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REYNOLDS NUMBER DEPENDENCE OF A FULLY DEVELOPED TURBULENT DUCT FLOW

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SUMMARY The streamwise velocity fluctuation u has been measured in a fully developed turbulent duct flow for Reynolds numbers in the range $3300 < R_d < 27100$ using two different hot wire lengths. The effect of imperfect spatial resolution of the wires on the probability density function of u and its associated moments is documented with respect to the skewness and rms value of u in the near-wall region extending to $y^+ \simeq 50$. The high frequency part of the u spectrum scales well on viscous variables for $y^+ \leq 20$. In this region, conditional averages of u and the frequency of occurrence of events detected using the VITA technique also scale on viscous variables.

NOMENCLATURE

A,B	log-law constants
d	duct half width
F_u	flatness factor of u fluctuation = $\overline{u^4}/\overline{u^2}^2$
ℓ	wire length
$P_{u/u'}(\alpha)$	probability density function corresponding to u/u'
R_d	Reynolds number = $U_0 d/\nu$
S_u	skewness of u = $\overline{u^3}/\overline{u^2}^{3/2}$
T	integration time used in VITA
U	local mean velocity
U_0	mean velocity at centreline
U_τ	friction velocity $(\tau_w/\rho)^{\frac{1}{2}}$
u	longitudinal velocity fluctuation
u'	$\overline{u^2}^{\frac{1}{2}}$
x	longitudinal co-ordinate with origin at the boundary layer trip
y	lateral co-ordinate normal to duct wall with origin at the aluminium plate
$\phi_u(f)$	spectrum of u, normalised such that $\int_0^\infty \phi_u df = 1.0$
ν	kinematic viscosity
+	denotes normalisation by viscous variables

INTRODUCTION

Mean and turbulent quantities have been measured in a fully developed two-dimensional duct flow by several investigators (e.g. Laufer, 1952; Comte-Bellot, 1963; Clark, 1968; Hussain and Reynolds, 1975). Zaric (1972) reported a strong Reynolds number dependence of the probability density function (pdf) in the buffer region ($5 \lesssim y^+ \lesssim 40$). Johansson and Alfredsson (1982a) [hereafter referred to as I] investigated the effect of Reynolds number on the turbulence structure in a fully developed water duct. Specifically, they considered u', the skewness S_u, the flatness factor F_u and obtained conditional averages of u using the variable interval time averaging (VITA) technique. They reported a slight Reynolds number dependence for S_u and F_u in the near-wall region. In the near-wall region, conditional averages of u and the frequency of occurrence and duration of VITA events were found (Alfredsson and Johansson, 1982) to scale not on outer variables such as U_0 and d but on a mixture of outer and viscous variables (velocity scale U_τ, length scale ν/U_τ). This result differs from already conflicting trends obtained in the near-wall region of a boundary layer. Rao et al (1971) concluded that the bursting frequency scales on outer variables. More recently, Blackwelder and Haritonidis (1980) suggested that the bursting frequency scaled on viscous variables.

As a first step towards resolving the apparent conflict with regard to scaling in the near-wall region, it seemed desirable to consider a wide selection of statistical quantities (pdf, spectra, conditional averages) over as wide a Reynolds number range as practicable. In this paper, the Reynolds number range ($3.3 \times 10^3 < R_d < 2.7 \times 10^4$) is twice as large as that considered in I. Willmarth and Bogar (1977), Sharma (1980), Johansson and Alfredsson (1982b) have indicated that the turbulence measurements near a wall may be seriously affected due to the existence of turbulence structures of much smaller scale than the size of the probes normally used. Similar spatial resolution effects have been observed with respect to the measurement of turbulent wall pressure fluctuations with transducers of different diameters (e.g. Willmarth and Bogar, 1977). In the light of the previous investigations, it was important that, in the present work, the length of the hot wire was chosen in order to minimise the problem of spatial resolution.

The effect of imperfect spatial resolution on the pdf of u and, more specifically, on u' and S_u is first discussed prior to considering the appropriate scaling for the u spectrum. As in I, the influence of Reynolds number on conditional averages of u and on the average frequency of events is investigated using VITA.

EXPERIMENTAL CONDITION

The duct working section is 7.32 m long, 0.76 m high and 42 mm (= 2d) wide. One of the side walls of the working section is made of 19 mm thick perspex panels. The other is made of 11 mm thick aluminium panels. The aluminium panels can be heated. For the present measurements, the hot wire was adjacent to the polished aluminium surface. A theodolite was used to estimate the distance between the wire and its image. The uncertainty in this measurement is estimated to be ±0.02 mm. The inclination of the wire prongs to the wall was less than 5° to minimise the aerodynamic interference (e.g. Krishnamoorthy et al, 1983). The probes were constructed using Wollaston wires of two different diameters and lengths : 5 μm (Pt-10% Rh), 1 mm (9Ω) and 1.23 μm (Pt), 0.2 mm (15Ω). The length to diameter ratio (≃ 200) was about the same for the two wires. The hot wires were operated with a DISA 55M10 constant temperature anemometer at an overheat ratio of 0.8. The output from the anemometer was conditioned and recorded on a FM tape recorder (HP3960), the tape speed being dictated by R_d. Records of about 60 s were digitised for further processing on a PDP 11/34 computer. Prior to digitising, the signal was low-pass filtered using a Krohn-Hite filter (the -3 dB cut-off frequency was selected in the range 1.3 - 5 kHz for the present R_d range). The sampling frequency was equal to twice the cut-off frequency. The hot wire signal was linearised on the computer before processing.

The inlet boundary layers were tripped at 40 mm from the entrance to the working section using 1.6 mm dia.

rods spanning the complete height of the duct. The turbulence level at the inlet was in the range 0.25 - 0.35% for the present R_d range.

ESTABLISHMENT OF FULLY DEVELOPED FLOW

The static pressure decreased linearly with x for $x/d > 85$ for all values of R_d. The establishment of a linear pressure distribution is not a sensitive test for fully developed flow. Comte-Bellot (1963) found that a useful indication of the state of flow development is provided by the variation of S_u and F_u on the duct centreline. For a fully developed flow she found that S_u and F_u were independent of x. In the present work, the velocity fluctuation u was measured near the duct centreline ($0.7d < y < 1.1d$) at 9 axial stations. The magnitudes of S_u, F_u and normalised higher order moments up to order eight of u were practically constant for $x/d > 90$, 100 and 130 corresponding to $R_d = 7000$, 17100 and 27100 respectively. The magnitudes of S_u and F_u on the duct centreline are in good agreement with those reported by Dean (1974) and Comte-Bellot (1963). Prior to considering the influence of Reynolds number on turbulence quantities, mean velocity profiles were measured in the fully developed part of the flow. They indicated a substantial logarithmic region of the form $U^+ = A\log y^+ + B$. The variations in A and B were of the same order as reported in other studies (e.g. Dean, 1974; I) and did not indicate a systematic Reynolds number influence.

PROBABILITY DENSITY FUNCTION OF u AND ASSOCIATED MOMENTS

Significant differences are noticeable in $p_{u/u'}$ at $y^+ = 15$ obtained using hot wires of different lengths (Figure 1). Whereas $p_{u/u'}$ is positively skewed for $\ell^+ = 58$, it is negatively skewed for $\ell^+ = 11$. This seems consistent with the strong likelihood that the shorter wire is more capable to resolve relatively small scale ejection events than the longer wire. Similar differences in $p_{u/u'}$ were noted by Johansson and Alfredsson (1982b). In view of these results, it is important to evaluate the error due to inadequate wire length before considering the effect of Reynolds number. For the longer wire used here, ℓ^+ varied between 9 and 58 as R_d increased from 3000 to 27100. The corresponding variation in ℓ^+ for the shorter wire is 2 - 11.

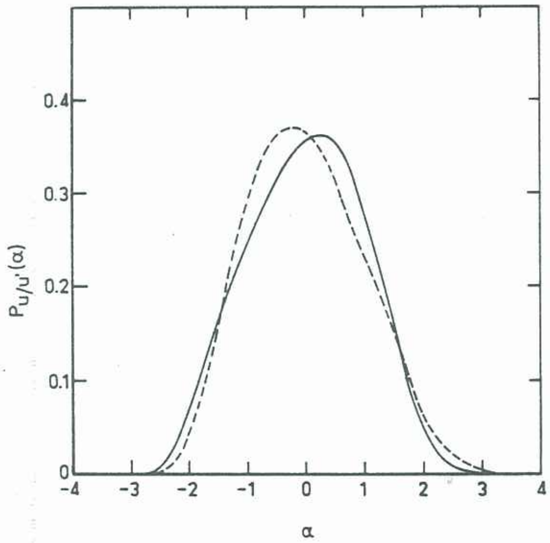

Figure 1 Probability density function of u at $y^+ = 15$ and $R_d = 27100$. ——, $\ell^+ = 11$; ---, 58.

The variation of u'^+ with ℓ^+ is largest at $y^+ \simeq 15$ where u'^+ is maximum and decreases for larger and smaller y^+. Johansson and Alfredsson (1982b) collated values of u'^+ obtained by several investigators at $y^+ \simeq 15$. These values are plotted with the present values in Figure 2. This figure suggests an apparently universal dependence of u'^+ on ℓ^+.

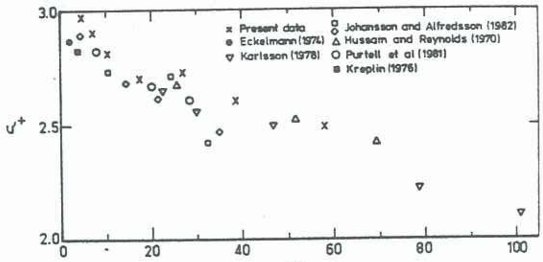

Figure 2 Dependence of u'^+ on wire length ℓ^+ at $y^+ \simeq 15$.

Significant differences in the skewness measured with the two wires are highlighted in Figures 3a,b. The results obtained with the longer wire (Figure 3a) indicate a Reynolds number trend qualitatively similar to that reported in I and Andreopoulos et al (1983). In particular, the value of y^+ corresponding to $S_u = 0$ increases as the Reynolds number increases. This apparent Reynolds number dependence is absent in the results (Figure 3b) obtained with the shorter wire. It should be noted that the effect of wire length should be small when R_d is small. In this context, the agreement between the present distribution at $R_d = 3300$ and $\ell^+ \simeq 9$ and that of Kreplin (Eckelmann, 1974) at $R_d \simeq 3550$ and $\ell^+ \simeq 3$ is good (Figure 3a). The conclusion in I that S_u and F_u exhibited a slight Reynolds number dependence was modified later by Johansson and Alfredsson (1982b) based on the differences in the pdf measured using two different probes and in view of the inadequate wire length, especially for larger R_d, used in I.

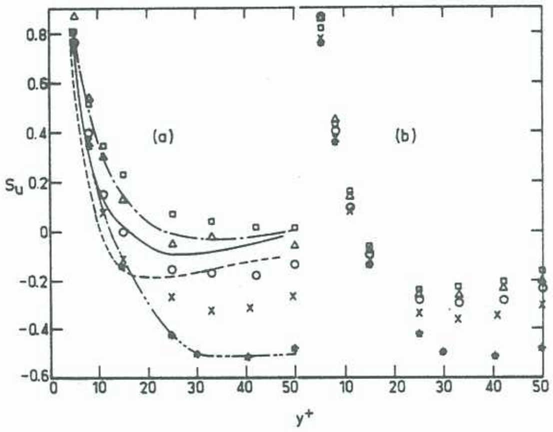

Figure 3 Distributions of S_u in the inner region. (a) $\ell^+ = 58$ at $R_d = 27100$. (b) $\ell^+ = 11$ at $R_d = 27100$. Symbols as for Figure 5. — - —, $R_d = 6900$; ——, 17300; ---, 2500, ($\ell^+ = 32$ at $R_d = 25000$) Ref. I. — -- —, 3550, Kreplin (Eckelmann, 1974).

Like u'^+, S_u also exhibits an apparently universal trend (Figure 4) when plotted as a function of ℓ^+. The results of Figure 4 are for $y^+ \simeq 15$, where the change of S_u with respect to ℓ^+ is expected to be larger than at other values of y^+. Figure 4 provides a means to correct other results in the literature at this particular value of y^+. Although the present results showed little wire length effect on S_u for $R_d \le 7000$ and $\ell^+ \le 17$, more data at small Reynolds numbers would be desirable. Sharma (1980) reported values of $S_u = 0.12$ and -0.25 for $\ell^+ = 0.4$ and 2.5 respectively obtained using very short hot wires at $y^+ \simeq 15$ in a turbulent boundary layer. Unfortunately, the difference between these two

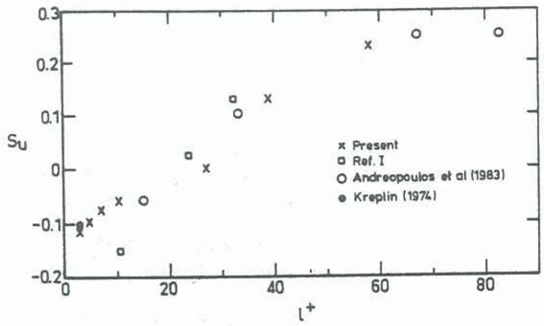

Figure 4 Dependence if S_u on ℓ^+ at $y^+ \simeq 15$.

values of S_u is too large vis-à-vis the relatively small values of ℓ^+ with which they are associated.

The distribution of F_u (not shown here) measured with the longer wire indicated a definite Reynolds number dependence in the region $8 < y^+ < 25$. A Reynolds number dependence was not noticeable for the F_u data obtained with the shorter wire.

Distributions of u', S_u and F_u in the outer region were found to be consistent with the concept of Reynolds number similarity.

SPECTRA

The spectrum ϕ_u of u was calculated by means of a standard FFT routine. The frequency is made non-dimensional using either a viscous (ν/U_τ^2), an inner (y/U), an outer (d/U_0) or a mixed $(\nu d/U_\tau^2 U_0)^{\frac{1}{2}}$ time scale. To highlight the relative contribution of different frequencies to u'^{+2} or u'^2/U_0^2, a semi-log presentation is preferred here. Spectra obtained only with the shorter wire are presented.

At $y^+ = 5$, viscous scaling and inner scaling are identical in view of the sublayer relation $U^+ = y^+$. These two types of scaling remain related in the region where the law of the wall $U^+ = f(y^+)$ applies. When viscous scaling is used (Figure 5), spectra at different R_d collapse onto a unique distribution for $R_d \geq 7000$ and for frequencies higher than $2\pi f\ \nu/U_\tau^2 > 0.1$. They also tend towards a unique distribution at low frequencies. This result is consistent with the behaviour reported by Thomas (1977) for wall shear stress spectra in a turbulent boundary layer. It is however in conflict with the conclusion of I that no part of the spectrum scales on viscous variables. Scaling on inner variables yielded a similar result as viscous scaling but with slightly more scatter in the data. Outer scaling was completely unsatisfactory for both low and high frequency parts of the spectrum at $y^+ = 5$. This is at variance with the results of Thomas (1977) in a boundary layer and those of Rajagopalan and Antonia (1979) in a duct which indicated that outer scaling seemed appropriate for low frequencies. Alfredsson and Johansson (1982) suggested that the governing timescale in the near-wall region may be the geometric mean of viscous and outer timescales. This new timescale was interpreted to be an estimate of the smallest timescale for turbulent fluctuations. Scaling on this mixed timescale is also unsatisfactory (Figure 6) for the present data at $y^+ = 5$. (Results similar to $y^+ = 5$ were obtained at $y^+ = 8$ and 15). In the outer region $(y/d > 0.2)$, the spectra scaled on outer variables for $R_d \geq 7000$. This scaling was also found to be appropriate in the outer region of a duct (e.g. Rajagopalan and Antonia, 1979; I).

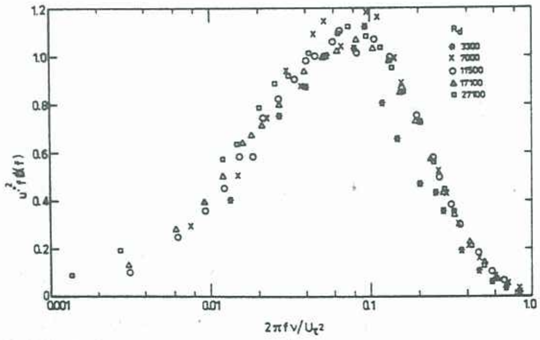

Figure 5 Spectra of u at $y^+ = 5$ using viscous scaling.

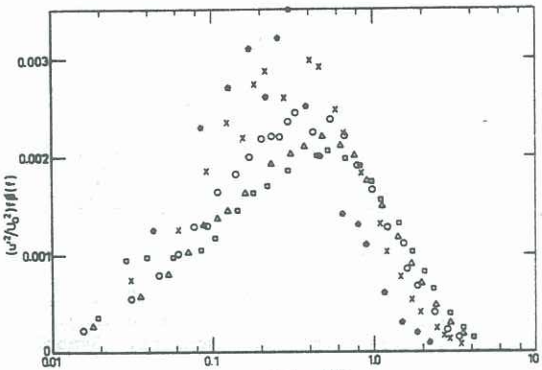

Figure 6 Spectra of u at $y^+ = 5$ using mixed scaling. Symbols as for Figure 5.

CONDITIONAL AVERAGES

Since the main interest here is in the effect of Reynolds number on conditional averages, a relatively simple one point technique has been used. Limitations of one point techniques are discussed by Subramanian et al (1982). VITA was found by Subramanian et al (1982) to be one of the "better" one-point techniques. For the particular form of VITA used here, detection occurs when $\widetilde{u^2}-\tilde{u}^2 > k\,u'^2$ and $\dot{\tilde{u}} > 0$ are first satisfied (~ denotes a moving average, • denotes a time derivative and k is a threshold parameter). Once the reference times t_j are detected, the conditional average is given by

$$\langle u(\tau)\rangle = \frac{1}{n}\sum_{j=1}^{n} u(t_j+\tau) .$$

The exponential decrease of n with increasing k was obtained at all values of y^+ considered here is similar to that reported in I. VITA was applied to the fluctuation u obtained with the shorter wire at $y^+ = 15$ for $k = 1.4$ and a range of integration times T. The average frequency n of VITA events normalised by either viscous or mixed timescales, is shown in Figure 7. Scaling on viscous variables is more satisfactory than scaling on mixed variables. Scaling on outer variables was found to be unsatisfactory.

Viscous scaling is satisfactory (Figure 8) for conditional averages obtained at $y^+ = 15$. Outer scaling was also tried and was found to be unsatisfactory. In the outer region $(y/d > 0.2)$, outer variables provide satisfactory scaling for conditional averages of u, corresponding to Reynolds numbers greater than about 7000.

CONCLUSIONS

Inadequate spatial resolution of the hot wire seriously impairs the measurement of u in the range $5 < y^+ < 35$, the largest error occurring near the location of maximum turbulence intensity. The high frequency part of the u spectrum scales well on viscous variables for $y^+ \lesssim 20$ and $R_d \geq 7000$. For the low frequency part of

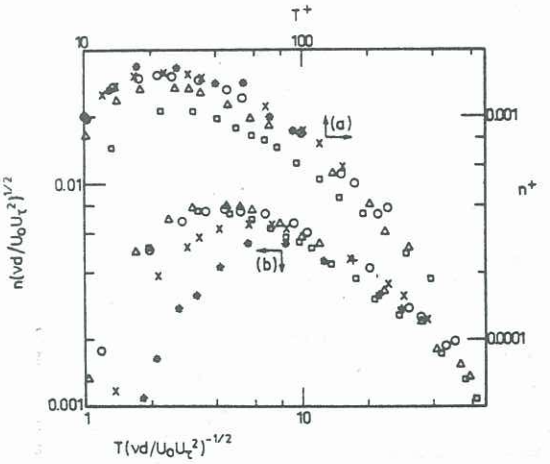

Figure 7 Frequency distributions of VITA events (k = 1.4, $y^+ = 15$) using viscous and mixed scaling. (a) viscous scaling; (b) mixed scaling. Symbols as for Figure 5.

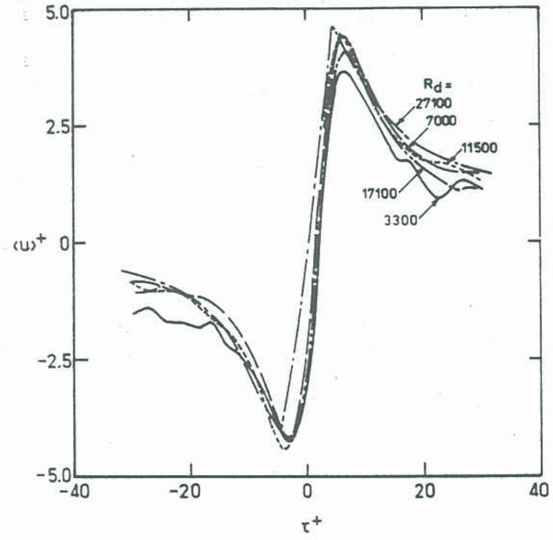

Figure 8 VITA conditional averages of u at $y^+ = 15$ using viscous scaling. k = 1.4, $T^+ \simeq 13$.

the spectrum, scaling on viscous variables is also more satisfactory than one based on outer variables. Satisfactory scaling on outer variables is exhibited by the whole spectrum for $y/d \geq 0.2$ and $R_d \geq 7000$. At variance with the conclusion of Alfredsson and Johansson (1982), scaling on mixed variables is not appropriate in the near-wall region. The frequency distribution of VITA events and VITA conditional averages of u scale reasonably well on viscous variables in the near-wall region. As was found by I, scaling on outer variables is satisfactory in the outer region. The discrepancy with regard to scaling in the near-wall region remains to be explained.

ACKNOWLEDGEMENT

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