Eighth Australasian Fluid Mechanics
Conference, University of Newcastle,
N.S.W., 28 November/2 December 1983.

THE SHAPE OF LOW REYNOLDS NUMBER JETS

R.I. TANNER

DEPARTMENT OF MECHANICAL ENGINEERING

UNIVERSITY OF SYDNEY, SYDNEY, N.S.W. 2006 AUSTRALIA

SUMMARY New experiments and new finite element computations with surface tension of jet shapes at Reynolds numbers up to 100 were made. Good agreement between experiment and computations was obtained.

1 INTRODUCTION

The shapes of free Newtonian jets emerging from a long circular tube (diameter d) depend only on the Reynolds number (ρ v d/η) and the capillary number ($\sigma/\eta v$) where ρ is the fluid density, v is the mean velocity in the tube, η in the fluid viscosity and σ is the interfacial tension. Goren and Wronski (1965) published experimental shapes and Reddy and Tanner (1978) used a finite element program to compute the shapes. The agreement in final downstream jet diameter (d_∞), or equivalently, swelling ratio χ (equal to d_∞/d, see Table 1) was excellent, but the shapes of the experimental and computed jets did not agree. The Reynolds number range was up to 50, and the object of this article is to report further experiments and calculations on this matter.

TABLE 1.

COMPARISON OF EXPERIMENTAL AND THEORETICAL JET SWELLING

Reynolds Number	Capillary Number	χ(theory)	χ(experiment)
0.0	0.0	1.134	1.135^c
4.09	0.0627	1.099	1.096
12.5	0.1696	1.011	0.998
17.2	0.1411	0.981	0.979
27.3	0.0888	0.945	0.959
47.4	0.2803	0.913	0.925^a
100.0	0.0	0.880	0.883^b
∞	0.0	0.866	0.874^b

[a] Data of Goren and Wronski (1965)

[b] Data of Middleman and Gavis (1961)

[c] Data of Batchelor and Horsfall (1971)

2 RESULTS

Experiments and calculations were made at Reynolds numbers of 4.09, 12.5, 17.2, 27.3 and 47.4. Calculations alone were made for Reynolds numbers of 0.0, 0.0494 and 100. The capillary numbers and final swelling ratios are given in Table 1. Capillary jets were produced by means of a pressure vessel/reservoir supplied with air at a pressure of up to 3 atmospheres driving the liquid through a long, horizontal tube and extruding into air. The instantaneous jet temperatures were obtained by means of a thermocouple inserted into the fluid near the tube entry. Glycerol/water mixtures were extruded into air from a sharp-edged tube of inside diameter 2.10 mm. The jet was photographed with a single lens reflex camera (60 mm × 70 mm format with extension tube) placed approximately 40 mm from the jet. Care was taken to ensure the lens axis was perpendicular to the jet direction. Flow rates were measured by collecting the extruded liquid in a measuring cylinder over a measured interval of time. The photograph was taken half-way through this interval. The temperature of the extruded liquid was measured with a thermocouple and was in the range 23°C to 39°C. The liquid used was glycerol (~96% purity).

Photographic negatives were then examined with a profile projector to obtain measurements of the jet diameter as a function of axial distance from the orifice to the point where jet curvature due to gravity rendered further measurement inaccurate. Experimental swelling ratios are subject to errors of about ±0.01.

The calculations were made with the program used previously (Reddy & Tanner, 1978) with a 6 × 13 element mesh interpolated quadratically in velocities and linearly in pressures (see Fig. 1). Usually 10 iterations or less sufficed to achieve a state at which the velocity at any given point changed less than 0.1% in the last iteration; hence the errors in the profiles due to lack of convergence are expected to be of the order of 0.001, which is much less than the expected mesh error of about 0.006.

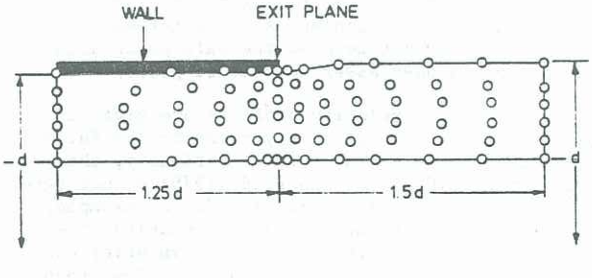

Figure 1 Problem definition and typical finite element grid.

For a Reynolds number of 4.2, the shape disagrees with that of Goren and Wronski (1965) by a scale factor in the axial direction of 2. We believe that the above curve of Goren and Wronski is plotted wrongly using diameter instead of radius as a scale factor and we accept the new experimental results as valid. For a comparison of the two sets of experimental results and the computed results see Fig. 2. We see that with the factor of 2 all three curves are in agreement.

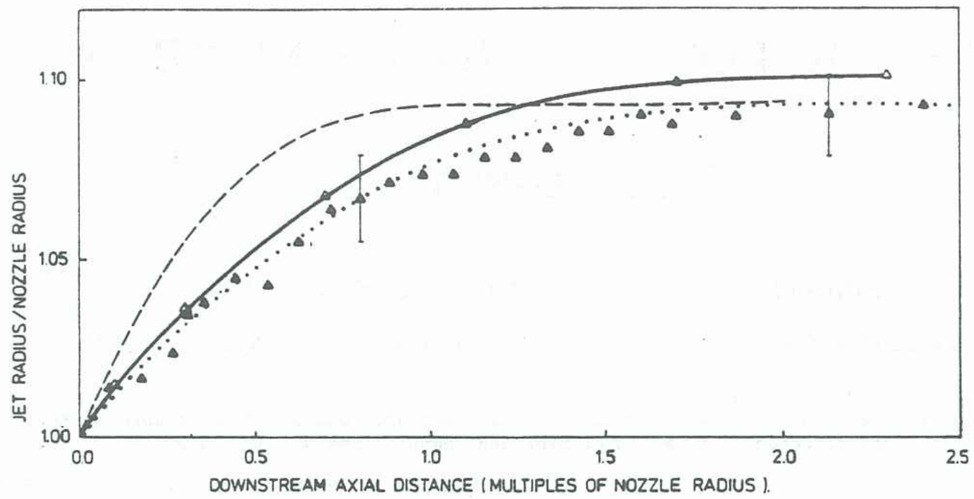

Figure 2 Comparison of two sets of experimental jet shapes for glycerol and one calculation all at Re = 4.09

— — — —	Data of Goren and Wronski (1965)
. . . . . .	Data of Goren and Wronski adjusted by scale factor, 2.0.
——△——	Finite element results
▲	Current experiments.

Comparison of the shapes for further Reynolds numbers is then shown in Fig. 3. The agreement of final diameters is good as found previously, and the agreement in shapes is fair. It should be mentioned that at around a Reynolds number of 15 the shapes are computationally and experimentally rather unstable due to necking. The wave or neck in the profiles for Re = 12.5, 17.2, is noticeable; the remaining discrepancies in shapes are probably due to factors not modelled, such as variation of surface tension along the jet, slight rounding of the exit nozzle, and errors in measurement. Some typical experimental error bars are given in Figs. 2 and 3. The meshes used thus far were of inadequate length to reach an equilibrium value for a Reynolds number of 100, hence a mesh of axial length 14 units was used to give the result shown in Fig. 4.

We also show the centreline values of the dimensionless pressure $pd/\eta v$ in Fig. 5; these show a low pressure immediately downstream of the exit plane and also demonstrate for a Newtonian jet that the extrapolated wall pressure is not zero at the exit plane although this has often been asserted without proof.

Finally, the exit velocity profiles are given in Fig. 6. Note that one is very close to the fully developed profile for Re = 100. Curiously, the experiments of Gottlieb and Bird (1979) do not agree well with these shapes. They found, for example, at Re = 0.0494 that the centreline velocity non-dimensionalised with respect to mean velocity was 1.8. Our computations indicate a value less than 1.7 (Fig. 6). The most likely explanation is that the Gottlieb-Bird experiments were not truly free jets and effects of high interfacial tension between the jet and the bath fluid ($\sigma/\eta v \simeq 5$ possibly) and the drag of the bath fluid on the jet served to alter the exit profile. It is also possible that other effects (corner effect at exit , variation of interfacial tension along jet) were also important.

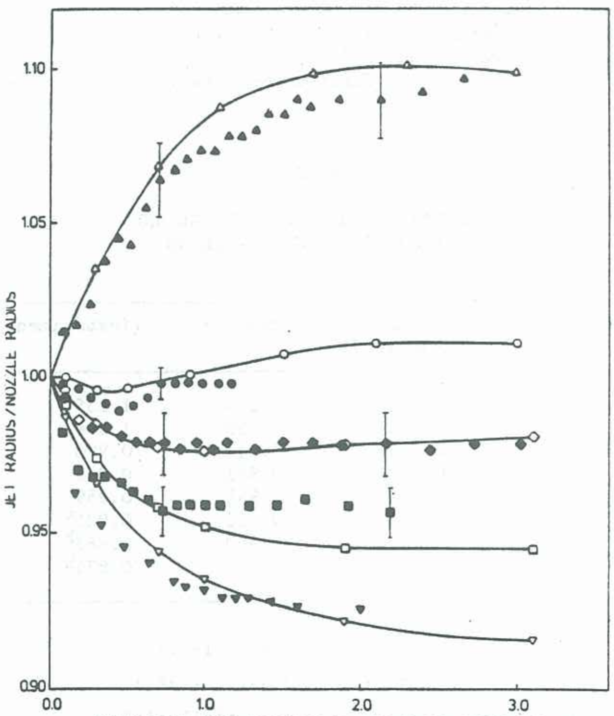

Figure 3 Jet shapes for various Reynolds and capillary numbers

Experimental	Reynolds numbers	Calculated
▲	4.09	—△—
●	12.5	—○—
◆	17.2	—◇—
■	27.3	—□—
▼	47.4	—▽—

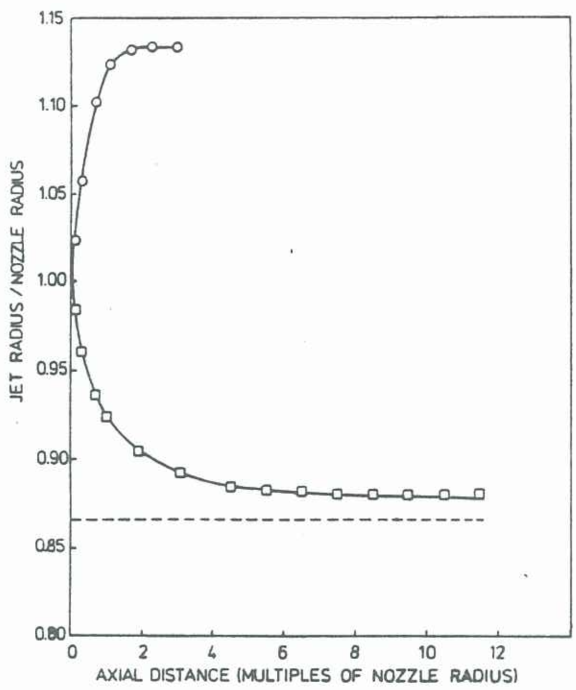

Figure 4 Computed jet shapes

Re = 0.0, ————
Re = 100 ————
Re → ∞ --------

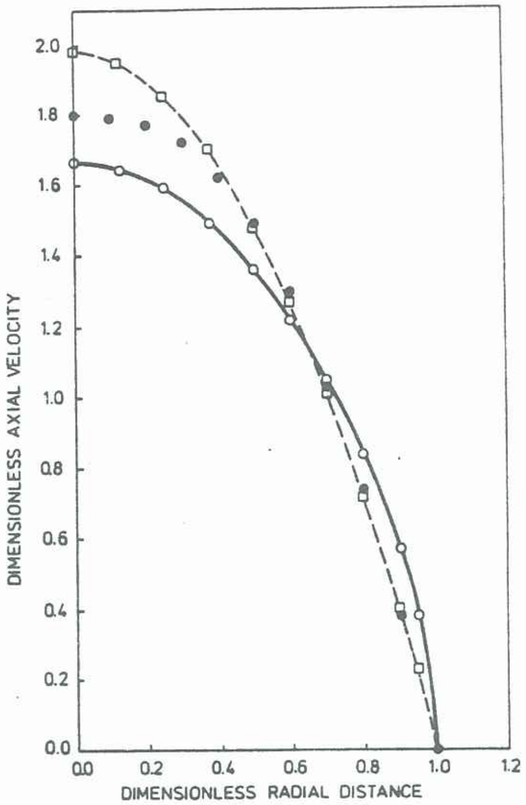

Figure 6 Axial velocities at mouth of tube

Reynolds numbers	Computed	Data of Gottlieb and Bird (1979)
0.0494	—○—	●
100	—□—	

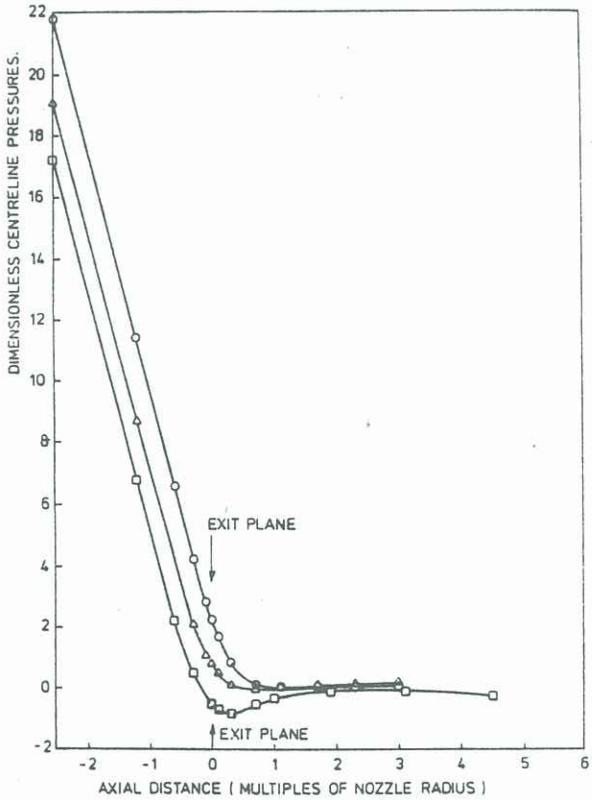

Figure 5 Normalized pressures along jet centreline for selected Reynolds numbers.

Re = 0.0 —○—
Re = 100 —□—
Re = 12.5 —△—

3 ACKNOWLEDGEMENT

This study was supported by the Australian Research Grants Scheme. This support is gratefully acknowledged.

4 REFERENCES

GOREN, S.L. and WRONSKI, J. (1965) J. Fluid Mech., 25. p.185.

REDDY, K.R. and TANNER, R.I. (1978) Computers and Fluids, 6, p.83.

GOTTLIEB, M. and BIRD, R.B. (1979) I. & E.C. Fundamentals, 18, p.357.

MIDDLEMAN, S. and GAVIS, J. (1961) Phys. Fluids, 4, p.355.

BATCHELOR, J. and HORSFALL, F. (1971) Die-Swell in Elastic and Viscous Fluids. Research Report No. 189, Rubber and Plastics Research Assoc. of Great Britain.