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Simulation of Swirling Diffuser Flow Using k-ε and Algebraic Reynolds Stress Turbulence Models

S. W. ARMFIELD and C. A. J. FLETCHER

University of Sydney, Australia.

ABSTRACT

Using a reduced set of Navier-Stokes equations, diffuser flow with large swirl and nearly reversing axial velocity is simulated using k-ε and algebraic Reynolds stress turbulence models. To simulate such flow in axisymmetric co-ordinates a multi-sweep algorithm is required in which the non-linear instability arising from high swirl in regions of low axial velocity is controlled by under-relaxation (Armfield, 1986).

INTRODUCTION

Swirling diffuser flow in which axial diffusion is small may be modelled by solving a reduced form of the Navier-Stokes equations in which axial diffusion terms, and all cross stream velocity terms in the cross stream momentum equation, are dropped on an order of magnitude basis (Armfield and Fletcher, 1986a). The resulting equation set is then parabolic (Armfield and Fletcher, 1986b) and may be solved using a single sweep algorithm marching in the dominant flow, axial, direction (Armfield and Fletcher, 1985a). Turbulent quantities in such flows have been modelled using both Clauser/mixing length and k-ε models (Armfield and Fletcher, 1985b). For flows with high swirl in which the centre-line axial velocity is close to reversal, the use of such a single sweep method is not possible due to a non-linear instability arising from the presence of large swirl in regions of low axial velocity (Armfield, 1986).

To enable such flows to be modelled a multi-sweep algorithm is required, which allows the non-linear instability to be controlled by under-relaxation. When such high levels of swirl are present the k-ε model gives poor prediction of turbulent quantities (Armfield and Fletcher, 1986c). In the present paper an algebraic Reynolds stress model, A.S.M., is presented for swirling flow in a conical diffuser at a Reynold's number of 380,000 with significant reduction in centreline axial velocity. It is shown that the A.S.M. gives a moderate improvement in the prediction of such flows.

EQUATIONS

The flow is considered to be axisymmetric, incompressible and of constant viscosity. The reduced Navier-Stokes equations written in the Reynolds-stress form using spherical co-ordinates (x,θ,ϕ) and corresponding velocity components (u,v,w), Figure 1, are

$$uu_x + vu_\theta/x - (v^2+w^2)/x = -p_x + \frac{1}{Re}\,[u_{\theta\theta}/x^2 + u_\theta/(x^2 \tan\theta)] - \frac{(\overline{u'v'})_\theta}{x} - (\overline{u'v'})/(x \tan\theta), \quad (1)$$

$$p_\theta/x = w^2/(x \tan\theta) \quad (2)$$

$$uw_x + vw_\theta/x + uw/x + \frac{vw}{x\tan\theta} = \frac{1}{Re}\left[w_{\theta\theta}/x + \frac{w_\theta}{x^2\tan\theta} \frac{-w}{(x^2\sin^2\theta)}\right] - \frac{(\overline{v'w'})_\theta}{x} - 2\,\frac{(\overline{v'w'})}{x\tan\theta}, \quad (3)$$

$$(x^2u)_x/x^2 + (x\sin\theta\, v)_\theta/(x^2\sin\theta)\ , = 0 \quad (4)$$

where u, v, and w are the mean velocities and u', v', w' are the fluctuating velocities, subscripts denote partial differentiation, p is the pressure, and the Reynolds number, $Re = \rho D u^1/\nu$, D the diffuser entrance diameter and u^1 the mean inlet velocity.

$(\overline{u'v'})$ and $(\overline{v'w'})$, the two Reynolds stresses present in equations (1) to (4), account for turbulent flucuations in the flow and are obtained using the k-ε model as,

$$(\overline{u'v'}) = -\nu_t\, u_\theta/x\ , \quad (5)$$

$$(\overline{v'w'}) = -\nu_t\,\left(w_\theta/x - w/(x\tan\theta)\right), \quad (6)$$

where ν_t, the turbulent eddy viscosity, is $C_\mu k^2/\varepsilon$, with k, the turbulent kinetic energy, and ε, the turbulent dissipation, obtained from their own transport equations (Rodi, 1972), reduced in the same manner as equations (1) to (4) (Armfield, 1986).

In the near wall region anisotropic eddy viscosities ν_x and ν_θ are obtained from a mixing length formulation as in Armfield and Fletcher, 1986a, which are then used to provide boundary values for the k and ε fields. In the A.S.M. Reynolds stresses are obtained by solving a series of six algebraic equations which are an approximation of the full Reynolds stress equations. The full Reynolds stress equations may be written in the following co-ordinate free tensor form, with U the velocity, and τ the Reynolds stress.

$$U^j\tau_{i\ell,j} = -(\tau^j_\ell\, U_{i,j} + \tau^j_i\, U_{\ell,j}) + \Phi_{i\ell} - \frac{2}{3}\, g_{i\ell}\,\varepsilon + D_{i\ell}\ , \quad (7)$$

where Φ is the pressure strain correlation, g the metric tensor and D the diffusion. The modelling approximations made to equation (7) to obtain the A.S.M. are as follows

$$U^j\tau_{i\ell,j} - D_{i\ell} = [-\tau^{jh}\, U_{h,j} - \varepsilon]\,\frac{\tau_{i\ell}}{k}\ , \quad (8)$$

due to Rodi, 1972.

$$\Phi_{i\ell} = -C_1\frac{\varepsilon}{k}\,(\tau_{i\ell} - \frac{2}{3}\, g_{i\ell}\, k) - C_2[-(\tau^j_\ell\, U_{i,j} + \tau^j_i\, U_{\ell,j}) + \frac{2}{3}\, g_{i\ell}(-\tau^{jh}\, U_{h,j})]\ , \quad (9)$$

the first part of the right hand side of equation (9) is due to Rotta, 1951, the second part is due to Naot, Shavit and Wolfshtien, 1970.

With the above approximations, the general form of the equation for the Reynolds stress components, $\tau_{i\ell}$, is,

$$\tau_{i\ell} = \left[\frac{k}{(-\tau^{jh}U_{h,j} + \varepsilon(C_1-1))}\right]\left[\frac{2}{3}\, g_{i\ell}\{C_2(-\tau^{jh}U_{h,j}) + \varepsilon(C_1-1)\} + (-\tau^j_\ell U_{i,j} - \tau^j_i U_{\ell,j})\,(1-C_2)\right]\ . \quad (10)$$

Once the individual equations for each $\tau_{i\ell}$ are obtained they may be reduced in the same manner as equations (1) to (4), Armfield 1986.

Equations (1) to (4) together with the k-ε equations and, when the A.S.M. is used, equation (10), are solved in the domain $x_1 \leq x \leq x_2$, and $0 \leq \theta \leq \theta_w$, where x_1 and x_2 are the spherical radii at the diffuser entrance and exit, and θ_w is the diffuser half angle, i.e. θ at the wall.

Boundary conditions for u,v and w are,

(i) inlet (x_1,θ): $u = u^1$, $v = v^1$, $w = w^1$,

(ii) diffuser wall (x,θ_w): $u = v = w = 0$,

(iii) diffuser centreline $(x,0)$: $v = w = u_\theta = 0$,

(iv) diffuser exit (x_2,θ) $u_{xx} = v_{xx} = w_{xx} = 0$.

Boundary conditions for pressure are dealt with in the next section.

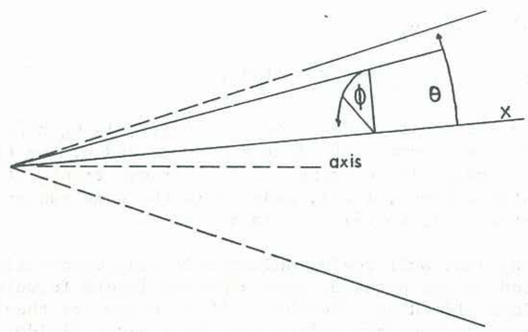

Fig. 1: Spherical Co-ordinates

METHOD OF SOLUTION

The domain is discretized using a variable mesh as shown in Figure 2.

All derivative terms are discretized using finite differencing in the following way, convective terms; hybrid central/upwind, p_x in equation (1); forward. all other terms central.

Unknowns u,w, k and ε are obtained from their transport equations using a tridiagonal method. The left hand side tridiagonal matrix is constructed from the convective terms and the viscous terms, including, for equations (1) and (3), the turbulent terms. When the A.S.M. is used a few initial sweeps are made using only the turbulent viscosity formulation, and then the difference between the Reynolds stress evaluated from the turbulent viscosity and that evaluated from the A.S.M. is included on the right hand side, at all points other than the mixing length near wall region. Thus the tridiagonal nature is retained.

Equation (4) is used to obtain v. To obtain p, the pressure is split into θ dependent and θ independent components. The θ dependent component is evaluated using equation (2), and is set to zero at the axis. The θ independent component is evaluated using a mass flow constraint (Briley, 1974). Since p_x in equation (1) is forward differenced a downstream boundary condition is required for the pressure. At the exit the axial derivative of the θ dependent component is set to zero. The θ independent components downstream condition is evaluated implicitly.

Repeated sweeps of the domain are made in the axial direction, with at each station x^n u,v,w,p,k and ε being evaluated in the manner indicated above, and those values being used to relax the stored values. The most recent corrected values at x^n are then used to obtain values at x^{n+1}.

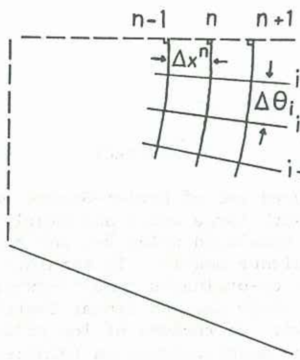

Fig. 2: Discretization

RESULTS

Solutions are presented in Figures 3 to 5 for a conical diffuser with a total angle of 8° and a Reynolds number of 3.8×10^5. The experimental results of So, 1964, at X/D = 0.6, which is just downstream of the diffuser entry, are used to generate the initial data for the computational solution. The computational solutions have been obtained on a non-uniform grid $40(x) \times 40(\theta)$. At the diffuser wall $x\Delta\theta = 0.001$, and at the axis $x\Delta\theta = 0.02$.

The k-ε model underpredicts the centreline axial velocity drop (Figure 3), whereas the A.S.M. predicts it very well. Both methods give very similar near wall profiles. Figure 4 shows that the A.S.M. also gives moderately better prediction for the swirl, although both methods appear to behave in a similar fashion. Both methods give satisfactory and near identical solutions for the pressure (Figure 5).

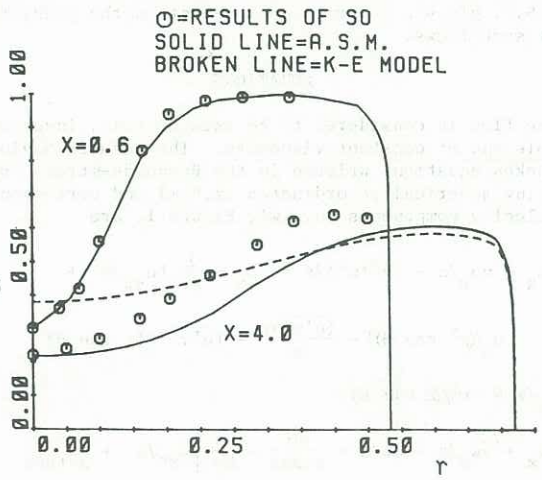

Fig. 3: Radial distribution of axial velocity.

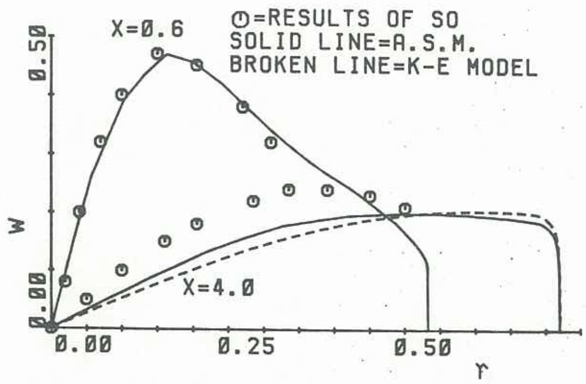

Fig. 4: Radial distribution of swirl velocity.

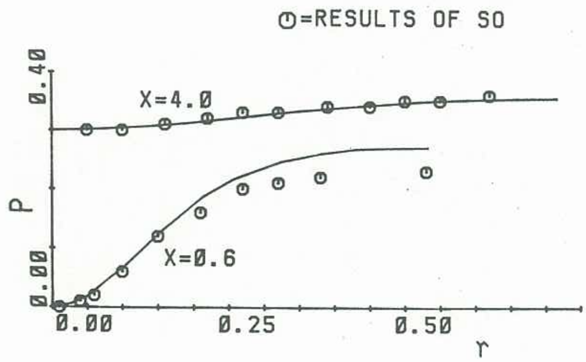

Fig. 5: Radial distribution of pressure.

The constants in the k-ε model are given the universal values commonly used; $C_\mu = 0.09$, $C_{\varepsilon 1} = 1.44$, $C_{\varepsilon 2} = 1.92$, $\sigma_\varepsilon = 1.2$. A range of values for C_1 and C_2 in the A.S.M., equation (10), were tested, the results presented are for the values; $C_1 = 2.8$, $C_2 = 0.8$.

DISCUSSION

The purpose of the present paper is to compare solutions of the reduced Navier-Stokes equations, (1) to (4), obtained using k-ε and algebraic Reynolds stress turbulence models. Reduction of the equations is based on an order-of-magnitude analysis presented in Armfield and Fletcher, 1986a valid in all but the near entrance region of diffusers with less than 15° total internal angle.

On the basis of the satisfactory prediction of pressure it appears that neglecting the secondary terms in the cross stream momentum equation, (2), is a valid approximation for the flow considered. Owing to the simplicity of the reduced equation (2), the θ dependent pressure component may be obtained directly from the swirl profile. This leads to a substantial saving in computing time, as otherwise the pressure is obtained from a form of Poisson's equation in which, typically, several sweeps of the domain must be made to obtain a suitably converged solution.

It has been shown, Armfield, 1986, that inclusion of all secondary terms makes very little difference, less than 10%, to solutions obtained using the k-ε model for the present flow. A similar comparison of reduced and non-reduced forms of the A.S.M. has not as yet been made. However results obtained with all secondary turbulent stress terms included in equations (1) and (3) showed their effect was small. Hence it is suggested that inclusion of all secondary terms in the A.S.M. will not have a large effect for this class of flow.

Armfield and Fletcher, 1985b, have shown that for flows with low swirl the k-ε model gives satisfactory predictions, however it is apparent for flows with high swirl, such as that considered here, the k-ε model gives poor results. Various correction factors may be included into the k-ε model to improve its performance for this class of flows. Such correction factors are generally derived from the full Reynolds stress equations using the same modelling approximations as the A.S.M. Since, as has been demonstrated, the A.S.M. does give improved solutions, it is likely that simple correction factors for the k-ε model can be derived. However, it is suggested that as the A.S.M. does not greatly increase the computational requirements, it is a better choice for such high swirl flows, since deriving an eddy viscosity formulation from it necessarily leads to further approximations, and hence further loss of information.

Although the A.S.M. leads to improved prediction, it is evident that, particularly in the swirl solution, further development is required. Broadly speaking, such development can take place in two areas. Firstly, there is the question of the approximations required to enable an unclosed set of partial differential equations, (7), to be modelled by a closed system of algebraic equations, (10). Similar approximations must also be made to the k and ε equations, to enable them to be solved. These approximations are based on the behaviour of thin shear flows, near to local equilibrium, and with minimal streamline curvature. Streamlines in the present flow follow a helical path. In regions where the axial velocity is much larger than the swirl, the helix is not very tightly wound, hence the streamlines are relatively straight. However in regions where the axial velocity is not large with respect to the swirl, as is the case in the near axis region of the present flow, the effect is to wind the helix tightly, and increase the degree of streamline curvature. It is quite possible that when this is the case important effects are being lost in the approximations made to equation (7). The ε equation presents additional problems due to the presence of quantities such as the production of dissipation, and the dissipation of dissipation, which are physically impossible to measure, and difficult to conceive. At present these terms are modelled inthe same way as terms in the k equation. Some authors, such as Hah, 1982, consider the ε equation to be the primary cause of poor results, and modify it to allow for the effects of streamline curvature.

The second area is the choice of the constants C_1 and C_2. It is suggested by Gibson and Younis, 1986, that with the present modelling assumptions, satisfactory results may be obtained by making a suitable choice for these constants. By considering experimental results for such flows as homogeneous grid turbulence downstream of a contraction and flows in local equilibrium, they obtained the values of $C_1 = 2.8$ and $C_2 = 0.3$. With these values they obtained satisfactory predictions for swirling jets.

The values of C_1 and C_2 used in the present paper were obtained by trial and error, and although a $C_1 = 2.8$, the same as Gibson and Younis, 1986, is used, the C_2 value is quite different. Reasons for this may be that the present flow is complicated by the effect of the wall and the near reversing axial velocity, making the assumptions of Gibson and Younis, 1986, less applicable.

CONCLUSIONS

For the class of diffuser flows with high swirl and nearly reversing centreline axial velocity where the diffuser total angle is small, pressure may be suitably modelled by relating the cross-stream pressure variation to the swirl profile, as in equation (2), and obtaining the streamwise pressure variation from a mass flow constraint, after Briley, 1974. The poor predictions for mean axial and swirl velocities of the standard k-ε may be improved by using an algebraic Reynolds stress model. However, it is apparent that further development of the A.S.M. is required, and it is suggested that in particular the effect of streamline curvature needs to be considered.

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