

GAS HOLDUP IN SIMPLE MULTIPHASE PROCESS VESSELS

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SUMMARY In the design and performance prediction of process vessels, fractional gas content, called gas holdup, is often an item of interest. Prediction of this gas holdup is difficult because many factors have to be considered. In this report, a non-correlative method for prediction of gas holdup in twophase systems is proposed. Agreement with observed and reported results is satisfactory.

NOTATION

C_D = drag coefficient
 D = characteristic bubble diameter L
 d = vessel diameter L
 D_o = diameter of particle at formation L
 D_E = diameter of maximum stable particle L
 E = power input per unit mass of fluid contents $L^2 t^{-3}$
 $Eö$ = Eötvös number = $\Delta\rho g D^2 / \sigma$
 e = volume fraction of dispersed phase
 g = gravity acceleration $L t^{-2}$
 N = number of bubble formation sites or bubbles
 Q = volumetric gas flowrate $L^3 t^{-1}$
 Re = particle Reynolds number
 Re_o = Reynolds number of gas flow at entry into liquid
 S = vessel cross-section normal to mean flow of dispersed phase L^2
 t_r = residence time of gas in vessel contents t
 U = terminal velocity of single particle in infinite uncontaminated fluid $L t^{-1}$
 V = volume of fluid L^3
 v_s = superficial velocity = Q/S $L t^{-1}$
 v_t = terminal velocity of dispersed phase $L t^{-1}$
 z = height of fluid L
 Δ = difference
 μ = viscosity $M L^{-1} t^{-1}$
 ρ = density $M L^{-3}$
 σ = interfacial tension $M t^{-2}$

Subscripts :

c = continuous phase
d = dispersed phase
o = at gas entry orifice

INTRODUCTION

Gas holdup depends on flowrates, component properties and fluid motion. Despite an extensive literature that proposes equations based on correlated experimental data, valid prediction of gas holdup in process systems of industrial size is still difficult (Hikita et al, 1980).

DISCUSSION

The fractional gas content e , here simply called hold-

up, is defined by

$$e = \frac{V_{TOT} - V_c}{V_{TOT}} \quad (1)$$

where V_{TOT} = total volume of fluid

V_c = volume of continuous phase

In dispersions of gas in liquid, which includes boiling at submerged surfaces, there are not only bubbles, but often an undesired cellular foam layer forms on top of the liquid, at times intruding into the next process stage. Cellular foams are not considered to be part of the total fluid volume V_{TOT} , or V_d , the volume of dispersed phase within the total volume of fluid.

In a vessel of uniform horizontal cross-section S , containing liquid or a quasi-continuous phase at rest at uniform height z_c , gas, say, is admitted at floor level; the residence time t_r of the gas in the liquid is then

$$t_r = \frac{z_c}{v_t} = \frac{e V_{TOT}}{v_s S} = \frac{e z_{TOT}}{v_s} \quad (2)$$

where v_t = representative terminal velocity of the gas with respect to the continuous phase.

v_s = superficial velocity of the gas

S = volumetric gas flowrate/S

z_T = total height of fluid

z_d = notional clear height of dispersed phase

$$z_{TOT} = z_c + z_d \quad (3)$$

Substituting in eq(2) and using eq(3)

$$e = \frac{z_d}{z_c + z_d} = \frac{v_s}{v_s + v_t} \quad (4)$$

Appropriate values of v_t have to be specified. This is difficult, as is evident from the apparent absence of any published quantitative criterion for transition between laminar and turbulent conditions in bubble beds.

Even the motion of single bubbles in infinite liquid is complex, part of the relevant literature is cited in reviews and references eg. Brodkey (1967), Wallis (1969) and Clift et al (1978).

In addition, it is required to distinguish between systems in which there is mainly

- (a) bubble coalescence only,
- (b) bubble coalescence and breakup,
- (c) maintenance of approximately even bubble size downstream of the formation region.

It has been observed that the terminal velocities of single particles in bounded fluids are smaller than the

terminal velocities of the same particles in fluid volumes that can be regarded as effectively "infinite". Velocity correction factors for various occasions were listed by Wallis (1969). An additional factor based on analogy with velocity of surface waves was proposed by Maneri and Mendelson (1968); the factor is of the type :

$$\left[\tanh\left(\frac{\text{vessel perimeter}}{\text{bubble perimeter}}\right) \right]^{1/2}$$

In evaluation of gas holdup, it is not the velocity of individual bubbles that is of interest, but the mean velocity of the particle bed. However, limiting relations between terminal velocity of a single particle and gas holdup are :

$$v_t = U \text{ when } e = 0, \quad (5)$$

U = terminal velocity of a single bubble in infinite fluid.

In bubble dispersions, as distinct from slug-to-annular flow and beyond, and also distinct from cellular foam,

$$v_t > v_s \quad (6)$$

so that from eqs (4) and (6),

$$e_{\text{MAX, BUBBLE DISPERSION}} = 0.5 \quad (7)$$

Eqs (5), (6), (7) are satisfied by

$$v_t = (U - v_s) \left[\tanh\left(\frac{d}{D\sqrt{N}}\right) \right]^{1/2} + v_s \quad (8)$$

Eq (4) is then written

$$e = \frac{v_s}{2v_s + (U - v_s) \left[\tanh\left(\frac{d}{D\sqrt{N}}\right) \right]^{1/2}}, \quad (9)$$

$U > v_s$

The appropriate terminal velocity U is to be estimated.

Definition of the terminal velocity U of a single bubble in infinite liquid is based on a steady-state force balance involving gravity-buoyancy and drag forces, i.e.

$$\frac{\pi D^3 \Delta \rho g}{6} = \frac{C_D \pi D^2 \rho_c U^2}{4(2)} \quad (10)$$

where $\Delta \rho = |\rho_c - \rho_d|$

$$U^2 = \frac{4 \Delta \rho g D}{3 \rho_c C_D} \quad (11)$$

Hence the appropriate values of D and C_D must be found.

To begin, uncontaminated fluids are assumed. Then in motion of single bubbles (and drops) at terminal velocity in infinite liquid, there are four regimes of motion. These are, in order of increasing particle Reynolds number Re :

(a) viscosity-dependent without internal circulation,

$$C_D = 24/\text{Re} = 24 \mu_c / \text{UD} \rho_c \quad (12)$$

(b) viscosity-dependent with internal circulation,

$$C_D = \frac{24}{\text{Re}} \frac{3\mu_d + 2\mu_c}{3\mu_d + 3\mu_c} \quad (13)$$

(Hadamard's and Rybczynsky's equation (1911), cited by Wallis, 1969, arranged explicitly for C_D), increasing to (Lehrer, 1980)

$$C_D = \frac{48(\mu_c + \mu_d)}{\rho_c \text{UD}} \quad (14)$$

(c) viscosity-independent, in clean fluids velocity depending also on interfacial tension σ

$$C_D = \frac{8}{3} \frac{E\ddot{o}}{(E\ddot{o}+6)}, \quad E\ddot{o} = \frac{\Delta \rho g D^2}{\sigma} \quad (15)$$

(Lehrer, 1976)

(d) velocity depends on particle size only

$$C_D = \frac{8}{3} \text{ or } \frac{8}{3} \frac{E\ddot{o}}{(E\ddot{o}+6)} \quad (16)$$

The particle size at transition from viscosity-dependent to viscosity independent regime is found by equating the velocities resulting from use of (Lehrer, 1980)

$$C_D = \frac{48(\mu_c + \mu_d)}{\rho_c \text{UD}} = \frac{8}{3} \frac{E\ddot{o}}{(E\ddot{o}+6)} \quad (17)$$

When dealing with gas dispersions in uncontaminated liquids, viscosity μ_d is insignificant at ordinary temperatures. Using eq (17) with omission of μ_d , the bubble size D_{trans} at which transition from viscosity-dependent to viscosity-independent bubble motion occurs is defined by

$$D_{\text{trans, minimum}} = \left| \frac{3,888\sigma}{\rho_c} \left(\frac{\mu_c}{\Delta \rho g} \right)^{2/5} \right|^{1/5} \quad (18)$$

$$D_{\text{trans, maximum}} = 2^{1/5} D_{\text{trans, minimum}} \quad (19)$$

Usually, the values are close enough to use $D_{\text{trans, minimum}}$. Otherwise, a better value can be found by iteration.

The drag coefficient given by eq (15), i.e. $C_D = 8 E\ddot{o} / [3(E\ddot{o}+6)]$ can be used for regimes (c) and (d).

When $E\ddot{o} > 6$, the transition between viscosity-dependent and viscosity-independent motion becomes indistinct, the same applies when the liquid is contaminated, i.e. contains additives. In both cases the C_D versus Re curve in regimes (b) and (c) moves up towards a line $C_D = (24/\text{Re}) + (8/3)$.

Equations (12) to (15) can be used in eq (11) to find U when D is found.

The characteristic particle diameter D depends on flow-rates, fluid properties and equipment geometry. Wallis (1969) cites a number of equations that pertain to bubble formation in a viscous liquid. These would apply if there is no coalescence. For constant flow of gas into inviscid liquid at say $\text{Re}_o > 200$, with a bubble chain model (Lehrer, 1971).

$$D_o = \left(\frac{6}{\pi} \frac{Q}{N} \right)^{2/5} \left(\frac{3C_D}{4g} \right)^{1/5} \quad (20)$$

where Q = volumetric gas flowrate
N = number of bubbling sites.

At high flowrates, $C_D \rightarrow 8/3$,

$$D_o = \frac{1.488}{g^{1/5}} \left(\frac{Q}{N} \right)^{2/5} \quad (21)$$

This value is similar to the one resulting from the Davidson and Schüler model (1960) for the same conditions.

With closely spaced gas inlets there may be coalescence near the gas inlet, so that the number of effective bubble generation sites $N_{\text{effective}}$ is less than N (Lehrer, 1971).

Another characteristic size is D_D , the maximum stable size in isotropic turbulence. An estimate of this is based on balancing surface tension force against drag force, yielding (Lehrer, 1968),

$$D_E = \left(\frac{8\sigma}{D\rho_c}\right)^{0.6} E^{-0.4} \quad (22)$$

where E = power input/mass of fluid. Alternatively,

$$\frac{4\sigma}{D} = \frac{\rho_c v^2}{2} \text{ yields } D_E = \left(\frac{8\sigma}{\rho_c}\right)^{0.6} E^{-0.4} \quad (23)$$

The choice of characteristic particle size requires judgment.

For a system containing coalescing gas bubbles, the characteristic size may be

$$D = (D_{O,N} D_{O,N=1})^{1/2} \quad (24)$$

$$\text{or } D = (D_{O,N \text{ effective}} D_{O,N=1})^{1/2} \quad (24a)$$

When there is rapid coalescence to a single bubble,

$$D = D_{O,N=1} \quad (25)$$

In pure liquids, when there are cycles of coalescence and breakup,

$$D = (D_{O,N} D_E)^{1/2}, \quad D_E \text{ from eq(22)} \quad (26)$$

$$\text{or } D = (D_{O,N \text{ effective}} D_E)^{1/2} \quad (26a)$$

This would apply to "churn-turbulent" regimes.

In non-coalescing systems,

$$D = D_E, \quad D_E \text{ from eq(22)} \quad (27)$$

or

$$D = D_O,$$

calculated by one of the relevant bubble formation equations found in the literature. The chosen value of D is then used in eq(11), hence U can be calculated, and with this, the holdup e can be found by eq(9).

If there is a choice of D , it is probably best to use that which yields the lower value of e , based on the argument that a system will tend towards a state of minimum potential energy.

However, if a higher value of e is concomitant with a less favourable condition, it is advisable to be pessimistic and use this higher value of e in evaluations. Example 1. shows possible differences.

Ex. 1. A vessel operates at $v_g = 0.10 \text{ ms}^{-1}$. Power input is such that $D_E = 6 \text{ mm}$, (eq(22)). Gas is sparged into clean water through 16 holes, 20 mm apart on a tube. $D_O = 50 \text{ mm}$, calculated by eq(20) with $N = 16$. With this, $D = [(50)6]^{1/2} = 17.3 \text{ mm}$ eq(26), eq(15) is appropriate to find C_D , which is 2.32. Say $d = 0.50 \text{ m}$. Hence from eq(11), $U = 0.312 \text{ ms}^{-1}$ from eq(9), $e \approx 0.24$.

Bubble, coalescence is probable when plan area per bubble is less than $2\pi D_O^2/4$. A criterion is then

$$N_{\text{effective}} = \frac{N(\text{orifice pitch})^2}{2 D_O^2} \quad (28)$$

for orifice pitch $< D_O/\sqrt{2}$

For holes in a single tube, pitch $< D_O/\sqrt{2}$

$$N_{\text{effective}} = N(\text{orifice pitch})/D_O\sqrt{2}.$$

Using this in the example,

$$N_{\text{effective}} = 16(20)/50\sqrt{2} = 4.525.$$

For D_O , eq(20) applies, the value of D_O for the coalesced bubble is

$$50(16/4.525)^{0.4} = 82.86 \text{ mm}$$

$$D = [(82.86)6]^{1/2} = 22.3 \text{ mm}$$

$$U = 0.345 \text{ ms}^{-1}, e \approx 0.23$$

If the 16 holes were arranged at the same pitch on, say, a shower rose,

$$N_{\text{effective}} = \frac{16(20^2)}{2(2500)} = 1.28$$

D_O for the coalesced bubble is

$$50(16/1.28)^{0.4} = 137.3 \text{ mm},$$

$$D = [(137.3)6]^{1/2} = 28.7 \text{ mm},$$

$$U = 0.385 \text{ ms}^{-1}, e \approx 0.21$$

In the presence of coalescence inhibitors, there will be no coalescence after bubble fracture, and use of $D = D_E$ seems suitable for finding e_{MAXIMUM} .

Using the other data in Example 1,

$$C_D = 1.19, \quad U = 0.257 \text{ ms}^{-1},$$

$e_{\text{max}} \approx 0.32$. It may also be argued that when there is no coalescence, bubbles of diameter D_O are still formed at all 16 sparger holes, but break up without re-coalescence. Thus in presence of coalescence inhibitors in this system,

$$0.28 < e < 0.39$$

The method shown here for estimating gas holdup in liquids could be applied to idealised fluidised beds. It is difficult to compare reported experimental results with prediction because there is not only a multitude of systems, but also of parameters (e.g. Miyauchi et al 1981).

It can be said that in bubbling fluidised beds, small bubbles tend to disappear, so that only the motion of large bubbles needs to be considered. Therefore, the relevant quantities are, with D_O from eq (21).

$$U = (gD/2)^{0.5} \quad (29)$$

$$D_{\text{MINIMUM}} = (D_{O,N \text{ effective}} D_{O,N=1})^{1/2} \quad (30)$$

$$D_{\text{MAXIMUM}} = D_{O,N=1} \quad (30a)$$

CONCLUSION

The method shown here for estimation of gas holdup in the bubble regime of two-phase mixtures in plain vessels is based on the physical phenomena, in contrast to predictions based on correlation. Agreement with observed results is satisfactory.

The velocity correction involving tanh could be replaced by other suitable asymptotic functions.

It is evident that at moderate gas flowrates in vessels of large cross-section normal to mean gas flow, the use of $v_t = U$ in eq.(4) is justifiable for reasons of simplicity, but inclusion of the velocity correction factor is advisable in first estimates.

A modification of eq(8) to allow description of the whole two-phase flow spectrum to $e = 1$ would be a desirable extension.

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