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MIXING TIME PREDICTION

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SUMMARY. Mixing times observed in aqueous systems of low viscosity and simple configuration are listed. Agitation was by impellers, by recirculation via jets and by gas sparging. For the cases of impeller agitation and recirculation via jet, equations for mixing time could be formulated directly without correlation. The listed data can be a basis for further thought.

NOTATION

b = width of impeller blade normal to direction of motion, L
b_o = half width of plane jet nozzle, L
c = concentration, ML^{-3}
d = vessel diameter, L
d_o = nozzle diameter of axisymmetric jet, L
D = impeller blade diameter, L
N = revolutions per second, t^{-1}
p = pressure, $ML^{-1} t^{-2}$
P = power input to liquid $ML^2 t^{-3}$
$\hat{R}$ = gas constant per unit mass, $L^2 t^{-2} {}^{-1}$
R = entrainment ratio,
Re = impeller Reynolds number
u = mixing rate, t^{-1}
v_o = nozzle exit velocity, $L t^{-1}$
v_s = superficial velocity, $L^3 t^{-1} L^{-2}$
V = liquid volume, L^3
w_o = nozzle exit flowrate, Mt^{-1}
x = jet length, L
Z = agitator or sparger submergence, L
Z_L = maximum height of liquid at rest, L
ϕ = power number
ρ = fluid density ML^{-3}
θ = mixing time t

1 INTRODUCTION

The function of mixing equipment is to achieve uniformity of properties in an initially non-uniform material in finite time. The time required to achieve a required degree of uniformity is the mixing time θ. Whilst the molecular diffusion coefficient in gases is 10^{-5} to 10^{-4} $m^2 s^{-1}$ for gas pairs at ambient conditions, for liquid pairs it is of the order 10^{-9} $m^2 s^{-1}$. Therefore, the break-up of bulk liquids is important not only for physical uniformity in small volumes (medicines, paints, drinks) but also for creation of short paths for molecular diffusion preceeding chemical reaction.

There are many types of liquid mixing equipment; this, together with their use in mainly turbulent conditions makes quantitative analytical interpretation of performance difficult.

Here, experimental mixing times are listed, they were obtained by use of various simple agitators operating in aqueous solutions. The experimental results can be used to compare agitator efficiency and to test the practicality of predicting mixing time without use of empirical correlations.

2 EXPERIMENTAL

Physical and chemical methods can be used to find mixing times in experimental equipment. The method used here was a chemical decolouration method, said to be the only method that follows the process to the molecular level (Zlokarnik, 1967) and also avoids the omission of, possibly intermittent, stagnant regions.

Procedure and choice of dosage have been discussed earlier (Brennan and Lehrer, 1976); briefly, procedure is : A transparent, strongly illuminated vessel is charged with clear tapwater. Agitator speed or flowrate are set at the desired value, using stroboscope and tachometer, or rotameters respectively. Methyl red indicator is added, followed by the dose of 2M HCl solution. After several minutes, the dose of 2M NaOH solution is added quickly to the uniformity red-coloured solution. The solution turns pale yellow as the neutralisation reaction proceeds. The time between pulse addition of NaOH and disappearance of the last trace of red colour is measured by stop-watch and taken as the mixing time θ. For the batch sizes used in TABLE I, i.e. 57 to 58 liters, quantities were 75 ml indicator for a series of tests, and per test, 25 ml 2M HCl, 25.2 ml 2M NaOH. The tank contents were renewed usually after six tests. Excess 2M NaOH used is 0.2 ml in 25.2 ml, required to compensate for retention of more viscous liquid in beaker and for achievement of a definite end point. The resultant shortening of mixing time is not significant. For the batch sizes shown in TABLES II and III, reagent quantities were adjusted in proportion to volume.

Mixing time tests were done by nine different groups of observers, the consistency of measured values is satisfactory. Accuracy has been estimated to be 10% at $\theta \cong 10s$, at very large values, say $\theta \cong 1800s$, it depends strongly on the observer (Zlokarnik, 1967).

MIXING TIME EQUATIONS

A basic relation between degree of uniformity $\overset{*}{c}$ and time t from start of a mixing process is

$$\overset{*}{c} = 1 - \exp(-ut) \qquad (1)$$

In discussion of diffusion of mass,

$$\overset{*}{c} = \left(\frac{c_{\text{mean, calculated}} - c_{\text{measured}}}{c_{\text{mean, calculated}} - c_{t=0}}\right) \qquad (2)$$

where c = concentration of dispersed material.

From eqs(1), (2) $- \log(1-\overset{*}{c}) = ut$ (3)

The quality of mixing is given by $\overset{*}{c}$, and if θ is the concomitant mixing time, u is a mixing-rate,

$$\theta = -\frac{\log(1-\overset{*}{c})}{u} \tag{4}$$

It seems that the chemical decolouration method is reliable to $\overset{*}{c} = 0.95$ to 0.99. Considering the use of, even clear, tapwater and variety of observers, $\overset{*}{c} = 0.95$ is used here. The mixing rate u is to be found.

IMPELLER MIXING

Based on dimensional and phenomenological considerations, it has been proposed that

$$\theta = \theta_T + \theta_M \tag{5}$$

θ_T = time for turbulent dispersion,
θ_M = time for molecular diffusion (Brennan and Lehrer, 1976). Using the relevant equations in this reference, it was found that for the conditions discussed here, θ_M was insignificant and can be omitted in this case. The time for turbulent dispersion is

$$\theta_T = -\frac{Z^2}{\psi ND[\phi Z_L(d-D)D]^{1/3}} \left(\frac{d}{D}\right)^2 \log(1-\overset{*}{c}),\ Z > \frac{Z_L}{2} \tag{6}$$

At the low speeds met here, speed factor $\psi = 1$ can be assumed. Eq.(6) can be simplified without sacrifice of parameters by replacing (d-D) with d. The equation used for comparison with observed values of θ in TABLE I is then

$$\theta_T = -\frac{Z^2}{ND[\phi Z_L dD]^{1/3}} \left(\frac{d}{D}\right)^2 \log(1-\overset{*}{c}),\ Z > \frac{Z_L}{2} \tag{7}$$

the mixing rate is thus

$$u = \frac{ND[\phi Z_L dD]^{1/3}}{Z^2} \left(\frac{d}{D}\right)^2 \tag{8}$$

MIXING BY JET INJECTION OF RECIRCULATING FLUID

Considering batch-mixing of a fluid volume V_{TOT}, turnover time = ρV_{TOT}/pumping rate. With jet injection, the pumping rate w_o through the jet nozzle is multiplied by the jet entrainment ratio R, so that for a circulating liquid of constant density,

$$\text{turnover time} = \frac{\rho V_{TOT}}{w_o R} \tag{9}$$

With constant and equal density of both injected and infinite bulk fluids,

$$R_{MAX} = x/3d_o,\quad x \gg d_o,\quad \text{for an axisymmetric jet,} \tag{10}$$

$$R_{MAX} = 0.52\,(x/b_o)^{\frac{1}{2}},\quad x \gg b_o \quad \text{for a plane jet} \tag{11}$$

x = distance between nozzle exit and bounding surface along the jet axis (Lehrer, 1981).

It can be argued that eq(9) should allow for the decrease of fluid that has not been mixed, i.e.

$$\text{turnover time} = \frac{V_{INITIAL} - \int_o^\theta w_o\, R\, dt}{w_o R} \tag{12}$$

Equating turnover time to mixing time, with eq (12),

$$\theta = \frac{-\rho V_{TOT}}{2\, w_o\, R_{MAX}} \log(1-\overset{*}{c}) \tag{13}$$

Eq.(13) is used for comparison with observed values in TABLE III.

MIXING BY MOTION OF AIR BUBBLES

Formulation of a direct equation for θ has not yet been achieved.

The power input to the liquid is (Lehrer, 1968)

$$P = w_o\left[(\text{factor})\, \frac{v_o^2}{2} + \hat{R}T \log \frac{p_1}{p_2}\right] \tag{14}$$

the appropriate factor for mixing of the whole liquid here is 0.06.

CONCLUSION

Optimum mixing arrangements depend on circumstances. Two clear limiting criteria are minimum mixing time Θ and minimum energy requirement ($P_{TOTAL}\ \theta/\rho\ V_{TOTAL}$). The data shown in Tables I, II and III together with the given equations can help to find optimal mixing equipment for a given set of criteria. However, when power input is considered, calculations should take into account:

1. With impeller agitation and jet injection of recirculating liquid, the respective terms $P = \phi\rho N^3D^5$ and $w_o v^2/2$ represent the power input to the vessel contents only. Comparisons require the evaluation of P_{TOTAL}, the power that is required to drive the mixing equipment under load, i.e. whilst delivering the required power input to vessel contents.

2. With gas sparging, the factor for the $v_o^2/2$ term in eq.(14) depends on configuration and purpose.
In heat and mass transfer, large interfacial area generated locally may justify taking the factor = 1 when using power input to vessel contents as parameter.
When mixing throughout the vessel contents is considered, the relevant power input may justify factor = 1 for shallow pools of liquid such as vapor/liquid contacting trays. When the distance over which the injected gas slows down to bubble rise velocity is small compared with height Z_L of liquid at rest, the kinetic energy contribution to mixing in the whole of the vessel is small and hence the factor is small. P_{TOTAL} should be evaluated as called for in 1. above.
In Table I, power numbers ϕ have been taken from Uhl and Gray (1966), Perry (1963) and a Chemineer instruction booklet. The values listed in the literature are not always clearly identified with vessel configuration but are satisfactory in the context of this work.
The observed values of θ in the cases shown in Table III, E clearly indicate insufficient flowrate, with unsuitable location of injection opening and/or position of outlet. They also indicate the difficulty of determining a definite end point with the method used, when there is a small change over a long period. Using the criterion $\overset{*}{c} = 0.95$, regarded as a suitable lower limit (Nagata, 1975), equations (7) and (13) provide a satisfactory estimate of mixing time in the observed systems. They do so without the use of empirical correlation factors other than ϕ. Eq.(13) could be modified to incorporate a length term that allows for distance of jet nozzle from furthest liquid boundary. For gas mixing, similar direct formulation of mixing time is still outstanding.

Equations such as (6), (7) and (13), though rational, were formulated with some hindsight. They can be modified to suit a particular set of conditions. At the same time, they exemplify a simple and direct prediction method.

TABLE I COMPARISON OF OBSERVED AND CALCULATED MIXING TIMES

A. Vessel without baffles, dished end, d = 420 mm, Z_L = 462 mm, Z = 262 mm, liquid volume = 0.058 m^3 approx.

D, mm	N, s^{-1}	Re	ϕ	θ, observed, s	$N\bar{\theta}$	θ_T eq. 7 $c^* = 0.95$	$\phi \rho N^3 D^5$, W
Propeller, 3 blades, pitch = diameter, downward discharge.							
76.2	$3.\overline{3}$	19,354	0.29	196, 185	535	151	0.027
	$6.\overline{6}$	38,709	0.27	68, 63	437	77	0.205
	10	58,064	0.26	49, 47	480	52	0.667
101.6	$3.\overline{3}$	34,408	0.27	87, 82	282	59	0.108
	$6.\overline{6}$	68,817	0.26	28, 33	203	30	0.834
	10	103,226	0.25	18, 20	190	20	2.70
45° pitched blade turbine, 6 blades, D/b = 9.4, downward discharge.							
63.5	$3.\overline{3}$	13,441	0.8	142, 138	467	198	0.031
	$6.\overline{6}$	26,882	"	61, 61	407	99	0.245
	10	40,322	"	33, 37	350	66	0.825
101.6	$3.\overline{3}$	34,408	"	35, 35	117	41	0.321
	$6.\overline{6}$	68,817	"	19, 21	133	21	2.566
Straight-bladed turbine, 6 blades, D/b = 8.44							
76.2	6.0	34,839	0.78	51, 51	306	48	0.433
	10.0	58,064	"	16, 23, 18	190	35	2.0
	11.7	67,935	"	17, 14, 14	176	31	3.21
	14.2	82,451	"	14	199	25	5.74
Straight-bladed turbine, 6 blades, D/b = 5							
63.5	$3.\overline{3}$	13,441	1.3	115, 129	407	169	0.050
	$6.\overline{6}$	26,882	1.2	60, 58	393	87	0.367
	10	40,322	1.15	37, 40	387	58	1.19
101.6	$3.\overline{3}$	34,408	1.2	47, 54	168	36	0.48
	$6.\overline{6}$	68,817	1.15	22, 18	133	18	3.69
B. Baffled vessel, dished end, four radial baffles 39 mm wide to within 102 mm of lowest point of vessel, d = 420 mm, Z_L = 458 mm, Z = 258 mm, liquid volume = 0.057 m^3 approx.							
Propeller, 3 blades, pitch = diameter, downward discharge							
76.2	$3.\overline{3}$	19,354	0.41	111, 118	382	131	0.039
	$6.\overline{6}$	38,709	"	46, 49	317	66	0.312
	10	58,064	"	19, 20	195	44	1.053
101.6	$3.\overline{3}$	34,408	"	41, 40	135	57	0.164
	$6.\overline{6}$	68,817	"	16, 11	90	28	1.32
	10	103,226	"	9, 10	95	18	4.44
45° pitched blade turbine, 6 blades, D/b = 9.4, downward discharge							
63.5	$3.\overline{3}$	13,441	1.3	230, 195	708	164	0.049
	$6.\overline{6}$	26,882	"	53, 42	317	82	0.398
	10	40,332	"	33, 34	335	56	1.342
101.6	$3.\overline{3}$	34,408	"	27, 31	97	34	0.521
	$6.\overline{6}$	68,817	"	14, 12	87	17	4.17
	10	103,226	"	10, 10	100	11.	14.1
	$13.\overline{3}$	137,634	"	7, 5.4	83	9	33.4
Straight-bladed turbine, 6 blades, D/b = 8.44							
76.2	21	121,935	2.6	7, 7, 7	147	11	61.9
	14	81,290	"	11, 11, 14	168	18	18.3
	6	34,839	"	26, 26	156	39	1.44
Straight-bladed turbine, 6 blades, D/b = 5							
63.5	$6.\overline{6}$	26,882	4	13, 14	90	56	1.22
	$13.\overline{3}$	53,764	"	9, 11	133	28	9.79
	20	80,645	"	9, 9	180	19	33.0
101.6	$3.\overline{3}$	34,408	"	16, 20	60	23	14.4
	15	154,838	"	7, 8	113	5	146
	20	206,451	"	5, 5	100	4	346

TABLE II. OBSERVED MIXING TIMES USING AIR AGITATION IN A VESSEL WITH DISHED END, d = 458 mm, Z_L = 456 mm, Z = 400 mm, LIQUID VOLUME 0.068 m^3 APPROX.

w_{AIR}, $kgs^{-1}(10^3)$	v_s, at 20°C, 1.013 bar $mm\ s^{-1}$	P, eq, (14) W	θ, s
Sparger: 16 holes, 1.6 mm dia, on 280 mm dia. ring, facing upward.			
0.558	2.71	1.78	20, 22, 21
1.23	5.95	3.93	15, 21, 17
2.80	13.6	9.34	11, 11
3.87	18.8	13.4	13, 13
6.18	30.0	24.1	12, 17, 13
Sparger: 16 holes, 1.6 mm dia, on 280 mm dia. ring, facing downward.			
0.558	2.71	1.78	17, 12, 19, 18
1.23	5.95	3.93	10. 11
2.80	13.6	9.34	7, 6
3.87	18.8	13.4	5, 4
6.18	30.0	24.1	4, 4
Sparger: 8 holes, 1.6 mm dia, on 80 mm dia. ring, facing upward.			
0.222	1.08	0.72	25, 24
0.558	2.71	1.79	17, 17, 14, 16
1.12	5.42	3.66	13, 15, 12, 14
1.50	7.26	4.99	12
2.29	11.1	8.17	28, 29
Sparger: 8 holes, 1.6 mm dia, on 80 mm dia. ring, facing downward.			
0.222	1.08	0.72	20, 21
0.558	2.71	1.79	17, 19
1.12	5.42	3.66	16, 12, 12
2.29	11.1	8.17	8, 7, 8
Sparger: 2 sparge rings above combined, 24 holes facing upward.			
0.781	3.79	2.49	13, 12
1.45	7.04	4.71	10, 8, 9
2.76	13.4	8.96	8, 6, 6
3.52	17.1	11.6	6, 5, 5
5.97	29	20.8	4, 5
Sparger: 2 sparge rings above combined, 24 holes facing downwards.			
0.781	3.79	2.49	16, 12, 13
1.45	7.04	4.71	8, 7, 6
2.20	10.7	7.09	6, 6
3.52	17.1	11.6	4, 5, 5
5.97	29	20.8	4, 4

TABLE III. OBSERVED MIXING TIMES USING JET INJECTION OF RECIRCULATING LIQUID IN VESSEL WITH FLAT FLOOR WITH CENTRAL OUTLET, d = 610 mm

v_o ms^{-1}	wv_o $kg\ ms^{-2}$	θ observed s	θ eq. (13) c^* = 0.95 s
A. Jet discharging diametrally at 45°, 80 mm from vessel floor and 150 mm inward along diameter D_o = 17 mm, Z_L/d = 1.0.			
0.37	0.036	144, 212	232
0.73	0.122	109, 103	117
1.10	0.275	59, 64	78
1.47	0.490	37, 42	58
0.81	0.148	102, 99, 128	106
1.84	0.765	48, 47	47
0.40	0.037	149, 153	214
0.79	0.143	73, 87, 74	109
1.19	0.321	60, 67, 57	72
0.73	0.122	102, 112	118
0.88	0.176	97, 99	98
1.18	0.313	66, 60	73
1.32	0.397	55, 65	65
B. Jet discharging at 80 mm above floor level, parallel to floor, along diameter, Z_L/d = 1.0.			
0.73	0.122	158, 163	118(4/3)*
0.88	0.176	143, 129	98(4/3)
1.18	0.313	91, 88	73(4/3)
1.32	0.397	80, 75	65(4/3)
tangential discharge:			
1.32	0.397	540, 660	
C. As for A. above, but Z_L/d = 1.7			
0.37	0.036	300, 331	394
0.73	0.122	123, 136	199
1.10	0.275	92, 92	132
1.47	0.490	78, 61	99
D. Jet discharging radially outward from a slot, 100 mm above vessel floor, slot dimensions are 53 mm along periphery, 4 mm wide, Z_L/d = 1.0.			
0.37	0.036	429, 454	502
0.73	0.122	138, 150	251
1.10	0.275	94, 95	167
1.47	0.490	99, 90, 84	126
E. As for D. above, but Z_L/d = 1.7			
0.37	0.036	299, 1168,	854
0.73	0.122	593, 689	426
1.10	0.275	374, 315	284
1.47	0.490	429, 240	213

*(4/3): based on subtended angle.

REFERENCES

BRENNAN, D.J. and LEHRER, I.H. Impeller Mixing in Vessels Experimental Studies on the Influence of some Parameters and Formulation of a General Mixing Time Equation. Trans. Instn. Chem. Engrs., 54, pp 139-152.

CHEMINEER, INC. Power data attached to specifications and instructions for ELB experimental agitator kit.

LEHRER, I.H., Gas Agitation of Liquids, I & EC Process Design and Development, 7, pp. 226-239.

LEHRER, I.H., A New Model for Free Turbulent Jets of Miscible Fluids of Different Density and Jet Mixing Time Criterion, Trans. Instn. Chem. Engrs., 59, pp. 247-252.

NAGATA, S., 1975, Mixing, Principles and Applications, Kodansha Ltd. Tokyo, Halsted Press (A Division of John Wiley, New York).

PERRY, J.H., ed., (1963), Chemical Engineers' Handbook, 4th ed., McGraw-Hill, New York.

UHL, V.W., and GRAY, J.B., (1966), Mixing, Theory and Practice, Vol. I, Academic Press.

ZLOKARNIK, M., Eignung von Rührern zum Homogenisieren von Flüssigkeitsgemischen (Suitability of Stirrers for Homogenisation of Mixtures of Liquids), Chemie-Ing. Techn., 39, pp. 539-548.

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