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# AN INVESTIGATION OF THE THIN-JET MODEL OF THE UNSTEADY JET FLAP

D.I. PULLIN AND J.M. SIMMONS

DEPARTMENT OF MECHANICAL ENGINEERING

UNIVERSITY OF QUEENSLAND, ST. LUCIA, QLD. 4067 AUSTRALIA

SUMMARY We study the thin-jet model of the jet flap which has been used widely in the analysis of both steady and unsteady jet flap behaviour. Evidence is presented based on linear stability analysis and on nonlinear numerical simulations which indicates that the thin-jet model is ill-conditioned, leading to unconditionally unstable growth of all length scales in unsteady flow. These results seem to cast some doubt on the suitability of the present form of the model for aerodynamic applications.

## 1 INTRODUCTION

A jet flap results if air is ejected from a spanwise slot at or near the trailing edge of an aerofoil. It produces lift both by the component of thrust in the lift direction and by the modification of circulation around the aerofoil. The steady jet flap, for which flow parameters are constant, has received considerable attention because of interest in V/STOL aircraft. It was analysed on the basis of two-dimensional inviscid flow, thin aerofoil, and thin-jet theory by Woods (1961) and Spence (1956).

Interest in unsteady jet flaps appears to have begun when W.R. Sears suggested the use of jet flaps for fast-acting lift control (Spence 1965). Potential applications to helicopter rotors and aircraft vibration mode stabilisation systems have led to experimental studies of two-dimensional incompressible flow past unsteady aerofoil-jet flap configurations (Trenka and Erickson 1970, Simmons 1976 A,B, Simmons et al 1978). Attempts to extend steady thin jet flap theory to unsteady configurations are due to Erickson (1962) and Spence (1965), the latter considering three cases; namely, a fixed flat-plate aerofoil with an oscillating jet deflection angle at the trailing edge, a flat-plate aerofoil oscillating in plunge with the jet tangential to the plate at the trailing edge, and a flat-plate aerofoil oscillating in pitch about its trailing edge with a tangential jet at the trailing edge. With an approximation of small jet-momentum flux Spence found complete solutions, including evaluation of instantaneous jet shape and lift, for the first two cases. For the third case he evaluated only the jet shape, the lift being later determined by Trenka and Erickson (1970). Since comparisons of the lift predictions of the unsteady theory with experiment have proved unsatisfactory (Simmons, 1976B), a study of the thin-jet model behaviour was undertaken.

## 2 THIN-JET THEORY

In the thin-jet theory of the jet flap the fluid motion is assumed to be incompressible, inviscid two-dimensional and irrotational. The jet flap is modelled by a thin fluid sheet of thickness order $\delta$ containing fluid moving with speed $U_J$. Letting $U_J \to \infty$, $\delta_J \to 0$ such that the jet momentum flux $M_J = \rho U_J^2 \delta_J$ remains finite ($\rho$ is the fluid density) gives the thin-jet limit. The jet becomes effectively a vortex sheet which may support a local pressure difference given by (Erickson, 1962, Spence, 1965)

$$p_2 - p_1 = M\kappa \ , \qquad (1)$$

where $\kappa$ is the instantaneous jet curvature and $p_2$ is the pressure on the convex side. Note that (1) is an essential part of the thin-jet dynamics and holds in unsteady flow.

### 2.1 Linearized Behaviour of The Thin-Jet

Some insight into the thin-jet model may be obtained from a linearized analysis. Consider the unsteady motion of an infinite jet in an unbounded fluid. In x - y co-ordinates the jet shape is $y = \eta(x,t)$ and the fluid motion above and below the jet is described by velocity potentials $\phi_1$, $\phi_2$ satisfying $\nabla^2\phi_1 = \nabla^2\phi_2 = 0$. The boundary conditions on the jet, which are linear in $\eta$ and $(\phi_1, \phi_2)$ are

$$\partial\eta/\partial t = \partial\phi_1/\partial y \ , \qquad (2a)$$

$$\partial\phi_1/\partial y = \partial\phi_2/\partial y \ , \qquad (2b)$$

$$p_2 - p_1 = -\rho(\partial\phi_2/\partial t - \partial\phi_1/\partial t) . \qquad (2c)$$

These are respectively the kinematic and normal velocity continuity conditions across the jet and the pressure difference as obtained from the linearized unsteady Bernoulli equation. We seek normal-mode solutions representing periodic motion in the x direction of the form

$$\eta = \eta_0 \, e^{\sigma t} e^{ikx} \ , \qquad (3a)$$

$$\phi_1 = a_1 \, e^{\sigma t} e^{ikx} e^{-ky} \ , \quad y > 0 \ , \qquad (3b)$$

$$\phi_2 = a_2 \, e^{\sigma t} e^{ikx} e^{ky} \ , \quad y < 0 \ , \qquad (3c)$$

where $\eta_0$, $a_1$ and $a_2$ are complex constants, $k = 2\pi/\lambda$ is the wave number with $\lambda$ the x - wavelength and $\sigma$ is an unknown time constant. The physical disturbance corresponds to the real part of (3).

We now use $\kappa \simeq \partial^2\eta/\partial x^2$ in (1), substitute into (2c) and satisfy (2) on y = 0 using (3) to evaluate derivatives, giving the dispersion relationship

$$\sigma^2 = \frac{M}{2\rho}\left(\frac{2\pi}{\lambda}\right)^3 . \qquad (4)$$

Since $M > 0$, (4) and (3) show that the jet is always unstable to infinitesimal x-periodic disturbances, and moreover that the disturbance growth rate increases without limit as $\lambda^{-3/2}$ when $\lambda \to 0$. Thus in the linearized approximation the jet behaviour is similar to but more pathological than the vortex sheet subject to a constant velocity difference where $\sigma \sim \lambda^{-1}$, $\lambda \to 0$. Note that (4) appears to cast doubt on the physical validity of the model as used by Spence (1965) in which the jet returns in the frequency domain to an undisturbed flat configuration far downstream of the wing.

### 2.2 Nonlinear Vortex Sheet Model

In view of the success of numerical vortex sheet-models of unsteady wakes (e.g. Fink and Soh 1978, Faltinsen & Pettersen, 1982) in which smoothing and rediscretization techniques (mimicking viscous smoothing) are used to control fine-scale instabilities, while producing a satisfactory description of the larger scale motions, we thought it worthwhile to investigate the vortex-sheet formulation of the thin jet in order to study its nonlinear behaviour in a simple configuration. We choose the case of the periodic plunging motion of a wing-jet flap combination, treated as an initial value problem so as to avoid difficulties associated with downstream boundary conditions. In what follows we nondimensionalize with respect to a length scale c, the wing chord, and time scale $c/U_\infty$ where $U_\infty$ is the x free-stream speed. At time t we describe the jet shape by the complex function $z(e,t) = x(e,t) + iy(e,t)$ and cumulative circulation $\Gamma(e,t) = \phi_2 - \phi_1$ where e is a Lagrangian label marking a material particle on the jet flap-vortex sheet. The velocity of this particle is the mean of the instantaneous fluid velocities on either side of the flap. Thus

$$u_p - i\,v_p = D\bar{z}/Dt = dW/dz\ , \tag{5}$$

where "—" denotes the complex conjugate and W is the complex velocity potential. The Lagrangian derivative following z(e,t) and unsteady Bernoulli equation at a field point Z are respectively

$$(D/Dt) = \partial/\partial t + u_p\partial/\partial x + v_p\partial/\partial y\ , \tag{6}$$

$$p/\rho + \tfrac{1}{2}|dW/dZ|^2 + \partial\phi/\partial t = \text{const}\ . \tag{7}$$

Applying (6) to $\Gamma(e,t)$, using (7) to evaluate $\partial(\phi_2 - \phi_1)/\partial t$ in terms of $p_2 - p_1$ and (1) to eliminate $p_2 - p_1$ leads to

$$\frac{D\Gamma}{Dt} = -C_J\,\frac{\mathrm{Im}[\partial\bar{z}/\partial e\ \partial^2 z/\partial e^2]}{|\partial z/\partial e|^2}\ . \tag{8}$$

In (8) $C_J = M(\tfrac{1}{2}\rho U_\infty^2 c)$ and the right hand side is $C_J$ times $\kappa$ expressed in terms of z, Im being the imaginary part of a complex argument. Thus the effect of the momentum jet is to generate local circulation following a particle at a rate proportional to $\kappa$.

### 2.3 Oscillating Plunging Wing

Equations (5) and (8) are an initial value problem for $[z,\Gamma]$, once we have specified dW/dZ on z. We consider the simplest case which will give a non-trivial jet flap motion; a flat wing of unit chord and zero thickness moving in a periodic plunging motion in a stream of x-speed $U_\infty$. At t = -0 the wing is in $\tfrac{1}{2} \geq x \geq -\tfrac{1}{2}$. The jet flap is turned on and immediately occupies (since $U_J = \infty$) $\infty > x > \tfrac{1}{2}$. The wing remains parallel to the x-axis and moves for t > 0, following a smooth initial transient (to avoid a strong starting vortex) with speed and y-elevation given respectively by

$$V(t) = \omega Y_0\cos(\omega t - \psi)\ ,\ Y(t) = Y_0\sin(\omega t - \psi)\ , \tag{9}$$

In (9) $\omega$ is the angular frequency, $\psi$ a phase angle associated with the initial transient and $Y_0$ is the amplitude. The jet emanates from the trailing edge at $(\tfrac{1}{2}, Y(t))$ always parallel to the x-axis.

At time t the velocity field on z(e,t) may be obtained through a conformal transformation of the wing to a circle of radius ¼ and centred at the origin in the $\zeta$ plane, together with a distribution of vortex singularities on the transformed jet, and its image in the circle. Thus we may write

$$\frac{dW}{dz} = \frac{d\zeta}{dz}\left[\left(\frac{dW}{d\zeta}\right)_a + \left(\frac{dW}{d\zeta}\right)_s\right] - iV(t)\ , \tag{10}$$

$$\zeta = \tfrac{1}{2}\left[z - iY(t) + \sqrt{(z-iY(t))^2 - \tfrac{1}{4}}\right]\ , \tag{11}$$

where the attached and separated flows are respectively

$$\left(\frac{dW}{d\zeta}\right)_a = [U_\infty + iV(t)] - [U_\infty - iV(t)]/16\zeta^2\ , \tag{12a}$$

$$\left(\frac{dW}{d\zeta}\right)_s = \frac{1}{2\pi i}\int_{\Gamma_0(t)}^{0}\left\{\frac{d\Gamma'}{\zeta - \zeta'} - \frac{d\Gamma'}{\zeta - 1/16\zeta'}\right\}. \tag{12b}$$

Equations (10-12) may be readily shown to satisfy the required boundary condition on the wing and at $|Z| \to \infty$. In (12b) primed quantities refer to integration variables on $\zeta(e,t)$. The total sheet circulation at the trailing edge is $\Gamma_0(t)$ and $\Gamma = 0$ on the jet far downstream. The Kutta condition determines $\Gamma_0(t)$ from

$$\left[\left(\frac{dW}{d\zeta}\right)_s + \left(\frac{dW}{d\zeta}\right)_a\right]_{\zeta=\frac{1}{4}} = 0\ . \tag{13}$$

Since vortex sheet numerical discretization is now standard in the literature (see previous references) we give only brief details. The jet is divided into order N segments with end points $z_j(t)$, j = 0 ... N and circulations $\Gamma_j(t)$, j = 0 ... N, and (5), (8) and (10-12) are discretized into 3N first order ODE's for these quantities. In (8) $\kappa_j$ is calculated using 5-point formulae for $\partial z/\partial e$, $\partial^2 z/\partial e^2$ at $z_j$ with e identified with j. To estimate (12b), the integral is first evaluated at the N segment midpoints assuming a linear variation of $\Gamma$ with $\zeta$ in each $(\zeta_j, \zeta_{j+1})$, j = 0,...N-1. The value at $\zeta_j$ is then obtained using linear interpolation and $\dot{z}_j$ evaluated using (10-12) and (5). Simple Euler integration with constant $\Delta t$ is used to march forward in time. The Kutta condition is implemented at each $\Delta t$, the contribution of the element adjacent to the trailing edge (in the $\zeta$ plane) being evaluated using a parabolic distribution of $\zeta - \zeta_0$ and $\Gamma - \Gamma_0$ with arc length s. This leads to a simple equation for $\Gamma_0(t)$. A new $(\zeta,\Gamma)$ in $(\zeta_0,\zeta_1)$ are then found by parabolic interpolation.

Downstream the calculation is terminated at X = 14.5 in the sense that particles with $x_j > X$ are eliminated from the calculation. This strategy can be justified by noting that it leads to negligible loss of circulation through convection past x = X over simulated time periods.

## 3 RESULTS AND DISCUSSION

The three cases reported here were calculated with N = 180, initial x-spacing of points $\Delta x \simeq .0778$ and $\Delta t \simeq .0778$. All cases have $\omega = 2$ (period = $\pi$) and $Y_0 = 0.2$, chosen to give significant nonlinearity. The initial transient period was $t_0 = 0.589$ ($\psi = 0.646$). Values of $C_J$ = 0, 0.05 and 0.1 were used.

For $C_J$=0, the flow is the vortex wake generated by a periodic plunging wing, which was used as a test case. Like other workers (e.g. Fink & Soh 1978) we found it necessary to smooth fine scale motion on the vortex-sheet wake which, owing to local shear instability would otherwise rapidly amplify and destroy the overall computation. In the present work we used a standard 5-point smoothing formula (Longuet-Higgins & Cokelet 1976) to smooth both $z_j$ and $\Gamma_j$ co-ordinates at each $\Delta t$. We accept that this procedure can only be justified heuristically as a numerical analogue of viscous action in real fluids. The calculated wake configuration after two periods in Figure 1 is similar to the comparable calculation of Faltinsen and Pettersen (1982). The familiar mushroom-like wake pattern evolves on the dominant length scale $\lambda \simeq 2\pi/\omega = \pi$ excited by the oscillating wing motion in the presence of the free stream.

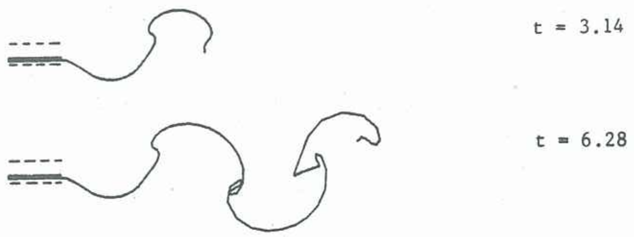


Figure 1 Vortex sheet at two non-dimensional times, t, with $C_J = 0$, $\omega = 2$ and $Y_0 = 0.2$. The dashed lines above and below the flat-plate indicate the extremes of its plunging oscillation.

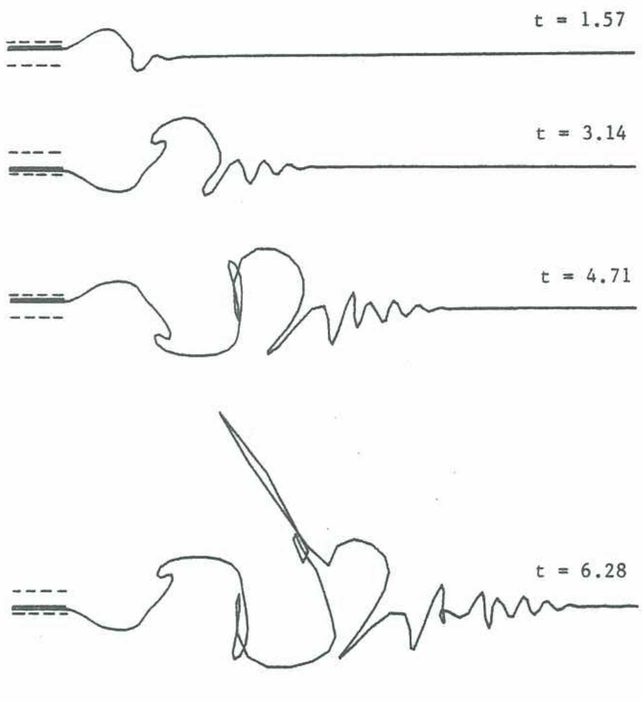


Figure 2 Vortex sheet at four non-dimensional times with $C_J = 0.05$, $\omega = 2$, and $Y_0 = 0.2$.

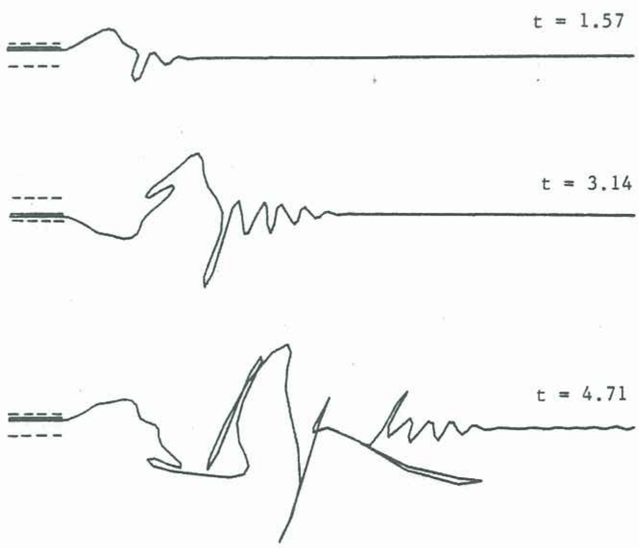


Figure 3 Vortex sheet at three non-dimensional times with $C_J = 0.1$, $\omega = 2$ and $Y_0 = 0.2$.

The calculated details of the "rolled-up" portions of the wake could be improved by use of a vortex-core amalgamation procedure (Hoeijmakers and Vaatstra 1983). Note that from (8), $D\Gamma/Dt = 0$ for $C_J = 0$. Hence only particles with finite $\Gamma$ obtained through separation at the trailing edge are shown in Figure 1.

Figures 2 and 3 illustrate graphically the dramatic effect of the thin momentum jet present in the wake. Here we show about 80% of the calculated jet-wake. Note that circulation can be generated locally at all the $z_j$, $j = 1 .. N$. At $t = 1.57$ and 3.14 in Figure 2, the jet wake remains fairly coherent although it is evident that instability on a spectrum of length scales is evolving through the combined effect of the momentum jet and local shear on the vortex sheet. By $t = 4.71$ and 6.28 disturbances are rapidly amplifying and the jet-wake flow has become pseudo-turbulent. The accuracy of the calculation is lost due to the randomly convoluted jet shape and the vortex sheet unrealistically crosses over itself. Note that the smallest amplified length scale on the relatively undisturbed downstream portion of the jet-wake is of order 5-6 vortex points, i.e. just above the smoothing scale. The behaviour shown in Figure 3 for $C_J = 0.1$ jet is qualitively similar but more "turbulent" due to the stronger jet.

A case with $C_J = -0.1$ was also calculated to test that (8) was giving the correct qualitative behaviour. This corresponds to an effective surface tension effect in the jet and, while quite unphysical, it should lead to stable behaviour. For $C_J = -0.1$, the jet behaved quite smoothly with no instability or tendency to roll-up over 3 wing oscillation periods.

## 4 CONCLUSIONS

We considered the thin-jet theory of some interest since it has been essentially the only jet-flap model studied extensively to date. The present results however, summarized in equation (4) and in Figures (2-3) clearly show that the model leads to violently unstable behaviour in the jet-wake. In view of the highly turbulent nature of jet flows this is not surprising physically, and perhaps indicates that at best the jet-flap wake flow is of comparable complexity (and intractability) to say bluff-body wake flows. In fact there may be a close similarity here in that classical free-streamline (thin-jet) theory, while mathematically elegant, has proved of limited value in the prediction of bluff-body (jet-flap) wake flows due essentially to initially unrecognized instabilities. Unfortunately it seems likely that the application of modern numerical techniques to the thin-jet model of the jet flap will not easily lead to results of comparable success to those achieved in recent numerical simulations of bluff-body wake flows.

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