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# SIMILARITIES BETWEEN TWO-PHASE FLOWS OF MODEL AND FULL-SIZE HORIZONTAL PIPELINES

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SUMMARY In this paper, similarities of two-phase flows in model and full-size horizontal pipe are discussed. According to the relationships among the energy equation, the equation of continuity and suspension condition of particles, the author points out that similarities between the two-phase flows in horizontal pipes should satisfy the following relations:

$$C_V^2/C_g C_l = 1 \quad ; \quad C_\lambda C_l / C_D = 1 \quad ; \quad C_\varsigma = 1$$

$$C_{(\gamma_t-\gamma)/\gamma_t} C_m C_n C_w / C_V = 1 \quad ; \quad C_{(\gamma_t-\gamma)/\gamma_t} C_m C_{(1-n)} C_f = 1 \quad ; \quad C_V / C_w = 1$$

The principles of calculation for two-phase flow in a model pipeline are given and checked by available experimental data. In the light of the conditions of similarity about additional losses of suspended particles and moving bed, two new criteria of similarity Fu and Tu are proposed.

The development of two-phase flow theory is not so perfect as that of uniform fluid theory. Nowadays, systematic theory about two-phase flow hasn't been established. The results obtained by researchers are quite different. Hence, for the purpose of developing two-phase flow theory, we must study the fundamental mechanism of two-phase flow on the one hand, and on the other hand, if we use the similarity to direct the analysis of experimental data, the comparatively correct conclusion may be obtained. Nowadays, the principles of similarity have been used in the research of sediment [1,2,3], but haven't been used fully in two-phase pipe flow problems. In this paper, based on the present two-phase flow theory, starting from comparatively rational theoretical model, some criteria of two-phase flow in horizontal pipe have been derived. Anyway, these analyses are limited, the material and shape of granula haven't been considered.

## I DERIVATION OF SIMILARITIES

Reference[7] has presented that the head loss of two-phase flow in the horizontal pipe can be expressed

$$i_s = i_o + \Delta i_1 + \Delta i_2 \tag{1}$$

$$i_o = \lambda V^2/2gD + (\Sigma \varsigma V^2/2g)/l \tag{2}$$

$$\Delta i_1 = K_1[(\gamma_t-\gamma)/\gamma_s] M n w/V \tag{3}$$

$$\Delta i_2 = K_2[(\gamma_t-\gamma)/\gamma_s] M(1-n) f \tag{4}$$

Where $i_s$ , $i_o$ —— head loss per unit pipe length of two-phase flow and pure water respectively, m(water column) /m (pipe length); $\Delta i_1$ , $\Delta i_2$ —— additional head loss of suspended particles and moving bed per unit pipe length respectively, m/m ; $\lambda$ —— resistance coefficient; $V$ —— velocity m/s; $g$ —— gravitational acceleration, m/s² ; $D$ —— pipe diameter, m; $l$ —— pipe length, m; $\varsigma$ —— local head loss coefficient; $K_1$ , $K_2$ —— head loss coefficient of suspended particles and moving bed respectively; $\gamma_t$ , $\gamma$ , $\gamma_s$ —— specific weight of solid particle, liquid and two-phase flow respectively, N/m³ ; $M$ —— volumetric concentration of two-phase flow ; $n$ —— fraction of suspended particles in solid phase; $w$ —— settling velocity of suspended particle, m/s; $f$ —— friction coefficient of solid particle along pipe surface.

According to Bernoulli's equation, we have

$$i_s = -\frac{de}{dl} = -\frac{1}{\gamma}\frac{d}{dl}\left(\frac{\rho V^2}{2} + \frac{\gamma}{\gamma_s} p + \gamma z\right) \tag{5}$$

where $e$ —— specific energy of two-phase flow, m; $p$ —— pressure, N/m² ; $z$ —— potential energy of two-phase flow, m; $\rho$ —— mass density of fluid, Kg/m³ . Notice that $M = m\gamma_s/\gamma_t$ , expressing equation (2), (3), (4) in differentiating type and substituting equation (2), (3), (4), (5) in equation (1), we obtain

$$-\frac{d}{dl}\left(\frac{\rho V^2}{2}\right) = \frac{d}{dl}\left(p\frac{\rho}{\rho_s}\right) + \frac{d}{dl}(\gamma z) + \frac{d}{dl}\left(\lambda\frac{l}{D}\frac{\rho V^2}{2} + \Sigma\varsigma\frac{\rho V^2}{2}\right)$$
$$+\frac{d}{dl}\left(K_1\gamma\frac{\gamma_t-\gamma}{\gamma_t} m l n \frac{w_1}{V}\right) + \frac{d}{dl}\left[K_2\gamma\frac{\gamma_t-\gamma}{\gamma_t} m l (1-n) f\right] \tag{6}$$

where $m$ —— weight concentration of two-phase flow. Let C denotes the ratio of corresponding quantities in full -size pipe and model, foot notes m and y denote model and full-size pipe respectively, then

$$V_y = V_m C_V \quad ; \quad l_y = l_m C_l \quad ; \quad \rho_y = \rho_m C_\rho \quad ; \quad \cdots \tag{7}$$

If two-phase flows were dynamically similar, they should satisfy the same differential equation . If flows of model and proto-type satisfy the same differential equation (6), the following relationships must be satisied:

$$C_p / C_{\rho_s} C_V^2 = 1 \tag{8}$$

$$C_\gamma C_z / C_\rho C_V^2 = 1 \tag{9}$$

$$C_\lambda C_l / C_D = 1 \tag{10}$$

$$C_\varsigma = 1 \tag{11}$$

$$C_{K_1} C_\gamma C_{(\gamma_t-\gamma)/\gamma_t} C_m C_l C_n C_{w_1} / C_\rho C_V^3 = 1 \tag{12}$$

$$C_{K_2} C_\gamma C_{(\gamma_t-\gamma)/\gamma_t} C_m C_l C_{(1-n)} C_f / C_\rho C_V^2 = 1 \tag{13}$$

Equation (8) expresses the Euler numbers Eu of model and proto-type are equal. Because pressure P is not independent, Eu is not the determining criterion. equation (9) expresses that Froude numbers Fr of model and proto-type are equal.

When the ratio of diameter $C_D$ and ratio of length $C_l$ are equal, equation (10) expresses that the resistance coefficients $\lambda$ of model and proto-type are equal. Equation (11) expresses that the local loss coefficients are equal. The $\lambda$ is the function of relative roughness $\varepsilon/D$ and Reynolds number Re, the $\varsigma$ is the function of geometrical form of local resistance and Re. In order to satisfy the conditions that $\lambda$ and $\varsigma$ of model and proto-type are equal, the absolute roughness $\varepsilon$ must be geometrically similar too, and the Re must be equal. If the Re of model and proto-type are in self-modeling zone, the equalization of Re is not necessary, if the model and proto-type are geometrically similar, their $\lambda$ and $\varsigma$ will be equal automatically.

Equation (12) represents the equality of the ratio of additional loss of suspended particles and inertia force in model and proto-type. The coefficients $K_1$ of model and proto-type are equal, $C_{K_1}=1$, notice equation (9) and $C_{w_1} = C_w$, then equation (12) will be

$$C_{(\gamma_t-\gamma)/\gamma_t} C_m C_n C_w / C_V = 1 \tag{14}$$

Let $\frac{\gamma_t-\gamma}{\gamma_t} m\, n\, \frac{w}{V} = Fu$ (15)

Fu is a new criterion, named suspended particle loss criterion. Therefore, if two-phase flows are similar, their Fu must be equal. Equation (13) represents the equality of the ratio of additional loss of moving bed and inertia force in model and proto-type. The coefficients $K_2$ of model and proto-type are equal, then equation (13) will be

$$C_{(\gamma_t-\gamma)/\gamma_t} C_m C_{(1-n)} C_f = 1 \quad (16)$$

Let $\frac{\gamma_t-\gamma}{\gamma_t} m(1-n) f = Tu$ (17)

Tu is another new criterion, named moving bed loss cirterion. Therefore, if two-phase flows are similar, their Tu must be equal.

For reducing the quantities of similarity conditions, if the most of particle's motion is in the suspended form, equation (16) will not necessarily be satisfied. Similarly, if the most of particle's motion is in the form of moving bed, equation (14) will not necessarily be satisfied.

In addition to the similarity of energy equilibrium, the initial condition of suspending of solid particle must be similar. Therefore, the two-phase flows of model and proto-type must satisfy the same equation of suspending velocity. In pipe line, the suspending velocity of solid particle $V$ may be calculated by following formula [8] :

$$w = 0.13\, V \sqrt{\lambda / k\, r_{u,v}}\, [\,1 + 1.74 (r/r_0)^{1.8}] \quad (18)$$

where k —— experimental coefficient, 1.5-2.0; $r_{u,v}$ —— correlation coefficient of pulsating component of velocity, which is the function of Re, when Re is sufficiently large (such as $> 10^5$ ), $r_{u,v} = 0.18$; $r_0$ —— pipe radius; $r$ —— distance of particle position from pipe center; $w$ —— settling velocity of solid particle. When there is settling layer of particles in the bottom of pipe, the suspending velocity calculated by equation (18) must be doubled. Generally, the primarily suspended particles are nearby the bottom of the pipe, i.e. $r = r_0$ . If model and proto-type satisfy equation (18) at the same time, we have

$$C_V C_\lambda^{1/2} / C_k^{1/2} C_{r_{u,v}}^{1/2} C_w = 1 \quad (19)$$

Suppose k of model and proto-type are equal, $C_k = 1$. When pipe roughness are geometrically similar and Re are equal or in the self-modeling zone, $C_\lambda = 1$, $C_{r_{u,v}} = 1$, so equation (19) may be simplified into

$$C_V / C_w = 1 \quad (20)$$

To sum up, if the flows of model and proto-type are similar, equations (9), (10), (11), (20) must be satisfied, or Fr 、$\lambda$ 、$\varsigma$ and $V/w$ of model and proto-type must be equal. If the most of particle's motion is in the suspended form, equation (14) must be satisfied, or Fu must be equal. If the most of particle's motion is in the form of moving bed, equation (16) must be satisfied, or Tu must be equal. It must be noticed that for satisfying equations (10), (11), and (20), Re must be in the self-modeling zone.

From previous analyses, we can understand that there are 5 criteria of similarity, when the most of particle's motion is in the suspended form, there are 11 constants of similarity: $C_V$ 、 $C_L$ 、 $C_\lambda$ 、 $C_\varsigma$ 、 $C_g$ 、 $C_w$ 、 $C_{(\gamma_t-\gamma)/\gamma_t}$ 、 $C_m$ 、 $C_\gamma$ 、 $C_{\gamma_t}$ 、 $C_d$ * (d represents particle diameter). In general, $C_g = 1$, and there only 3 out of 5 similarity constnats $C_w$ 、 $C_\gamma$ 、 $C_{\gamma_t}$ 、 $C_{(\gamma_t-\gamma)/\gamma_t}$ and $C_d$ are independent. So there are 8 independent constants in all. i.e. the degree of freedom of selecting scale is 8-5=3. If the most particle's motion is in the form of moving bed, besides the above 8 constants, $C_f$ must be involved. Varying the material of pipe surface may change the quantity of $C_f$ . Therefore, there are 9 independent constants and 4 degree of freedom of selecting scale.

II VERIFICATION OF SIMILARITY PRINCIPLES

For verifying the accuracy of similarity principles proposed in this paper, author quotes the experimental data of Ufen and Roer [4,6] . When all particles are in suspended form, author verifies 8 cases in following ranges by Ufen's data: Dy=300-500mm; $\gamma_{sy} = 1.03\times10^4 - 1.37\times10^4$ N/m³ ; dy = 0.15 - 10 mm ; $V_y$ : from critical velocity to 7 m/s ; selected dimension scale $C_L$ =1.5-5. In the following, we take one of the 8 cases as example to illustrate the verifying procedure. Original data: Pipe diameter Dy=500 mm ; conveied uniform sand diameter $d_y$ = 1.5 mm ; specific weight of two-phase flow $\gamma_{sy} = 1.08\times10^4$ N/m³ . Determine the pressure loss per unit pipe length at $V=7$ m/s by modeling test.

1. Selecting dimension scale $C_L$ =5:2, let $C_D = C_L$ , so the diameter of modeling pipe is

$$D_m = D_y / C_L = 500/2.5 = 200 \text{ mm}$$

2. According to equation (9) and noticing $C_\gamma / C_\rho = C_g = 1$, we have $C_V = \sqrt{C_L} = \sqrt{2.5} = 1.581$ ,

hence, the velocity in modeling pipe is

$$V_m = V_y / C_V = 7/1.581 = 4.43 \text{ m/s} .$$

The Reynolds number in proto-type is

$$Re_y = 700\times50/0.01 = 35\times10^5 > 10^5 .$$

in model is

$$Re_m = 443\times20/0.01 = 8.84\times10^5 > 10^5 ,$$

therefore, the flows of model and proto-type are all in self-modeling zone.

3. Sand and water are also used as testing materials in modeling test, so

$$C_\gamma = 1 , \quad C_{\gamma_t} = 1 \text{ and } \quad C_{(\gamma_t-\gamma)/\gamma_t} = 1 .$$

4. According to equation (20), we have $C_w = C_V$ =1.581. Refering to the literature [8], if sand diameter $d_y$ = 1.5 mm , the settling velocity $w_y$ =16.5 cm/s, hence,

$$w_m = w_y / C_w = 16.5/1.581 = 10.44 \text{ cm/s}$$

Refering to the literature [8] , for $w$ =10.44cm/s, the diameter of sand is 0.95 mm , this kind of sand has been selected as modeling sand.

5. According to equation (18), when $w$ =10.44 cm/s suspending velocity of sand $V$ =2.73 m/s, now the velocity in modeling pipe is 4.43 m/s, so all the sand particles are in suspended form. $C_m$ may be calculated by equation (14). Because $C_{(\gamma_t-\gamma)/\gamma_t} = 1$, in the light of equation (14), we have $C_m = 1$, while $C_\gamma = C_{\gamma_t} = 1$, so $C_{\gamma_s} = C_{\rho_s} = 1$.

6. For the case of D=200 mm , d=0.95 mm , $\gamma_s$ = 1.08X10⁴ N/m³ , $V$ =4.43 m/s, from Ufen's data, the head loss per unit pipe length is 0.093 m water column. Hence the pressure drop per unit pipe length is

$$\Delta P_m = \gamma \cdot i_{sm} = 9.81\times10^4 \times 0.093 = 912 \text{ N/m}^3 .$$

7. According to equation (8), we have

$$\Delta P_y = \Delta P_m (V_y/V_m)^2 (\rho_{sy}/\rho_{sm}) = 912 (7/4.43)^2 = 2277 \text{ N/m}^2 .$$

Because $C_L = 2.5$, $\Delta P_y$ is corresponding to the pressure drop in 2.5 m pipe length, therefore, the pressure drop per 1 m pipe is

$$\Delta P_y / L_y = 2277/2.5 = 910 \text{ N/m}^2/\text{m}$$

Refering to Ufen's data, the actual pressure drop per 1 m pipe is 893 $\text{N/m}^2/\text{m}$ ,so the error of pressure drop converted from modeling test is (910-893)/893=1.9%.

*Although $C_\gamma$ 、 $C_{\gamma_t}$ and $C_d$ haven't appeared in deriving procedure, they are included in $C_w$ and $C_{(\gamma_t-\gamma)/\gamma_t}$ , this three scales must appear in model design procedure.

For other velocity, the verification can also be carried on in the same way and compared with the actual value. Fig shows the comparison of calculated value and actual value, the dotted line denotes the pressure drop calculated from the modeling test, the solid line denotes the actual pressure drop.

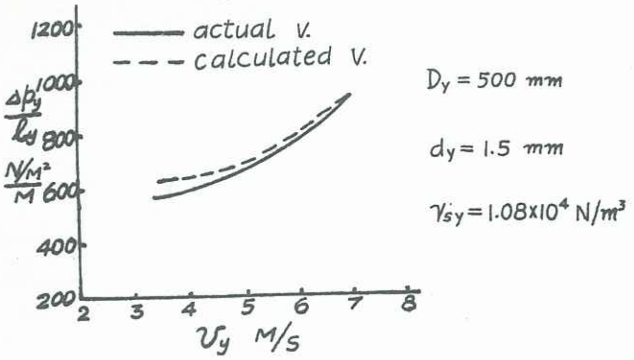


Fig. Comparison of calculated value from modeling test and actual value of pressure drop in full-size pipe

For the above mentioned 8 cases, the verification have also been carried out in the same way, we obtain that for $V > V_K$, the average error is +2.74%, while $V < V_K$, the average error is +9.21%.

When the most of particles move in moving bed, Roer's experimental data have been quoted to verify the similarity principles. The verifying procedure is the same as above. Since the most of particles move in moving bed, equation (14) must be replaced by equation (16). Proto-type pipe diameter $D_y$ =600 mm, average particle diameter $d_y$ =15 mm, non-uniformity coefficient of particle $\sigma$ =0.167, specific weight $\gamma_{sy}$ = $1.08 \times 10^4$ N/m³, velocity $V_y$ =3.95 m/s. Selecting dimention scale $C_l$ =3, so the parameters of model pipe may be calculated, they are: Dm=200 mm ; $V_m$ =2.28m/s; $\gamma_{sm}$ =$1.08 \times 10^4$ N/m³ ; head loss per 1m model pipe $i_{sm}$=0.0516, so the pressure drop per 1 m pipe length in proto-type can be calculated, $i_{sy}$ =506 $\frac{N}{m^2}/m$ . Refering to Roer's data, the actual pressure drop per 1 m pipe is 458 $\frac{N}{m^2}/m$, so the error is +10.5%.

From this it will be seen that in the case of $V > V_K$, according to the similarity principles presented in this paper, the error of $i_{sy}$ calculated from $i_{sm}$ is relatively small. But in the case of V closed to $V_K$ and moving bed exists, the error is larger than that of all particles are suspended.

## III CONCLUSION

1. The principles of similarity proposed in this paper are correct and have been verified by the experimental data, in the case of $V > V_K$, the calculated results are sufficiently accurate.

2. The similarity of two-phase flow in horizontal pipe fundamentally are determined by the following criteria: Fr, Fu 、Tu 、$V/w$ , $\lambda$ and $\varsigma$ . If two sorts of flow are geometrically similar and Re are in self-modeling zone and all criteria mentioned above are equal, this two sorts of flow will be similar.

3. Criterion Fu represents the ratio of additional resistance of suspended particles with inertia force of flow. Criterion Tu represents the ratio of additional resistance of particles in moving bed with inertia force of flow.

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