase and time dependent (turbulent) velocities to the power, $\dot{W}$. The equation for $\dot{W}$ as a function of the average, phase dependent, and turbulent components is derived. The flow is assumed to be symmetric every $2\pi/n$, where n is the number of blades. For our case n = 2, so symmetry should occur every 180°. The accuracy of the PLA results outside the blade wake is excellent. The results show the phase dependent contribution is small and the turbulent contribution an order of magnitude smaller. Comparison between PLA and the conventionally obtained results shows that the non-uniformities caused by slight differences between the blades are of greater significance than both the phase-dependent and turbulent contributions.

This work is a preliminary study of the flow development behind the turbine. The measurement position was chosen to ensure, hopefully, that the turbulence levels and deviations in the mean velocities were small enough for accurate measuring.

THEORY

In general, U_i, a velocity component at a fixed point in the wake can be written as

$$U_i = \tilde{\bar{U}}_i + \overline{u_i(\theta)} + u_i(\theta,t) \quad (1)$$

For convenience we define $U(\theta)$ as the sum of the first two terms on the right hand side of (1). The overbar denotes time averaging so that for any quantity, $q(\theta,t)$,

$$\overline{q(\theta)} = \lim_{t\to\infty} \int_0^t q(\theta,t)\ dt \quad (2)$$

and the tilde denotes phase averaging, so that

$$\tilde{q} = \frac{1}{2\pi}\int_0^{2\pi} q(\theta)\ d\theta \quad (3)$$

$q(\theta)$ is called the phase-locked average by Gostelow (1977); note that $\widetilde{\overline{q(\theta)}}$, the phase-average of $\overline{q(\theta)}$, is always zero. $\tilde{\bar{U}}_i$ is the conventional mean of U_i measured at a fixed point. The combination of the overbar and the tilde emphasises that this mean is both a time and phase average in a rotating flow. In principle, the order of averaging is unimportant as $\tilde{\bar{U}}_i = \bar{\tilde{U}}_i$. In practice, however, time averaging must be done first if measurements are made at a fixed point. This is the reason why q in (3) was not written as a function of time.

We assume that the integral in (2) is operationally equivalent to sampling at a constant, Ω, for N_t revolutions of the turbine so,

$$\overline{q(\theta)} = \frac{1}{N_t}\int_{i=1}^{N_t} q(\theta, t = i/\Omega) \quad . \quad (4)$$

Similarly, q was obtained by replacing the integral in (3) by the following sum for one blade

$$\tilde{q} = \frac{1}{\pi}\sum_{i=1}^{N_\theta} q(\theta = \pi i/N_\theta) \quad . \quad (5)$$

The integrals were approximated by the trapezoidal rule.

To obtain the equation for conservation of angular momentum, consider a small control volume of width $rd\theta$, height dr and radius r rotating at Ω. The control volume extends upstream and downstream of the turbine. The downstream axial and circumferential velocities are

$$U = U(\theta) + u(\theta,t) \quad \text{and} \quad W = W(\theta) + w(\theta,t) - r\Omega \tag{6}$$

V is assumed to be zero. Note that the velocities on the right hand side are relative to a fixed co-ordinate system.

Assuming no swirl upstream of the blades apart from that due to $-r\Omega$, the contribution to the torque, T, from the rotating streamtube at radius r is obtained by integrating over a circumferential distance of $2\pi r/n$. Thus

$$\frac{dT}{dr} = n \int_0^{2\pi r/n} \rho r\,[U(\theta) + u(\theta,t)]\,[W(\theta) + w(\theta,t)]\,rd\theta\,. \tag{7}$$

Time averaging gives

$$\frac{\overline{dT}}{dr} = n \int_0^{2\pi r/n} \rho r^2\,[U(\theta)W(\theta) + \overline{u(\theta,t)w(\theta,t)}]\,d\theta\,. \tag{8}$$

Phase averaging and using

$$\tilde{\tilde{U}} = \frac{1}{2\pi}\int_0^{2\pi} U(\theta)\,d\theta \quad \text{and} \quad U(\theta) = \tilde{\tilde{U}} + \overline{u(\theta)} \quad \text{etc}$$

gives

$$\frac{\overline{dT}}{dr} = n\,\pi\,r^2\,\rho\,[\tilde{\tilde{U}}\,\tilde{\tilde{W}} + \widetilde{\overline{u(\theta)w(\theta)}} + \widetilde{\overline{u(\theta,t)w(\theta,t)}}] \tag{9}$$

and

$$\frac{dP}{dr} = \Omega\,n\,\pi\,r^2\,\rho\,[\tilde{\tilde{U}}\,\tilde{\tilde{W}} + \widetilde{\overline{u(\theta)w(\theta)}} + \widetilde{\overline{u(\theta,t)w(\theta,t)}}]\,. \tag{10}$$

The first term in the brackets is the product of the conventional mean axial and circumferential velocities. It is the only term that would appear if the downstream flow was wholly irrotational, even if $u(\theta)$ and $w(\theta)$ were non-zero. Thus the other terms are due mainly to viscous effects in the blade wake acting on the mean velocity to give the second term, and producing turbulence to give the third. Just as Reynolds stresses in turbulent flow arise from the nonlinearity of the Navier Stokes equation, the second and third terms in eq. (10) are caused by the nonlinearity of the equation for conservation of angular momentum. The derivation ignores the viscous torques acting on the control volume. The effects of the viscous torque in the x,θ plane will, like that of non-zero V, be redistributive. That is, neither will contribute to the total power obtained by integrating eq. (10) from the hub to the tip. Furthermore the usual order of magnitude arguments for turbulent flow suggest that all the terms missing from eq. (10) wi 1 be small, apart from the transverse components of the wall shear stress acting on the hub and wall, which should appear in the equation for the total power. We did not measure the wall shear stresses and ignore them in what follows.

EXPERIMENTAL TECHNIQUE

The flow was investigated using a DISA 55P51 X-probe and the data acquired by the system described in Clausen (1986). The X-probe was calibrated using a King's law with an exponent of 0.45. The "cosine cooling" law, e.g. Bradshaw (1971), was used and the effective wire angles obtained from a yaw calibration of ±25° from the probe axis. No significant deviation from the average effective angle was found. The average flow angle at each radii, $\tan^{-1}(\tilde{\tilde{W}}/\tilde{\tilde{U}})$, was calculated using $\tilde{\tilde{U}}$ and $\tilde{\tilde{W}}$ conventionally acquired (behind both blades) at a sampling frequency of 1 kHz for 10 seconds.

PLA measurements were done with the X-probe in the x,θ plane to obtain U and W and in the x,r plane to obtain V. U was also obtained from the probe in the x,r plane. The X-probe was yawed to the average flow angle at each radii in both cases. This was done to minimise the transverse velocities, that is velocities at right angles to the plane of the wires, for the probe in the x,r plane and to keep the flow within the effective measuring cone of the wires for the probe in the x,θ plane. No correction was attempted for transverse velocity effects.

To obtain PLA measurements, a blade passing a fixed sensor in the pipe wall generated a pulse which triggered the data acquisition system. The system then acquired data at a selected rate for a total of approximately 200° of the flow. Sampling was done for 10000 revolutions (N_t = 10000). The maximum number of data points in a profile was 28, giving an angular spacing of about 7°; the exact spacing is a function of the turbine speed and data sampling rate. N_θ is approximately 25.

The turbine speed varied between 2500 and 5500 rpm and the corresponding variation in sampling frequency between 2.7 and 4.6 kHz.

RESULTS

Figures 1 and 2 are for X = 2.144 and U_0 = 17.16 m/s. Figure 1 is a profile taken close to the wall; Figure 2 is taken in the region unaffected by the wall and hub boundary layers. The wake is easily identified by the high turbulence levels.

When V and the deviations of $U(\theta)$ and $W(\theta)$ from the average flow direction are small, then ignoring the transverse components in the data analysis does not lead to any significant errors as shown by the close agreement between the two estimates of $U(\theta)$. Further, within the wake in Figure 2, where the transverse components are small, good agreement is found between the two estimates of $u^2(\theta)$. However, within the wake in Figure 1, where V is significant and the flow angle deviates by up to 20° from its mean direction, then the agreement between the two estimates of $U(\theta)$ and $u^2(\theta)$ is poor. To first order, transverse velocity effects lead to an over-estimation of the mean velocities as shown by $U(\theta)$ from the x,r plane in the wake of Figure 1. As V is small, outside of the blade wake, then the x,θ plane measurements are less affected by the transverse velocity. The terms in the square brackets of eq. (10) were obtained from the probe in the x,θ plane.

The steadiness of Ω during the measurements is evident from the near zero $u^2(\theta)$, outside of the blade wake, even when $\partial u(\theta)/\partial\theta$ is large, e.g. between 45° and 75° in Figure 1. If Ω was fluctuating, then to first order $U(\theta)$ would be unchanged but $u^2(\theta)$ would be increased by the "oscillation" in $U(\theta)$ whenever $\partial u(\theta)/\partial\theta$ is large.

The overall power contribution from the three components of eq. (10) are shown in Table 1. The contribution from the phase dependent mean velocity term is small; the turbulent term is at least an order of magnitude smaller. Generally, both terms reduce the total power output.

TABLE 1

X	U_0	θ_p	Output Power (Watts)				
			(a)	(b)	(c)	(d)	(e)
2.144	17.16	18.25	104	95.7	90.81	+0.89	-0.20
3.124	17.51	18.25	123	124.3	112.68	-0.78	-0.08
5.103	14.82	10.0	163	163.2	151.3	-3.09	-0.06
4.00	14.81	10.0	158	151.7	148.16	-2.60	-0.03
3.04	15.00	10.0	---	114.9	106.8	-0.006	-0.36
4.96	13.12	6.0	165	153.9	148.99	-3.00	+0.04
3.62	15.1	6.0	196	194.5	197.53	-1.30	-0.20

(a) measured using pump drive system, CPW
(b) evaluated using $\tilde{\tilde{U}}\,\tilde{\tilde{W}}$ from both blades
(c) evaluated using $\tilde{\tilde{U}}\,\tilde{\tilde{W}}$ from PLA measurements behind one blade
(d) contribution from $\widetilde{\overline{u(\theta)w(\theta)}}$
(e) contribution from $\widetilde{\overline{u(\theta,t)w(\theta,t)}}$

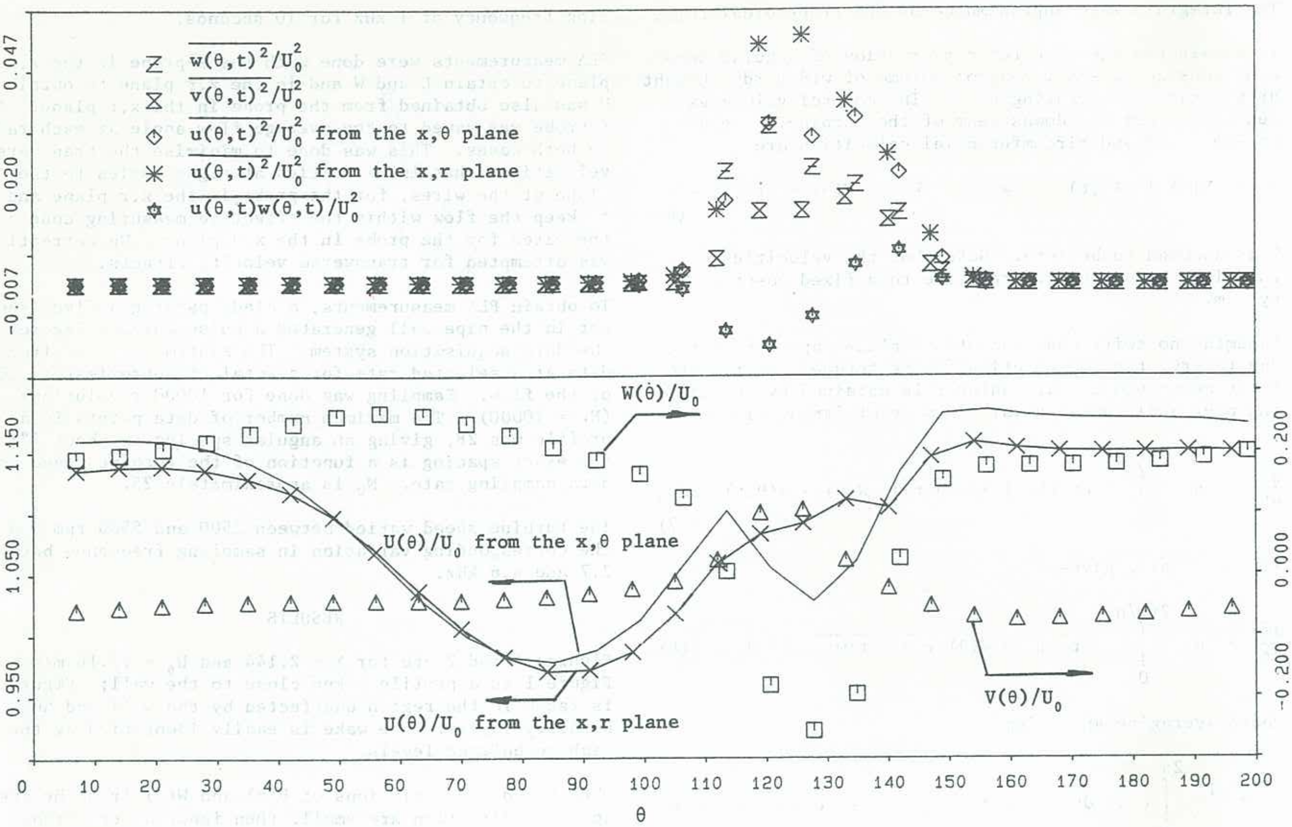

Figure 1 Phase-locked average measurements for r = 0.120 m, X = 2.144, U_0 = 17.16 m/s, θ_p = 18.25°

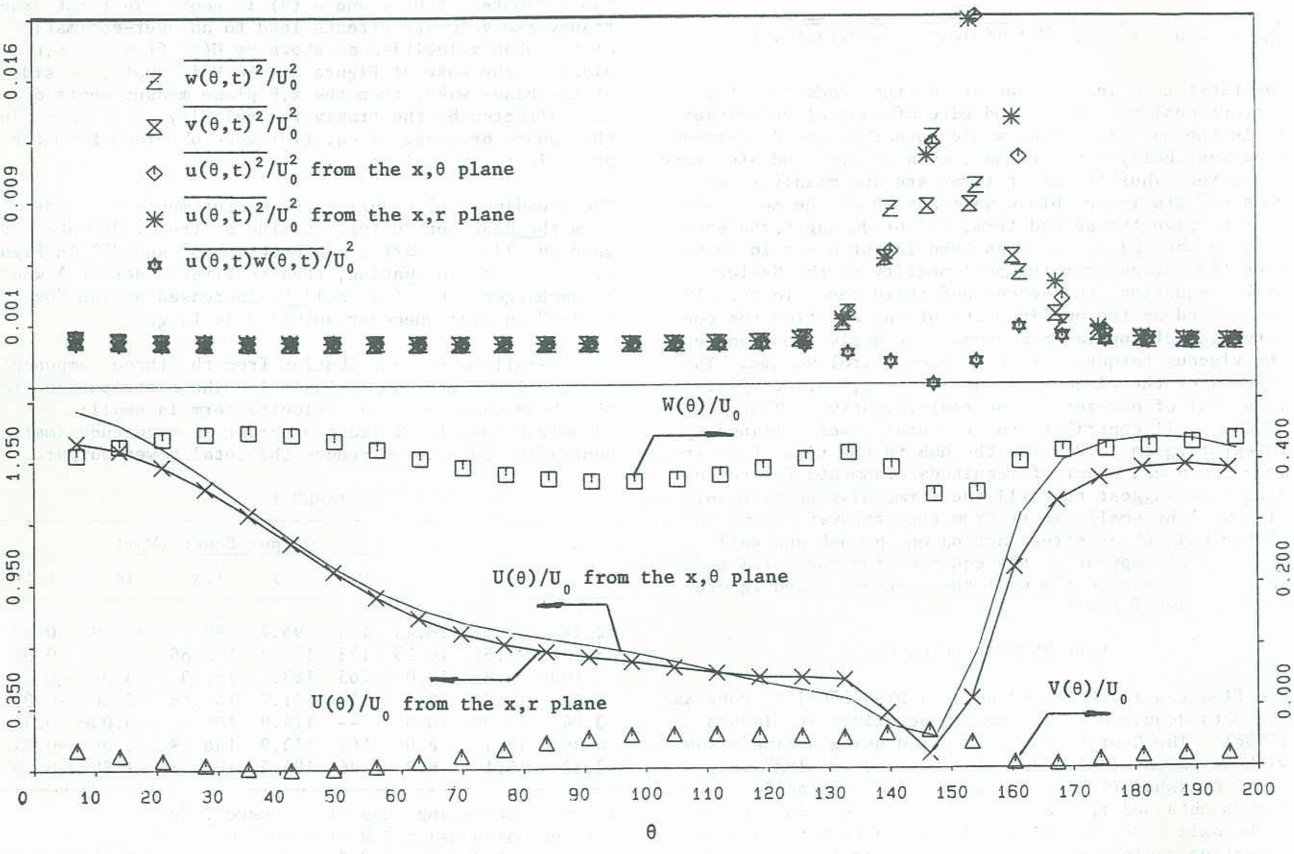

Figure 2 Phase-locked average measurements for r = 0.065 m, X = 2.144, U_0 = 17.16 m/s, θ_p = 18.25°

Table 2 shows a typical radial distribution of angular momentum flux for all three components of eq. (10) and $\bar{U}\,\bar{W}$ from conventional time and phase averaging behind both blades. The phase dependent terms have their greatest contribution near the hub and wall and generally reduced the power. Within the region unaffected by the hub and wall boundary layers, the phase dependent term is negligibly small. Contributions from $\overline{u(\theta,t)w(\theta,t)}$ are confined to the blade wake region. As shown in Figures 1 and 2 $\overline{u(\theta,t)w(\theta,t)}$ changes sign about half way through the wake region accounting for the small phase average contribution.

TABLE 2

r (m)	Contributions to eq. (10) [$m^2\ s^{-2}$]			
	(a)	(b)	(c)	(d)
0.045	114.55	110.66	-1.40	0.418
0.055	106.05	115.93	-1.008	0.125
0.065	93.89	104.70	0.141	0.108
0.075	91.84	94.77	-0.094	0.145
0.085	72.65	83.45	0.086	0.013
0.095	63.58	75.12	0.006	0.033
0.105	58.45	68.25	-0.060	-0.015
0.115	44.58	49.17	-2.446	-0.211
0.120	45.76	48.41	-4.34	-0.041

$X = 4.96$; $U_0 = 13.12$ m/s; $\theta_p = 6°$

(a) $\tilde{\tilde{U}}\,\tilde{\tilde{W}}$ from both blades
(b) $\bar{U}\,\bar{W}$ from PLA measurements behind one blade
(c) $\overline{u(\theta)w(\theta)}$
(d) $\overline{u(\theta,t)w(\theta,t)}$

DISCUSSION AND CONCLUSIONS

The contribution of phase-dependent and turbulent components lie within the uncertainty of determining the angular momentum flux at each radii. PLA results were obtained for one blade and conventionally averaged results from both blades. The difference between $\tilde{\tilde{U}}\,\tilde{\tilde{W}}$ from the two methods must be caused by slight geometric differences between the two blades, such as slight θ_p differences especially for small θ_p. Because the difference in $\tilde{\tilde{U}}\,\tilde{\tilde{W}}$ is greater than the contribution from either $\overline{u(\theta)w(\theta)}$ or $\overline{u(\theta,t)w(\theta,t)}$ then geometric variations are more significant than the viscous effects in the blade wakes. The neglecting of the phase-dependent and turbulence term in eq. (10) appears to be justifiable.

Transverse velocity effects can be ignored except within the blade wake. Here a first order correction to the mean quantities could be done iteratively using the results in one plane to correct those in another. By ignoring the transverse velocity in the data acquisition and analysis is simplified without seriously altering the x,θ plane results used in conjunction with eq. (10). However transverse velocity corrections may become unavoidable when measurements closer to the blade are required.

ACKNOWLEDGEMENTS

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Note added in proof : $\overline{u^2(\theta)}$ that appears in the second and third paragraphs of the Results should read $\overline{u(\theta,t)^2}$.