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EXPERIMENTAL TESTS OF HIGHER-ORDER CLOSURE ASSUMPTIONS FOR SCALAR TRANSPORT

M.R. RAUPACH

DIVISION OF ENVIRONMENTAL MECHANICS

CSIRO, CANBERRA, A.C.T. 2601 AUSTRALIA

SUMMARY This paper describes wind-tunnel experiments on passive scalar dispersion in a rough-wall turbulent boundary layer. The sources considered are an elevated lateral line source (EL) and a ground-level plane source (GP), the tracer being heat in both cases. Difference measurements of pressure terms in the tracer flux budgets are compared with existing parameterizations often used in higher-order closure models. Suitable parameterizations are then used to examine the predictions of second-order closure theory for the turbulent Prandtl number ρ. The analysis explains "anomalous" near-surface behaviour for ρ which has been observed in the atmosphere, and which is also seen in the wind-tunnel data considered here.

1 INTRODUCTION

For some years, higher-order closure methods have been suggested as viable ways of predicting the dispersion of scalar additives by turbulent flows; their use in the atmosphere boundary layer and in engineering is now widespread. In situations where the turbulence is far from local equilibrium, leading to a failure of first-order gradient-diffusion theory and (possibly) the appearance of countergradient fluxes, second-order closure of the Reynolds stress tensor and scalar flux equations represents the minumum physical input necessary to obtain even a qualitatively correct model. Examples occur in the upper part of the PBL (Deardorff, 1966) and within, or just above, rough surfaces such as vegetation canopies (Wilson and Shaw, 1977).

There is a need (which will continue for some time) for good experimental information against which to test closure parameterizations. As well as in "fundamental" flows (grid turbulence, plane and axisymmetric jets, etc.), observations are required in well-controlled flows which simulate those natural flows for which predictions are ultimately required, for example, boundary-layer dispersion. Such experiments, besides providing detailed understanding of natural phenomena worthy of study in their own right, establish the generality or otherwise of closure parameterizations.

This paper considers passive-scalar dispersion in an adiabatic turbulent boundary layer over a rough wall. In separate experiments, wind-tunnel data have been obtained for two source configurations: (i) an elevated lateral line source (EL) and (ii) a ground-level plane source (GP). The tracer was heat in both cases. Full descriptions of procedures and results are given by Raupach and Legg (1983) for EL, and (1984) for GP. Attention is focussed here on the pressure term in the tracer flux equation; suggested parameterizations are tested both directly, via the measured flux budgets, and indirectly, by examining second-order closure predictions for the turbulent Prandtl number.

2 THE TRACER FLUX BUDGET

In invariant form, the budget for tracer flux $\overline{u_i\theta}$ is

$$\frac{\partial\overline{u_i\theta}}{\partial t} = A_{\theta i} + P_{\theta i} + Q_{\theta i} + T_{\theta i} + \phi_{\theta i} + B_{\theta i} + M_{\theta i} \qquad (1)$$

where

$$\left.\begin{aligned} A_{\theta i} &= -\overline{U}_j\partial\overline{u_i\theta}/\partial x_j && \text{(advection)}\\ P_{\theta i} &= -\overline{u_iu_j}\partial\overline{\Theta}/\partial x_j && \text{(gradient production)}\\ Q_{\theta i} &= -\overline{u_i\theta}\partial\overline{U}_i/\partial x_j && \text{(shear production)}\\ T_{\theta i} &= -\partial\overline{u_iu_j\theta}/\partial x_j && \text{(transport)}\\ \phi_{\theta i} &= -\overline{\theta\partial p/\partial x_i} && \text{(pressure)}\\ B_{\theta i} &= -g_i\overline{\theta^2}/T_0 && \text{(buoyant production)}\end{aligned}\right\} \qquad (2)$$

Here $U_i = \overline{U}_i + u_i$ is the wind vector, $\Theta = \overline{\Theta} + \theta$ the temperature (we take the tracer to be heat), p the fluctuating kinematic pressure, g_i the gravity vector $(0,0,-g)$, T_0 the reference temperature and $M_{\theta i}$ the molecular terms, which vanish if the flow is locally isotropic (we assume this). The terms that must be parameterized in higher-order closure models are $T_{\theta i}$ and $\phi_{\theta i}$; we here focus on $\phi_{\theta i}$, for which a common parameterization is (Launder, 1976; Lumley, 1978; Zeman, 1981)

$$\phi_\theta = \phi^I_{\theta i} + \phi^S_{\theta i} + \phi^B_{\theta i} \qquad (3)$$

$$\phi^I_{\theta i} = -\overline{u_i\theta}/c\tau \qquad (4)$$

$$\phi^S_{\theta i} = \frac{4}{5}\overline{u_j\theta}\partial\overline{U}_i/\partial x_j - \frac{1}{5}\overline{u_j\theta}\partial\overline{U}_j/\partial x_i \qquad (5)$$

$$\phi^B_{\theta i} = \frac{1}{3}g_i\overline{\theta^2}/T_0 \qquad (6)$$

where I, S and B superscripts denote the turbulence-interaction, mean-strain and buoyant contributions to $\phi_{\theta i}$; c is a proportionality constant and τ is the timescale $\overline{q^2}/\varepsilon$ ($q^2 = u_iu_i$ and ε is the dissipation rate for $\overline{q^2}/2$). The expression (5) for $\phi_{\theta i}$ is derived by expansion about an isotropic state (Lumley, 1978; Wyngaard, 1981).

Lumley (1978) also showed that (4), (5) and (6) do not satisfy the requirement of realizability, and suggested the following realizable alternatives to (4) and (5):

$$\left.\begin{aligned} \phi^I_{\theta i} &= -\overline{u_j\theta}\,F_{ij}/\tau\\ F_{ij} &= \alpha_1\delta_{ij} + \alpha_2 D_{ij} + \alpha_3 D_{ik}D_{kj}\\ D_{ij} &= \overline{u_iu_j}/\overline{q^2} - \overline{u_i\theta}\;\overline{u_j\theta}/\overline{\theta^2}\,\overline{q^2}\end{aligned}\right\} \qquad (7)$$

$$\phi^S_{\theta i} = -Q_{\theta j}(\delta_{ij} - \overline{u_i\theta}\;\overline{u_j\theta}/(\overline{u_k\theta}\;\overline{u_k\theta})) \qquad (8)$$

In (7), α_1, α_2 and α_3 are functions of the invariants of the flow, for which Lumley suggested rather complex forms. The form of (7) stems from the realizability requirement that F_{ij} and D_{ij} have the same principal axes. Note that (7) reduces to (4) if $\alpha_1 = c^{-1}$, $\alpha_2 = \alpha_3 = 0$. Lumley obtained (8) for $\phi^S_{\theta i}$ by inspection from several realizability conditions.

In the following, we examine the parameterizations (4) and (7) for $\phi^I_{\theta i}$ and (5) and (8) for $\phi^S_{\theta i}$.

3 EXPERIMENTAL ARRANGEMENT

Figure 1 shows the experimental layout and definition of axes in the Pye Laboratory wind tunnel (CSIRO Division of Environmental Mechanics, Canberra), the working section of which is 11 m long, 1.8 m wide and 0.7 m high. The experimental rough surface was densely packed 7 mm road gravel, glued to baseboards. With the aid of an upstream trip (a 50 mm high fence), this surface generated a thick turbulent boundary layer,

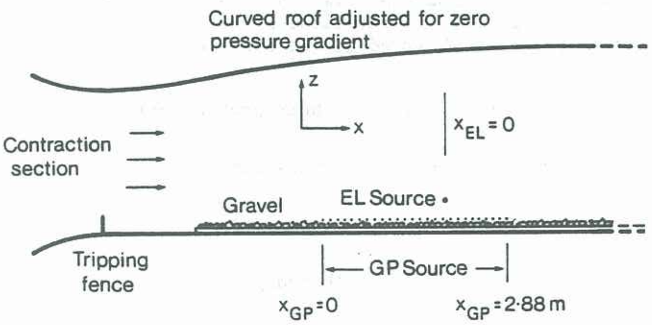

Figure 1 Experimental arrangement in the wind tunnel, with vertical axis not to scale. The height origin z = 0 coincides with the aerodynamic zero-plane of the surface.

designed to simulate the adiabatic atmospheric surface layer. Instantaneous streamwise and vertical velocity components were measured with an X-wire anemometer, and the temperature with a fine-wire (1.2 μm) resistance thermometer, these sensors being positioned close together on a traversing mechanism.

For EL, the source was a 0.9 mm diameter Nichrome wire, tensioned across the tunnel at height z = 60 mm. For GP, the source consisted of fine heating wires running laterally over the gravel, spaced at 2 cm intervals and resting on the gravel; these provided an effective plane heat source. The maximum power applied to either source was small enough for buoyancy forces to be negligible; thus, the heat acted as a passive scalar.

For presenting data, we use the notation $x_i = (x,y,z)$, $\overline{U}_i = (\overline{U}(z),0,0)$, $u_i = (u,v,w)$. The origin of the steamwise coordinate x is the source wire for EL, and the upstream edge of the heated section for GP; where necessary, the origin will be distinguished by a subscript (x_{EL} or x_{GP}).

The boundary layer mean wind profile $\overline{U}(z)$ was semi-logarithmic up to $z \approx 250$ mm, with a zero-plane (z = 0) 6 mm above the substrate surface to which the gravel was glued. A boundary-layer depth scale δ was defined as the height of maximum mean wind speed, and was δ = 540 mm, with $\overline{U}(\delta) = 11$ m s^{-1}. At δ, the flow was nearly, but not completely, nonturbulent (intermittency ≈ 0.1); thus, there was no classical free stream above the boundary layer. The friction velocity u_* (defined as $(-\overline{uw})^{\frac{1}{2}}$ in the constant-stress layer) was 0.50 m s^{-1} at $x_{GP} = 2.88$ m, from the X-wire data. Figure 2 shows profiles of $\overline{uw}$, $\sigma_u = (\overline{u^2})^{\frac{1}{2}}$ and $\sigma_w = (\overline{w^2})^{\frac{1}{2}}$ at two stations, normalized with this value of u_*; note the constant-stress layer and the slight streamwise evolution of the boundary layer.

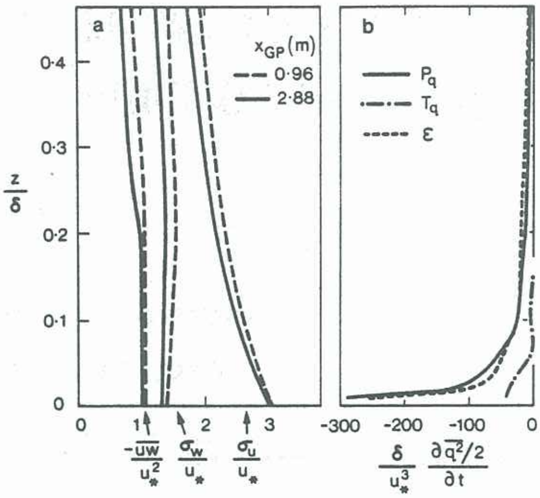

Figure 2 (a) Turbulence statistics. (b) Terms in the $\overline{q^2}/2$ budget at $x_{GP} = 2.88$ m. Note reflection of P_q.

Figure 2 also shows the major terms in the budget of turbulent energy, $\frac{1}{2}\overline{q^2} = \frac{1}{2}\overline{u_i u_i}$:

$$0 = \frac{1}{2}\frac{\partial \overline{q^2}}{\partial t} = A_q + P_q + T_q + \phi_q - \varepsilon \qquad (9)$$

where

$$\left.\begin{aligned} A_q &= -\tfrac{1}{2}\overline{U}_j \partial\overline{q^2}/\partial x_j \\ P_q &= -\overline{u_i u_j}\partial\overline{U}_i/\partial x_j \\ T_q &= -\tfrac{1}{2}\partial\overline{u_j q^2}/\partial x_j \\ \phi_q &= -\partial\overline{p u_j}/\partial x_j \end{aligned}\right\} \qquad (10)$$

All terms were directly measured except ϕ_q. There was excellent agreement between direct measurements of (made in the same boundary layer in the course of experiments by Chambers et al. (1983)) and the sum $A_q + P_q + T_q$. Below $z/\delta \approx 0.2$, the only significant terms were ε and the vertical contributions to P_q and T_q (these are the only terms shown); in this region, conditions were close to local equilibrium ($P_q = \varepsilon$), with $T_q/P_q \approx -0.2$.

The growth of the heated plumes for EL and GP is shown by the mean temperature profiles in Figure 3. For GP, (but, clearly not for EL) the profiles of $\overline{\theta}$ and of θ statistics ($\overline{\theta'^2}$, $\overline{w\theta}$, $\overline{u\theta}$, etc.) are approximately self-preserving (Raupach and Legg, 1984).

4 BUDGETS OF $\overline{u\theta}$ AND $\overline{w\theta}$

Figure 4 shows the $\overline{w\theta}$ and $\overline{u\theta}$ budgets for EL at $x_{EL} = 0.90$ m; figure 5 shows the same for GP at $x_{GP} = 2.88$ m. Terms are denoted as in (2), with individual components of summed terms being denoted by bracketed subscripts. The pressure term $\phi_{\theta i}$ was obtained by residual. Note that $B_{\theta i} = 0$ since the flow was adiabatic and the tracer passive. Terms not shown either vanish identically or are negligible by measurement.

Discussing the GP results first, it is clear that both the $\overline{u\theta}$ and $\overline{w\theta}$ budgets are close to local equilibrium, for which case advection and transport are negligible and the $\overline{u_i\theta}$ budget reduces to $P_{\theta i} + Q_{\theta i} + \phi_{\theta i} = 0$. The only place where this is not so is close to the surface, where transport acts as a small loss term for $|\overline{u_i\theta}|$. (Note that corresponding terms in the $\overline{u\theta}$ and $\overline{w\theta}$ budgets are of opposite sign because $\overline{u\theta}/\overline{w\theta} < 0$;

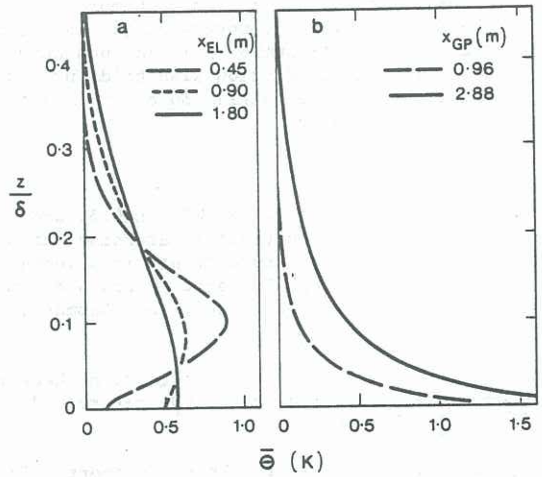

Figure 3 Development of mean temperature profiles in (a) EL and (b) GP.

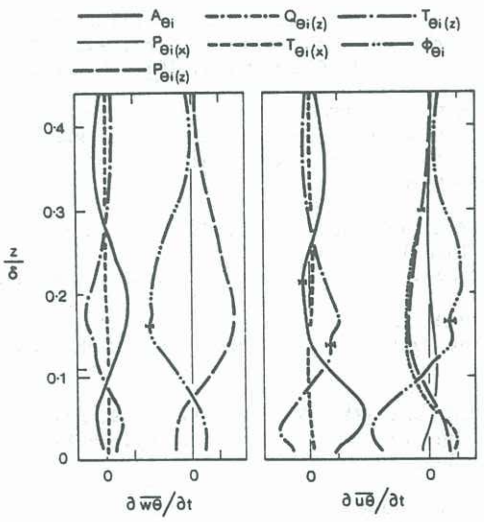

Figure 4 Terms in the budgets of $\overline{w\theta}$ and $\overline{u\theta}$, EL experiment, x_{EL} = 0.90 m. Error bars indicate error estimates where non-negligible. Abscissa units are arbitrary but conserved in this figure (see Raupach and Legg, 1983, for details). Source at z/δ = 0.11.

also, note that $Q_{\theta i}$ vanishes when i = 3, that is, in the $\overline{w\theta}$ budget). There is a strong similarity between the role of transport here and in the $\overline{q^2}/2$ budget (see Figure 2). The role of transport as a near-surface loss of $\overline{q^2}/2$ is known to be a roughness-dependent effect, which increases with surface roughness and effectively vanishes when the surface is smooth (Raupach, 1981); hence it is likely that the observed $\overline{u_i\theta}$ transport in Figure 5 is a consequence of the surface roughness.

For EL (Figure 4), advection and transport are far from negligible. However, they tend to balance one another, to quite a good approximation for $\overline{w\theta}$ and less accurately for $\overline{u\theta}$. Hence, the $\overline{w\theta}$ budget for EL is in a state of "quasi-local equilibrium", in which advection and transport can be ignored in sum even though each is large on its own. This is true at other values of x_{EL} as well as that shown in Figure 4 (Raupach and Legg, 1983).

The $\overline{u\theta}$ and $\overline{w\theta}$ budgets provide (by residual) measurements of the pressure term $\phi_{\theta i}$, which can be compared with the models (4) to (8). A detailed comparison for EL shows that F_{ij} in (7) cannot possibly be of the form

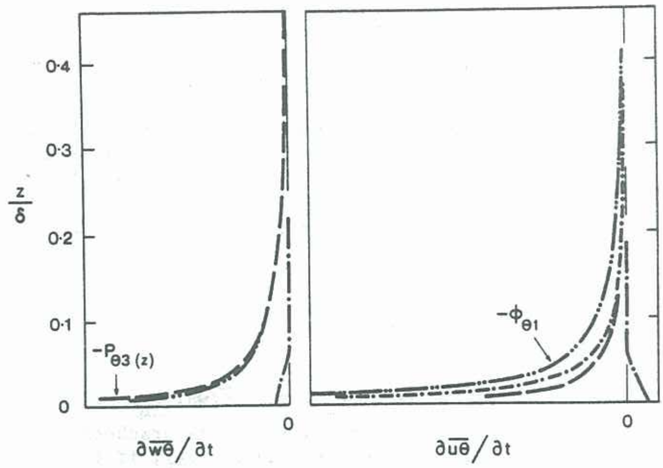

Figure 5 Terms in the budgets of $\overline{w\theta}$ and $\overline{u\theta}$, GP experiment, x_{GP} = 2.88 m. Abscissa units are arbitrary but conserved in this figure (see Raupach and Legg, 1984, for details). Legend as in figure 4. Note reflection of $P_{\theta 3(2)}$ and $\phi_{\theta 1}$.

$\alpha_1\delta_{ij}$, and hence that (4) cannot be valid (Raupach and Legg, 1983). The same is true for GP, as can be quickly shown thus: both suggested parameterizations for $\phi^S_{\theta i}$, (5) and (8), are of the order of $Q_{\theta i}$ and of opposite sign, so that $Q_{\theta i}$ and $\phi^S_{\theta i}$ tend to cancel each other. Neglecting $Q_{\theta i} + \phi^S_{\theta i}$ and assuming local equilibrium (both of which are reasonable approximations for GP), the $\overline{u_i\theta}$ budget reduces to $P_{\theta i} = \overline{u_i\theta}/c\tau$ if (4) is used. Ignoring streamwise production, this implies (taking the ratio of the $\overline{u\theta}$ and $\overline{w\theta}$ equations) that $\overline{u\theta}/\overline{w\theta} = \overline{uw}/\overline{w^2} \approx -0.6$. However, the observed ratio $\overline{u\theta}/\overline{w\theta}$ is near -3 at small z; hence c in (4) must depend on coordinate direction, necessitating an equation like (7).

Concerning (7), we note that D_{ij} is typically nearly diagonal in the boundary layer; its value at z = 20 mm, x = 2.88 m, for GP, is

$$D_{ij} = \begin{pmatrix} 0.32 & 0 & 0.01 \\ 0 & 0.33 & 0 \\ 0.01 & 0 & 0.09 \end{pmatrix} \qquad (11)$$

If we assume that D_{ij} is diagonal, then F_{ij} must be diagonal and (7) reduces to

$$\phi^I_{\theta\alpha} = -\overline{u_\alpha\theta}\, F_{\alpha\alpha}/\tau \qquad (12)$$

where the Greek index α suppresses the summation convention. Hence $\phi^I_{\theta\alpha}$ depends only on $\overline{u_\alpha\theta}$, which is a property useful in analysis. Clearly, this property is not invariant since it depends on the alignment of the axes.

An attempt was made to test (7) against our GP data using Lumley's (1978) suggested forms for α_1, α_2 and α_3. Unfortunately, the predictions for $\phi^I_{\theta i}$ were of the wrong sign, so it appears that the precise forms of α_1, α_2 and α_3 are still open.

5 A PREDICTION FOR THE TURBULENT PRANDTL NUMBER

An interesting feature of the GP data is the variation in the turbulent Prandtl number $\rho = (\overline{uw}\partial\bar{\Theta}/\partial z)(\overline{w\theta}\partial\bar{U}/\partial z)^{-1}$, which (at x_{GP} = 2.88 m) is about 1 except close to the surface (z < 60 mm) and high in the plume (z > 200 mm) The near-surface behaviour, and in particular the surface value $\rho \approx 0.5$, is reminiscent of hitherto unexplained atmospheric observations over very rough surfaces such as forests (Raupach and Thom, 1981). We here investigate the effect of the mean-strain term $\phi^S_{\theta i}$, and the transport terms $T_{\theta i}$ and T_q, on ρ. Consider the $\overline{u_i\theta}$ and $\overline{q^2}/2$ budgets in steady, adiabatic conditions, with the following extra assumptions: (i) the flow is essentially one-dimensional, so that $U_i = (\overline{U}(z),0,0)$ and x- and y-derivatives vanish in the budgets; (iii) $\phi^T_{\theta i}$ can be parameterized by (12) with a diagonal F_{ij}; (iii) $\phi^S_{\theta i}$ can be parameterized by either (5) or (8). We have shown that assumptions (i) and (ii) are reasonable for GP. Some algebra, starting from the $\overline{w\theta}$ equation, gives

$$\rho = \frac{a_{13}^2 p_q}{f_3 a_{33} p_{\theta 3}} - \frac{a_{13}}{a_{33} p_{\theta 3}} \left[\frac{r}{5}\right] \tag{13}$$

where $a_{ij} = \overline{u_iu_j}/q^2$, $f_\alpha = F^{-1}_{\alpha\alpha}$, $r = \overline{u\theta}/\overline{w\theta}$, $p_q = 1 + T_q/P_q$, $p_{\theta\alpha} = 1 + T_{\theta\alpha}/P_{\theta\alpha}$. The factor in square brackets is correct if (5) is used to parameterize $\phi^S_{\theta i}$; if (8) is used, this factor becomes $[r/(r^2 + 1)]$. Note that the factors $p_{\theta 3}$ and p_q account, respectively, for departures from local equilibrium in the $\overline{w\theta}$ and $\overline{q^2}/2$ budgets, assumed here to be due to transport. (In principle, the same analysis can handle other kinds of departure as well.)

Figure 6 shows, for x_{GP} = 2.88 m, the data for r and and the prediction (13) with both parameterizations for $\phi^S_{\theta i}$. The predictions were obtained using measured values for a_{13}, a_{33} and r, the height-independent value $f_3 = 0.04$, and with $p_{\theta 3} = p_q = 1$, implying local equilibrium for both the $\overline{w\theta}$ and the $q^2/2$ budgets. This is a better assumption than it appears at first, at least in the near-surface region where transport is significant, because (13) shows that losses of $\overline{w\theta}$ and $\overline{q^2}/2$ due to transport have opposite, compensating effects on ρ. The prediction for ρ fits the data fairly well if the form (5) (based on expansion about isotropy) is used for $\phi^S_{\theta i}$ rather than the form (8) (based on realizability). Similar results to Figure 6 are obtained from the GP data at other values of x.

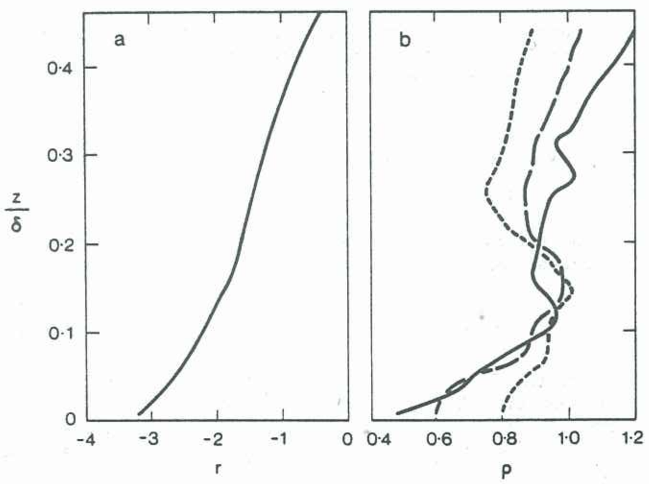

Figure 6 (a) The ratio $r = \overline{u\theta}/\overline{w\theta}$; (b) Observations and predictions for the turbulent Prandtl number ρ. Data from GP experiment, x_{GP} = 2.88 m.

——— Observed, — — Predicted with (5)
------- Predicted with (8).

6 CONCLUSIONS

The two main conclusions are, firstly, that F_{ij} in (7) is not of the form $\alpha_1\delta_{ij}$; it may well be modelled adequately by $\alpha_1\delta_{ij} + \alpha_2 D_{ij}$, but more work is needed to investigate α_1 and α_2. Secondly, the "expansion-about-isotropy" expression (5) for $\phi^S_{\theta i}$ is much more successful than the "realizability" expression (8) in predicting the turbulent Prandtl number ρ in the boundary layer, via (13). This prediction also holds promise for explaining the observed variation of ρ close to atmospheric rough surfaces.

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