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Wave Force on Compound Cylinder

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ABSTRACT

Theoretical investigation of wave force on compound cylinders is carried out by the linear diffraction theory. The force and moment functions are derived by the proper selection of a pressure modification function, such that the continuity of pressure profile at the common boundary of the two cylinders is satisfied. This provides a simpler approach to the problem as well as closed form solutions for force and moment functions due to dynamic component of pressure. Experimental data obtained are in good agreement with the theoretical results.

INTRODUCTION

A compound cylinder is a vertical axisymmetrical body comprising of a cylinder resting on a large cylindrical base. This particular configuration is often used as offshore storage tanks, gravity platforms, etc. This shape is also more suitable for structures in the Arctic environment as the smaller cylinder extending to the free surface will provide a smaller contact surface to the floating icebergs and the larger bottom cylinder provides a more suitable foundation against wind and wave loadings.

Considerable research has been done on vertical circular cylinders subjected to wave induced forces. The single cylinder has been extensively investigated under both submerged and surface piercing conditions. Evaluation of wave forces on compound cylinders was carried out on the basis of linear diffraction theory (Isaacson, 1979). This configuration also provided a useful reference in assessing the negligible influence of a slender top cylinder in calculating force on compound cylinders. The method was an extension of that used for calculating the wave forces on a circular dock (Garret 1971). Isaacson expanded the flow potential into different series in the regions above and exterior to the lower cylinder, and the potential and radial velocities were then matched along the common boundary between the two cylinders.

The paper provides a semi analytical solution for forces on a compound cylinder based on simplified assumptions of pressure function.

THEORETICAL FORMULATION AND SOLUTION

Consider the compound cylinder as shown in Fig. 1. The origin is chosen at the bottom centre of the base cylinder. The cylindrical coordinate system (r,θ,z) is formed with $r = 0$ lying around the body axis and $\theta = 0$ forming the positive x axis. The parameters are shown in Fig. 1.

From the assumptions, the velocity potential for the flow exists and the problem reduces to the determination of the velocity potential which satisfies the Laplace equation

$$\nabla^2 \phi = 0 \tag{1}$$

within the fluid region, and is subjected to the linearized kinematic and dynamic free-surface boundary conditions;

$$\frac{\partial^2 \phi}{\partial t^2} + g\frac{\partial \phi}{\partial z} = 0 \quad \text{at } z = d \tag{2}$$

$$\eta = -\frac{1}{g}\left(\frac{\partial \phi}{\partial t}\right)_{z = d} \tag{3}$$

where g is the acceleration due to gravity and η is the free-surface elevation measured above $z = d$.

The linearisation of the motion and the specification of an incident wave permits the velocity potential ϕ to be considered as the sum of components describing the incident (subscript i) and scattered (subscript s) wave motions

$$\phi = \phi_i + \phi_s \tag{4}$$

The free surface and boundary conditions apply to both ϕ and ϕ_i and therefore also to ϕ_s.

The expression of the velocity potential and associated variables in the form of Eq (4) and involving separation into undisturbed incident wave and scattered wave components constitute the basis for diffraction theory. The incident wave potential is specified in complex form as

$$\phi_i = \frac{-igH}{2\sigma}\frac{\cosh kz}{\cosh kd}e^{-i\sigma t}\sum_{m=0}^{\infty}\varepsilon_m i^m J_m(kr)\cos m\theta \tag{5}$$

where $\varepsilon_o = 1$ and $\varepsilon_m = 2$ for $m = 1, 2, 3, \ldots$ are the Jacobi symbols and $J_m(kr)$ is the Bessel function of the first kind of order m and argument kr.

Similarly the expression for ϕ_s is

$$\phi_s = \frac{-igH}{2\sigma}\frac{\cosh kz}{\cosh kd}e^{-i\sigma t}\sum_{m=0}^{\infty}\left(A\,\varepsilon_m i^m H_m(kr)\right)\cos m\theta \tag{6}$$

where A is an initially unknown (complex coefficient) and can be easily shown to be, for the bottom cylinder, given by

$$A = -\frac{J_m'(ka)}{H_m'(ka)} \tag{7}$$

where the prime denotes differentiation with respect to the argument. Similarly, for the top cylinder 'a' is replaced by 'b' in Eq. (7).

If ϕ_a and ϕ_b denote the complete potential solution for the bottom and top cylinders respectively, then

$$\phi_a = \frac{gH}{\sigma}\frac{\cosh kz}{\cosh kd}e^{-i\sigma t}\cos\theta\left(J_1(kr) - \frac{J_1'(ka)}{H_1'(ka)}H_1(kr)\right) \tag{8}$$

Similarly

$$\phi_b = \frac{gH}{\sigma}\frac{\cosh kz}{\cosh kd}e^{-i\sigma t}\cos\theta\left(J_1(kr) - \frac{J_1'(kb)}{H_1'(kb)}H_1(kr)\right) \tag{9}$$

To obtain the force and moment functions, it is necessary to derive the pressure functions first.

Pressure functions

Since the dynamic pressure is given by,

$$p = -\rho \frac{\partial \phi}{\partial t} \qquad (10)$$

The maximum pressure distribution on the surface of bottom cylinder (r=a) can be obtained as

$$P_a(a,\theta,z) = \frac{-2\rho gH}{\pi ka\, H_1'(ka)} \frac{\cosh kz}{\cosh kd} \cos\theta \qquad (11)$$

Similarly on the surface of top cylinder, P_b can be obtained by substituting a = b in Eq. (11).

For calculating forces and moments, the maximum pressure functions given by Eqn (11) will be used. The horizontal force and overtuning moment thus obtained will be due to the dynamic component of pressure.

The pressure functions P_a and P_b are those obtained if the cylinders of radii a and b were subjected to waves independently. For the compound cylinder considered, the following conditions should be satisfied: (i) Pressure distribution at the common boundary of the cylinders must be the same, and (ii) For continuous pressure distribution across the flow, the tangent of the pressure profile must be the same at z = h.

Therefore at z = h, we assume

$$\alpha P_a = \beta P_b \qquad (12)$$

and $$\frac{\partial(\alpha P_a)}{\partial z} = \frac{\partial(\beta P_b)}{\partial z} \qquad (13)$$

where α and β are unknown modification functions for the pressure distribution so that it is compatible to the compound cylinder. From Eq (12)

$$\beta = \frac{P_a}{P_b}\alpha = \frac{b}{a}\frac{H_1'(kb)}{H_1'(ka)}\alpha \qquad (14)$$

From Eq (13), it is evident that, at z = h

$$\alpha \frac{\partial P_a}{\partial z} = \beta \frac{\partial P_b}{\partial z} \qquad (15)$$

For a given configuration, the pressure distribution will vary with depth, so will α and β. It is assumed that the pressure at the base is due to bottom cylinder only, ($\alpha = 1$ at $z = 0$), and in elsewhere $\alpha = f(\frac{b}{a}, \frac{h}{d}, z)$.

Force functions

Consider a horizontal slice of unit height for the bottom cylinder. The element force per unit height in the direction of wave propagation is,

$$\frac{df_a}{dz} = -\int_o^{2\pi} \alpha P_a(a,\theta,z)\, a\cos\theta\, d\theta \qquad (16)$$

Substituting for P_a and integrating, we get the total horizontal force on the bottom cylinder as,

$$f_a = \frac{2a\rho gH}{kaH_1'(ka)} \frac{1}{\cosh kd} \int_o^h \alpha \cosh kz\, dz \qquad (17)$$

Similarly the total horizontal force on the top cylinder is,

$$f_b = \frac{2b\rho gH}{kbH_1'(kb)} \frac{1}{\cosh kd} \int_h^d \beta \cosh kz\, dz \qquad (18)$$

Substituting for β from Eq. (14), we obtain

$$f_b = \frac{2b\rho gH}{kaH_1'(ka)} \frac{1}{\cosh kd} \int_h^d \alpha \cosh kz\, dz \qquad (19)$$

Thus, the total horizontal force on the compound cylinder is, $f_t = f_a + f_b$. The forces can be written as dimensionless force coefficients by dividing them by $\rho gH\pi a^2/2$. The force thus obtained is the maximum, as the maximum pressure distribution function is used in calculating them.

Several empirical expressions for α were tested to verify the analytical expressions with the experimentally determined forces on the compound cylinders and it was found that the most appropriate expression is

$$\alpha = 1 - (1 - \frac{b}{a})\left(\frac{d - z}{h}\right)\left(\frac{z}{h}\right)^2 \qquad (20)$$

Using Eq. (20), the following expressions for the force coefficients are obtained for the compound cylinder.

$$F_a = f_a/\tfrac{1}{2}\rho gH\pi a^2 = \frac{4}{\pi akaH_1'(ka)} \frac{1}{\cosh kd} \Big(\frac{1}{k}\sinh kh + \frac{d}{k^3h^3}(\frac{b}{a} - 1)\,[(k^2h^2 + 2)\sinh kh - 2kh\cosh kh] + \frac{1}{k^4h^3}(1 - \frac{b}{a})\,[(k^3h^3 + 6kh)\sinh kh - (3k^2h^2 + 6)\cosh kh]\Big) \qquad (21)$$

$$F_b = f_b/\tfrac{1}{2}\rho gH\pi a^2 = \frac{4b}{\pi a^2kaH_1'(ka)} \frac{1}{\cosh kd} \Big(\frac{1}{k}(\sinh kd - \sinh kh) + \frac{d}{k^3h^3}(\frac{b}{a} - 1)\,[(k^2d^2 + 2)\sinh kd - 2\,kd\cosh kd - \{(k^2h^2 + 2)\sinh kh - 2kh\cosh kh\}] + \frac{1}{k^4h^3}(1 - \frac{b}{a})\,[(k^3d^3 + 6kd)\sinh kd - (3k^2d^2 + 6)\cosh kd - \{(k^3h^3 + 6kh)\sinh kh - (3k^2h^2 + 6)\cosh kh\}]\Big) \qquad (22)$$

Then, $FT = F_a + F_b \qquad (23)$

For the single cylinder, substitution of b = a into Eq. (23) yields

$$FT = \frac{4}{\pi akaH_1'(ka)} \frac{\tanh kd}{k} \qquad (24)$$

Eq (24) is the same expression obtained (MacCamy and Fuchs, 1954) for the total horizontal force on a single, large vertical cylinder extending to the free surface.

Substitution of b = 0 into Eqs. (21 and 22) will reduce F_b to 0 and all terms containing b in F_a will vanish, giving the maximum horizontal force on a submerged cylinder.

Moment functions

The moment due to the wave force about an axis parallel to y, passing through the bottom of the compound cylinder can similarly be obtained using the α function.

The moment due to wave force on the bottom cylinder is

$$m_a = \int_o^h z\frac{df_a}{dz}dz \qquad (25)$$

and that due to the top cylinder is,

$$m_b = \int_h^d z\frac{df_b}{dz}dz \qquad (26)$$

The total moment about the base of the compound cylinder is $m_t = m_a = m_b$.

Defining the dimensionless moment coefficients as $M = m/\tfrac{1}{2}\rho gH\pi a^3$, and substituting for α (Eq. 20), we can easily obtain

$$M_a = \frac{4}{\pi a^2 kaH_1'(ka)} \frac{1}{\cosh kd} \Big(\frac{1}{k^2} (kh \sinh kh -$$

$$\cosh kh + 1) + \frac{d}{k^4 h^3} (\frac{b}{a} - 1)[(k^3h^3 + 6kh) \sinh kh -$$

$$(3k^2h^2 + 6) \cosh kh + 6] + \frac{1}{k^5 h^3} (1 - \frac{b}{a})[(k^4h^4 +$$

$$12k^2h^2 + 24)\sinh kh - (4k^3h^3 + 24kh)\cosh kh] \Big) \quad (27)$$

$$M_b = \frac{4b}{\pi a^3 kaH_1'(ka)} \frac{1}{\cosh kd} \Big(\frac{1}{k^2}[(kd \sinh kd - \cos kd) -$$

$$(kh \sinh kh - \cosh kh)] + \frac{d}{k^4 h^3}(\frac{b}{a} - 1)[(k^3d^3 + 6\ kd)$$

$$\sinh kd - (3k^2d^2 + 6) \cosh kd - \{(k^3h^3 + 6kh)$$

$$\sinh kh - (3k^2h^2 + 6) \cosh kh\}] + \frac{1}{k^5 h^3} (1 - \frac{b}{a})$$

$$[(k^4d^4 + 12k^2d^2 + 24)\sinh kd - (4k^3d^3 +$$

$$24\ kd) \cosh kd - \{(k^3h^4 + 12k^2h^2 + 24) \sinh kh -$$

$$(4k^3h^3 + 24\ kh) \cosh kh\}] \Big) \quad (28)$$

Then $$MT = M_a + M_b \quad (29)$$

The moment function for the single cylinder can be obtained by substituting b = a into Eq. (47). Then,

$$MT = \frac{4}{\pi a^2 kaH_1'(ka)} \frac{kd \sinh kd - \cosh kd + 1}{k^2 \cosh kd} \quad (30)$$

EXPERIMENTAL WORK

All experiments were conducted in the Hydraulic Engineering Laboratory of the National University of Singapore. Tests were carried out on models of the compound cylinders in a wave flume which is 35.22 m long and 2 m wide, with a height of 1.3 m, and has an operating water depth of 1 m.

The models were made of rigid hollow PVC pipe sections. All experiments were conducted with a fixed bottom cylinder of 32 cm diameter (a). The diameters of the top cylindres (b) used were 6 cm, 11.4 cm and 16.4 cm. A single cylinder of 32 cm diameter was also used in the study. Experiments were conducted for two depths (d) of water, 32 cm and 68 cm. The heights of the bottom cylinders (h) used were 16 cm and 34 cm respectively, so that a h/d ratio of 0.5 was maintained.

THEORETICAL RESULTS

Force and Moment Functions

The amplitude of the force and moment coefficients were considered to depend on the diffraction parameter ka, and the ratios b/a, a/h and h/d [6]. Computer programmes were used for the computation of these coefficients for various values of a, b, d and h. As an example of numerical calculations, results are presented here for the particular case of a compound cylinder with h/d = 0.5 and a/h = 1.0 in Figs. 2 to 5. Comparison of the predicted results with the works of Isaacson [6] is shown in Fig. 6.

The variation of force coefficient FT with (ka) (Figs. 2 to 5) is similar to those obtained by Isaacson however the present study tends to predict a lower values of FT (Fig. 6). The difference in FT was found to increase as the b/a ratio decreases from 1 to 0. The single cylinder (b/a = 1) and the submerged cylinder (b/a = 0) can be taken as the limiting conditions of the study; the predicted results for which can be compared with known solutions. The forces exerted on the various compound cylinders will lie between these two extreme limits. Isaacson had presented his results for these limiting cases by showing good agreement with the predictions of MacCamy and Fuchs for the single cylinder.

Comparing Fig. 2 with Fig. 4 and Fig. 3 with Fig. 5 it can be been that for lower values of b/a ($\leq$ 0.25) the influence of the top cylinder on FT and MT is not significant.

EXPERIMENTAL RESULTS

The experimental data for the force and moment coefficient are plotted in Figs. 7 and 8. The theoretical curves can be seen to describe the data well, though the curves are generally found to overestimate the force and moment at their peak values. It may be noted that Isaacson's theoretical results were found to predict still higher forces and moments compared to the present theoretical results.

A possible reason for higher values of experimental data compared to the theoretical values, at large values of ka (>1), is probably due to the formation of steep waves at large ka which may result in higher order force contributions due to wave nonlinearity.

CONCLUSIONS

Theoretical investigation to determine wave forces on compound cylinders was carried out on the basis of linear diffraction theory. The approach was simplified by the selection of a pressure modification function, α by trial and error. This function conforms the pressure distribution with depth for a single cylinder to that of a compound cylinder and is given by the empirical expression

$$\alpha = 1 - (1 - \frac{b}{a}) (\frac{d-z}{h}) (\frac{z}{h})^2$$

Simple and closed-bound solutions for the horizontal force and overturning moment functions are established. The theoretical approach is verified by the well established MacCamy and Fuchs solution for the wave force on a single surface piercing cylinder. But comparison with the results of Isaacson show that the present study predicts lower force and moment values. Experimental investigations were carried out to measure forces exerted on laboratory models of the compound cylinders in a wave flume. The results show that the theoretical predictions for the peak values of FT & MT were higher, compared to the experimental values.

It is found that the effect of a slender top column to the contribution of total force and moment becomes negligible when b/a < 0.25.

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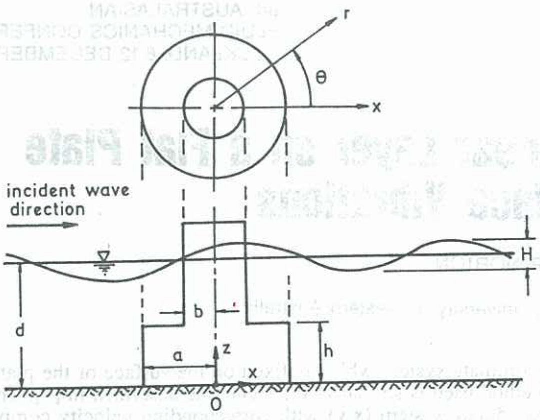

Fig. 1: Definition sketch and notations

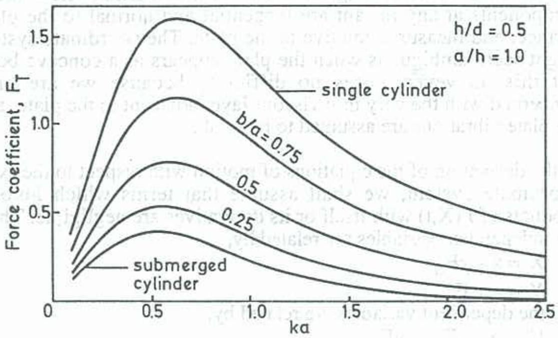

Fig. 2: Typical variation of force coefficient

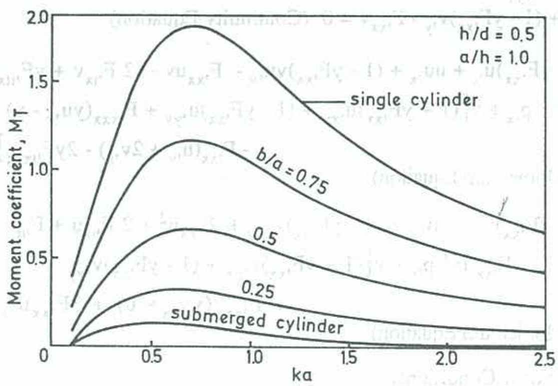

Fig. 3: Typical variation of moment coefficient

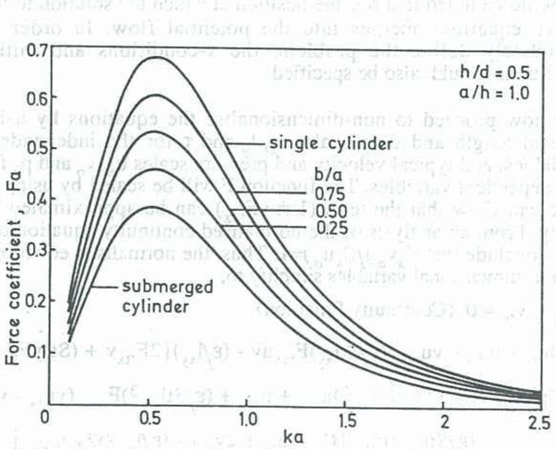

Fig. 4: Typical variation of force coefficient for the bottom cylinder

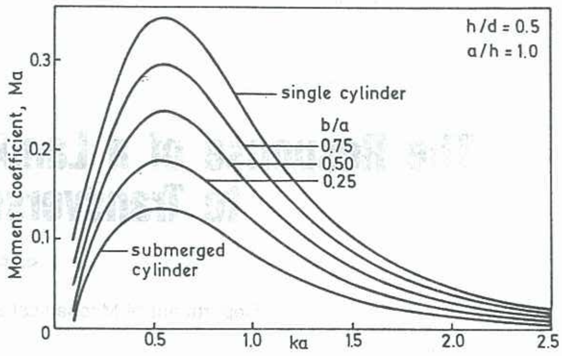

Fig. 5: Typical variation of moment coefficient for the bottom cylinder

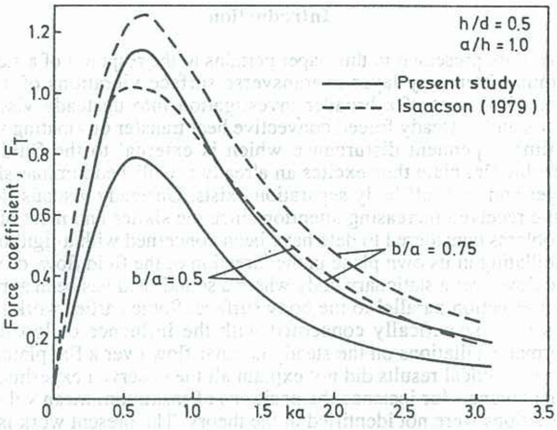

Fig. 6: Comparison of force coefficient for compound cylinders with Isaacson

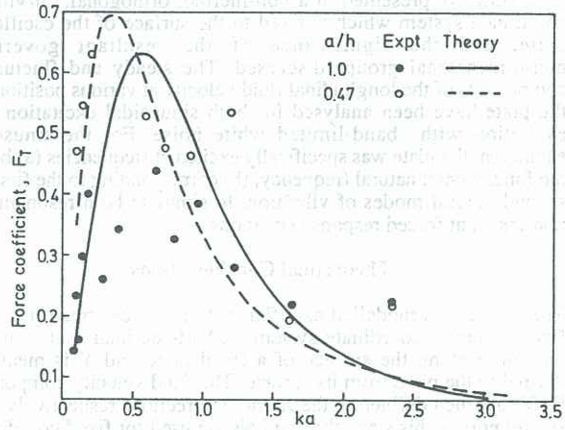

Fig. 7: Theoretical and experimental force coefficients for a compound cylinder (b/a = 0.3563, h/d = 0.5)

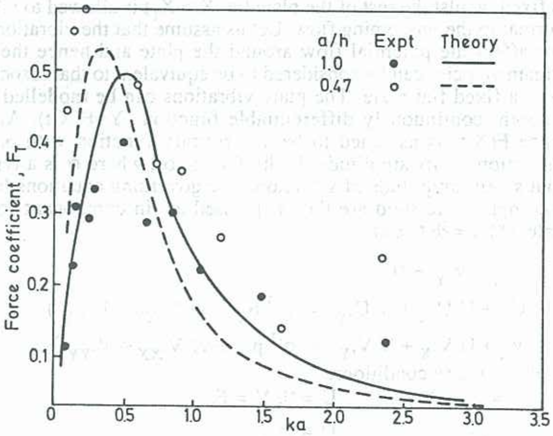

Fig. 8: Theoretical and experimental force coefficients for a compound cylinder (b/a = 0.1875, h/d = 0.5)