

Two Stage Thrust Ejections — Subsonic Flow Solution

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ABSTRACT

The results of theoretical assessment of two stage thrust augmenting ejectors are presented and compared with those of single stage ejectors. The mixing ducts were of constant cross-sectional area, the flows at the inlet and exit of each stage were assumed to be uniform, and friction effects were ignored.

With fully mixed subsonic flow at the final ejector exit, two distinct solutions can be described with reference to the completely mixed flow emanating from the first stage. The first solution is for a subsonic mixed flow and the other is supersonic. Calculations presented in this paper are for the subsonic flow solution.

It was found that staging the ejector increases thrust augmentation at all primary jet stagnation pressures, but is more effective in the low pressure range and with high area ratios for any gas combination. Heating the primary jet was found to reduce the effectiveness of staging.

1. NOMENCLATURE

A	Duct or flow cross sectional area
$F_{n,e}$	Nozzle and ejector thrust
H.R.G.	Hot rocket gas
M	Mach number
\dot{m}	Mass flow rate
P, P_0	Static and total pressure.
\bar{R}	Universal gas constant
T, T_0	Static and total temperature
V	Flow velocity
W	Molecular weight
γ	Ratio of specific heats
$\mu_{1,2}$	Secondary and tertiary mass flow ratios $= \dot{m}_2 / \dot{m}_1$
τ	Thrust augmentation ratio $= (F_n + F_e) / F_{na}$
ϕ	A function defined in text
ψ	Effectiveness of staging $= \tau_{2s} / \tau_{1s}$
ρ	Density
Γ	Improvement of mass augmentation by staging $(1 + \mu_1 + \mu_2)_{2s} / (1 + \mu_1)_{1s}$

Subscripts

1,2,3,4	Relating to stations 1,2,3 and 4 in figure 2
a	Relating to ambient conditions
p	Relating to primary jet flow
1s	Single stage ejectors
2s	Two stage ejector
*	Relating to nozzle throat

2. INTRODUCTION

The propulsive efficiency of high velocity jet issuing from a stationary or low forward speed vehicle (e.g. aircraft at take-off) is known to be poor, and can be improved by reducing the jet velocity. Without affecting the primary jet thrust, the simplest way to achieve this is by the use of an ejector. In this device, part of the energy in a relatively high velocity jet from a nozzle at the inlet is used to entrain a stream of low energy secondary fluid (e.g. ambient air) within a confining duct as shown in

figure 1. The degree of this entrainment process depends mainly on the effectiveness of kinetic, and thermal mixing between the two streams in the duct. The momentum of the relatively low velocity mixed flow emanating from the mixing duct is generally high compared with that in the primary jet, the increased mass flow outweighing the reduction in jet velocity.

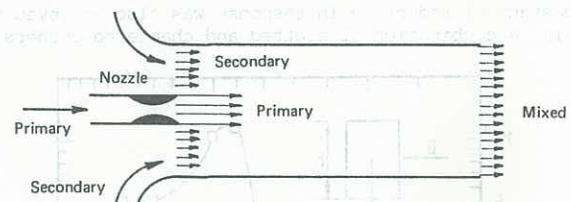


Figure 1: Single Stage thrust augmenting ejector

The present study forms part of a broader investigation of the use of ejectors for improving the static thrust of rocket motors. In terms of primary jet stagnation pressure and temperature, this application is well outside the experience of most previous workers "Viets 1975, Quin 1976", which is more applicable for STOL and V/STOL aircraft propulsion. Recent work with jets simulating those of rocket efflux "Fisher 1980, Fisher and Irvine 1981", was confined to single stage ejectors. Although multi-staging the ejector was found to increase the availability of thrust experimentally "Morrison 1942", and analytically "Nagaraja et al 1973", no further work to the best knowledge of the author has taken place. These two references were cited by "Viets 1975", but were not available to the present author. It is the purpose of the present work to evaluate theoretically the availability of thrust with two stage ejectors, with an emphasis on relatively high primary jet stagnation pressure ratios, such as those used in rocket motors.

3. THEORETICAL ASPECTS

One dimensional compressible flow theory is adopted with the two stage ejector configuration shown in figure 2. The primary jet flows through an area A_p and entrains the secondary ambient air through the annular area A_1 . The two streams mix in the first duct of cross-sectional area A_4 . This mixed flow entrains the tertiary air through the annular area A_2 and mixes with it in the second duct of cross-sectional area A_3 before emanating to the atmosphere.

The following are assumed:

- Both mixing ducts are of constant cross-sectional area.
 $A_3 = A_4 + A_2 = A_p + A_1 + A_2$
- Static pressure across the inlet and exit planes of each duct is uniform.
 $P_p = P_1, P_4 = P_2, \text{ and } P_3 = P_a = P_{01}$
- The flow distribution in each stream at the inlet and exit of each duct is uniform.

- d. Skin friction is neglected.
- e. The gases are compressible and satisfy the perfect gas law throughout the mixing process, with specific heats independent of temperature.
- f. The primary jet is correctly and isentropically expanded.

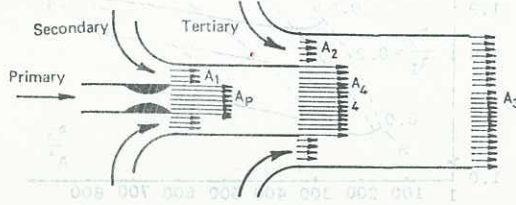


Figure 2: Two stage thrust augmenting ejector.

With the above assumptions, the conservation equations for the balance of mass, momentum, and energy at various ejector stations can be written as follows:

1. Conservation of Mass:

$$\Sigma(\rho AV)_{P,1,2} = (\rho AV)_3$$

$$\Sigma \left[PAM \sqrt{\frac{\gamma}{RT_0}} \left(1 + \frac{\gamma-1}{2} M^2 \right) \right]_{P,1,2} = \left[PAM \sqrt{\frac{\gamma}{RT_0}} \left(1 + \frac{\gamma-1}{2} M^2 \right) \right]_3 \quad (1)$$

2. Conservation of Momentum:

$$\Sigma(\rho AV^2 + PA)_{P,1,2} = (\rho AV^2 + PA)_3$$

$$\Sigma[PA(1 + \gamma M^2)]_{P,1,2} = [PA(1 + \gamma M^2)]_3 \quad (2)$$

3. Conservation of Energy:

$$\Sigma \left(\dot{m} \frac{\gamma}{\gamma-1} \frac{\bar{R}}{W} T_0 \right)_{P,1,2} = \left(\dot{m} \frac{\gamma}{\gamma-1} \frac{\bar{R}}{W} T_0 \right)_3 \quad (3)$$

The area ratios $A_{1,2}/A_p$ can be related to the mass ratios μ_1 and μ_2 by the following relationship:

$$\frac{A_{1,2}}{A_p} = \mu_{1,2} \frac{P_p}{P_{1,2}} \frac{M_p}{M_{1,2}} \sqrt{\frac{W_p}{W_{1,2}} \frac{\gamma_p}{\gamma_{1,2}} \frac{T_{01,2}}{T_{0p}}} \frac{2 + (\gamma_p - 1) M_p^2}{2 + (\gamma_{1,2} - 1) M_{1,2}^2} \quad (4)$$

Divide the momentum equation (2) by the mass equation (1), and substitute for the area ratios $A_{1,2}/A_p$ from equation (4), and after algebraically gathering terms:

$$\frac{\phi_p + \mu_1 \sqrt{\frac{W_p}{W_1} \frac{\gamma_p}{\gamma_1} \frac{T_{01}}{T_{0p}}} \phi_1 + \mu_2 \sqrt{\frac{W_p}{W_2} \frac{\gamma_p}{\gamma_2} \frac{T_{02}}{T_{0p}}} \phi_2}{1 + \mu_1 + \mu_2} = \sqrt{\frac{W_p}{W_3} \frac{\gamma_p}{\gamma_3} \frac{T_{03}}{T_{0p}}} \phi_3$$

$$\text{where } \phi = \frac{1 + \gamma M^2}{M \sqrt{1 + \frac{\gamma-1}{2} M^2}} \quad (5)$$

In our case, both entrained gases 1 and 2 are the same (atmospheric air) and at the same stagnation pressure and temperature. Squaring both sides of equation (5):

$$\phi_3^2 = \left[\frac{\phi_p + \sqrt{\frac{W_p}{W_1} \frac{\gamma_p}{\gamma_1} \frac{T_{01}}{T_{0p}}} (\mu_1 \phi_1 + \mu_2 \phi_2)}{1 + \mu_1 + \mu_2} \right]^2 \left(\frac{W_3}{W_p} \frac{\gamma_3}{\gamma_p} \frac{T_{03}}{T_{0p}} \right) \quad (6)$$

This yields the biquadratic equation for the fully mixed flow Mach number M_3 :

$$(\gamma_3^2 - \frac{\gamma_3-1}{2} \phi_3^2) M_3^4 + (2\gamma_3 - \phi_3^2) M_3^2 + 1 = 0 \quad (7)$$

$$M_3^2 = \frac{(\phi_3^2 - 2\gamma_3) \pm \sqrt{(2\gamma_3 - \phi_3^2)^2 - 4(\gamma_3^2 - \frac{\gamma_3-1}{2} \phi_3^2)}}{2(\gamma_3^2 - \frac{\gamma_3-1}{2} \phi_3^2)} \quad (8)$$

For M_3 to be real the following condition must be satisfied:

$$(2\gamma_3 - \phi_3^2)^2 - 4(\gamma_3^2 - \frac{\gamma_3-1}{2} \phi_3^2) \geq 0 \quad (9)$$

For the equality condition

$$\phi_3^2 = 2(\gamma_3 + 1)$$

This leads to $M_3 = 1$, or the second mixing duct is said to be choked.

With the inequality condition, it is clear from equation (8) that M_3 has two values, one of which is subsonic $M_{3(-)}$ and the other is supersonic $M_{3(+)}$. These two values are related to each other by the following equation which is derived from equation (8) and by substitution for the function ϕ_3 :

$$M_{3(-)}^2 = \frac{\frac{\gamma_3-1}{2} M_{3(+)}^2 + 1}{\gamma_3 M_{3(+)}^2 - \frac{\gamma_3-1}{2}} \quad (10)$$

which happens to be the relationship between the upstream $M_{(+)}$ and downstream $M_{(-)}$ Mach numbers for a normal shock wave.

For given P_{op}/P_a , $T_{op}/(T_{01}=T_{02})$, A_3/A^* and known properties of the primary and entrained gases, M_3 can be obtained by nominating M_1 and A_2/A_1 . The calculated M_3 value must satisfy the imposed condition that the static pressure at the exit of the mixing duct equals to the ambient value. This can be realised by fixing either of the nominated M_1 or A_2/A_1 and iterating with the other until the $(P_3 = P_a)$ condition is satisfied.

4. RANGE OF CONDITIONS CONSIDERED

4.1 Subsonic-Supersonic Flow Regime

The conceivable solutions for the governing equations derived in the previous section are grouped into four combinations defined with respect to the mixed flow Mach numbers M_4 and M_3 at the exit of the first and second mixing ducts respectively:

$$\begin{array}{ll} 1 - M_4 > 1, M_3 < 1 & 3 - M_4 < 1, M_3 > 1 \\ 2 - M_4 > 1, M_3 > 1 & 4 - M_4 < 1, M_3 < 1 \end{array}$$

However, with the assumption of constant area mixing and balanced static pressure across the 1st-2nd stage interface, which were made to limit the potentially enormous number of variables involved, the first of these combinations arose only with a narrow range of extremely low first stage area ratios, and the second and third combinations were not available. The following results are therefore confined to subsonic flow solutions ($M_4 < 1, M_3 < 1$). It is planned to explore the supersonic solutions further by permitting greater freedom in the ejector model, in terms of duct shape and relative pressures in the different streams.

4.2 Primary Gas Conditions

Calculations were made in turn with the primary jet consisting of unheated air and hot rocket efflux respectively. Interest in the former case arose both

from its relative simplicity and from the fact that many of the experiments in the broader investigation into high pressure ratio thrust augmenting ejectors have been performed with unheated air jets. Jet stagnation pressure and duct to nozzle area ratio were maintained as variables for the purpose of the calculations. In all cases the entrained secondary and tertiary flows were of ambient air.

5. RESULTS AND DISCUSSIONS

For a given gas combination, P_{op}/P_a , and $T_{op}/(T_{01} = T_{02})$ the effectiveness of staging $\psi = \tau_{2s}/\tau_{1s}$ is a measurement of the improvement in thrust augmentation obtained with the two stage ejector relative to that with an equivalent single stage ejector having the same area ratio A_3/A^* . The equivalent single stage ejector is a special case of the two stage ejector, and can be obtained by fixing the annular area ratio A_2/A_1 at either 0 or ∞ . In this case $\mu_2 = 0$ or $\mu_1 = 0$ and the duct of the first stage coincides either with the duct of the second stage or with the nozzle respectively.

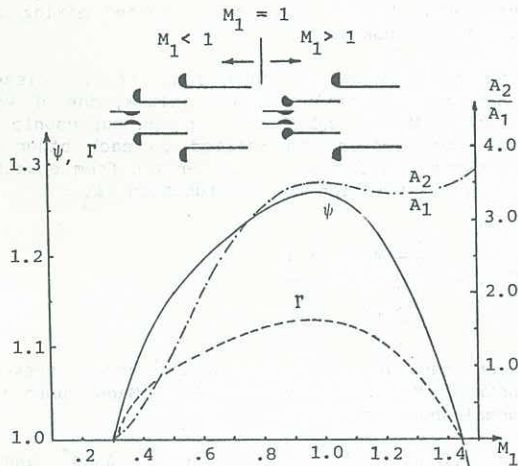


Figure 3: Performance of two stage thrust augmenting ejectors, for $P_{op}/P_a = 42$, $A_3/A^* = 800$

5.1 Air-Air Combination

Typical behaviour of ψ as a function of M_1 is shown in figure 3 together with the corresponding variation of A_2/A_1 and γ . This figure was obtained for fixed values of $P_{op}/P_a = 42$ and $A_3/A^* = 800$. Point A in the figure represents a single stage ejector $A_2/A_1 = 0$ and $\psi = \gamma = 1$. In the subsonic range of $M_1 = 0.299 - 1.0$, ψ is an increasing function of M_1 from $\psi = 1$ to a maximum $\psi = 1.268$ respectively. With further increase in M_1 , ψ reduces rapidly to the point where again $\psi = 1$ at $M_1 = 1.448$. Any increase in M_1 beyond this limit will further reduce the two stage ejector performance to be less than that of the equivalent single stage ejector. The supersonic range of $M_1 = 1.0 - 1.448$, at least for stationary conditions which are assumed in these calculations, would require special arrangements such as a sonic throat upstream of the first stage duct as shown in figure 3. As at present, our calculations are for the basic constant cross sectional area ejectors, the supersonic M_1 range will be discarded from the rest of the analysis.

With P_{op}/P_a fixed at a representative value of 42, and $T_{op} = T_{01} = T_{02}$, calculations were performed for different area ratios A_3/A^* . The results are shown in figure 4 in the form of τ_{2s}/τ_{1s} with M_1 and A_2/A_1 as parameters. Single stage ejectors in this figure are represented by the curve $A_2/A_1 = 0$. It is clear from figure 4 that for a given A_3/A^* the maximum thrust augmentation is always obtained at $M_1 = 1$. Point A in the figure represents the minimum $A_3/A^* = 159$ for which a solution is available with $P_{op}/P_a = 42$. This limit will be discussed further in section 5.3 below. Also for a given M_1 greater levels of thrust augmentation ratio become available with

increasing the overall area ratio.

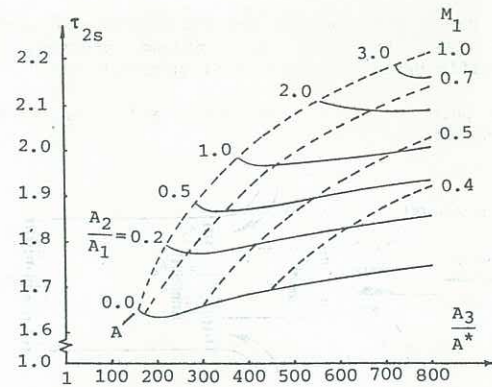


Figure 4: Performance of two stage thrust augmenting ejectors, for $P_{op}/P_a = 42$, $T_{op} = T_{01} = T_{02}$

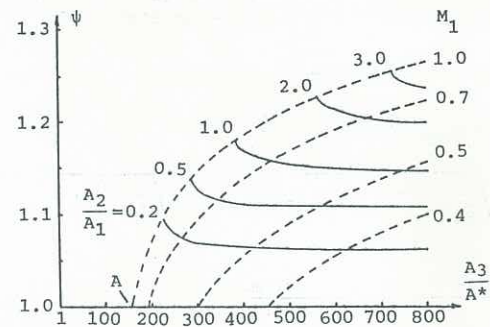


Figure 5: Performance of two stage thrust augmenting ejector, for $P_{op}/P_a = 42$, $T_{op} = T_{01} = T_{02}$

For clearer illustration of the effect of staging, the results of figure (4) are replotted as $\psi \sqrt{A_3/A^*}$ in figure 5. The single stage ejectors are then represented by the abscissa or $\psi = 1$, which is also $A_2/A_1 = 0$.

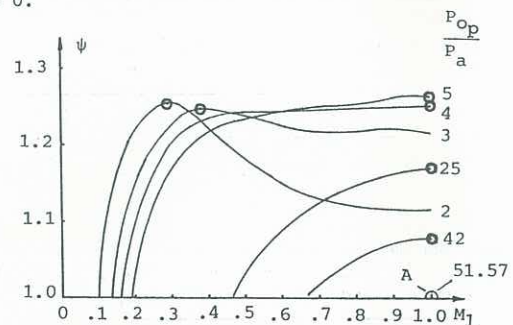


Figure 6: Results for two stage thrust augmenting ejectors for $A_3/A^* = 200 = \text{constant}$ and different P_{op}/P_a . $T_{op} = T_{01} = T_{02}$, θ ; maximum augmentation

With a fixed $A_3/A^* = 200$, and $T_{op} = T_{01} = T_{02}$ calculations were performed for different P_{op}/P_a . The results $\psi \sqrt{A_3/A^*}$ are shown in figure 6. In the range $P_{op}/P_a \geq 3.9$ the maximum value of ψ occurs at $M_1 = 1$, consistent with the previous observations. For $P_{op}/P_a < 3.9$, ψ (maximum) occurs in the subsonic range of M_1 .

5.2 Hot Rocket Gas-Air Combination

The above calculations were repeated but with the hot rocket gas as the primary fluid, entraining atmospheric air at ambient conditions. The properties of the gas are as follows

$$W = 22, \gamma = 1.24, \text{ and } T_{op} = 2400^\circ \text{K}.$$

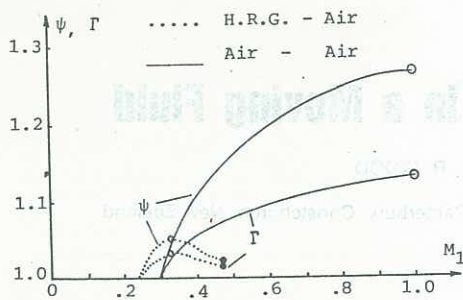


Figure 7 : Comparison of thrust and mass augmentation obtained with two stage ejectors. $P_{op}/P_a = 42$, $A_3/A^* = 800$. θ maximum augmentation, $\theta M_1 = 1$.

For a fixed $P_{op}/P_a = 42$, and $A_3/A^* = 800$, the results for H.R.G.-Air, $T_{01}/T_{0p} = .1208$, are compared with those for Air-Air, $T_{01}/T_{0p} = 1$ in figure 7. Clearly staging with H.R.G.-Air is potentially much less effective than with Air-Air in terms of both thrust and mass augmentation.

5.3 Optimum Configurations

In figures 5 and 6 for Air-Air the design criterion for a two stage ejector is the ψ (maximum) which can be obtained for any given P_{op}/P_a , A_3/A^* configuration. The ψ (maximum) values are plotted in figure 8 as a function of P_{op}/P_a , and A_3/A^* for both gas combinations. The parameter A_2/A_1 is also included for Air-Air gas combination. It is clear that the effectiveness of staging ψ increases with area ratio A_3/A^* and decreases with increasing stagnation pressure ratio P_{op}/P_a , for both gas combinations.

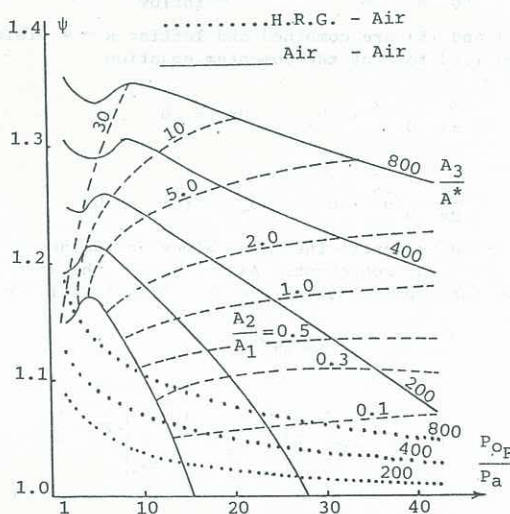


Figure 8: Results of two stage thrust augmenting ejectors with various P_{op}/P_a and A_3/A^* for Air - Air $T_{01} = T_{02} = T_{0p}$ and H.R.G. - Air, $T_{01}/T_{0p} = .1208$ gas combinations.

5.4 Limiting Conditions

5.4.1 Air-Air Combination

For any constant area ratio A_3/A^* as shown in figure 8, the effectiveness of staging decreases with increasing P_{op}/P_a until a point is reached on the pressure abscissa where $\psi = 1$ or $\tau_{2a} = \tau_{1a}$. Any further increase in pressure beyond this limit causes the two stage ejector thrust to be less than that of the equivalent single stage ejector. This limit corresponds to point A in figures 3 and 4 and figure 6, where the pressure curve collapses into a single

point A and $M_1 = 1$. For a given P_{op}/P_a and in the range $A_3/A^* < (A_3/A^*)_{limit}$ the only solutions which can be obtained are mathematical ones involving $M_1 > 1$ and $\psi < 1$. This limiting minimum area ratio can be defined as that at which $\psi = 1$ and $M_1 = 1$, and is shown as a function of P_{op}/P_a in figure 9.

5.4.2 Hot Rocket Gas-Air Combination

The limiting condition for the H.R.G.-Air combination is the exit choking condition for the mixing duct of the first stage $M_1 = 1$, which was discussed earlier and is also shown in figure 9.

Only above the limiting curves can a solution be obtained for a two stage ejector with $\psi \geq 1$.

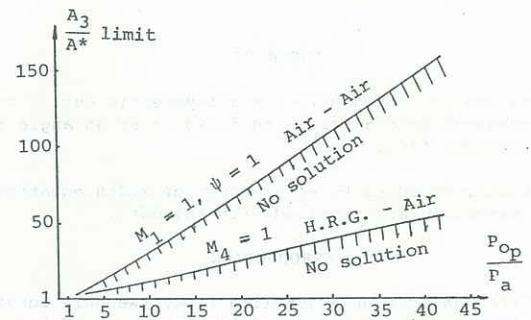


Figure 9: The limiting conditions for two stage thrust augmenting ejectors.

CONCLUSIONS

Calculations based on one dimensional flow theory have been performed for two stage ejectors having constant area mixing ducts. Within the limits imposed by the constraining assumptions - for example, supersonic duct flow solutions were not fully explored - it is concluded that the two stage ejector is not a viable alternative to the single stage ejector with the high levels of stagnation pressure and temperature pertaining in the efflux of rocket motors. Relative to a single stage ejector with the same overall diameter, staging provides reasonable improvement in thrust augmentation only with prohibitively large diameters.

The one dimensional flow assumptions could take no account of duct length. It is possible that in practice, where the degree of mixing is variable, a two stage ejector of given diameter could reduce the overall length required for a certain level of performance. However, this could be determined only by experiments, which appear barely justifiable on the basis of the above results.

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